

A Sub-pixel Registration Approach Based on Powell Algorithm

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Abstract: A novel motion parameters estimation approach for sub-pixel accuracy is presented which is based on Powell optimization and the pyramid decomposition method of the images. The non-linear least square formula for the difference of reference and sensed images is seen as the object function for Powell algorithm in which motion parameters are used as variations. Based on multi-resolution decomposition from fine to coarse representation of images, sub-pixel parameters can be accurately estimated. Numerical experiment results indicate that the proposed algorithm can be used to perform accuracy and robust sub-pixel parameters estimation. The drawback of this algorithm is the longer waiting time compared with that of other algorithms.

Key words: Image registration; Powell algorithm; Gaussian Pyramid; Super-resolution reconstruction; Image fusion

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0 Introduction

In image processing fields, image registration is one of the most important research directions. In GIS, registration is used to align different digitalized maps about the same scene. In super-resolution reconstruction, the image sequences about one object taken from different viewpoint or different sensors should be registered first. In medicine image processing, computer tomography (CT) and NMR data should be fused to obtain more complete information about the patient, monitoring tumor growth, treatment verification, comparison of the patient's data with anatomical atlases^[1]. Image registration is used to match two or more images that show the same scene from different viewpoints, from different sensors, or at different time^[2]. So, it is very important to provide accuracy registration parameters. During the past twenty years, more attentions have been paid on this topic and many effectively algorithms have been proposed. More details about the registration algorithms can be refers to literatures^[1-3].

Recently there has been a lot of interest in image registration technique. The proposed algorithms mainly classified into two categories in terms of the domain of the transformation: spatial domain algorithm and frequency domain based algorithm. More attentions have been paid on the

frequency domain based motion parameters estimation problem for the reason that fast Fourier transform provide a fast calculation and effective solutions. Phase correlation approach is the mainly motion estimation methods in frequency domain which is used in many application fields such as image fusion and image super-resolution reconstruction. Another method used in frequency domain is cross-correlation method which can also provide effective estimation of registration parameters.

Usually, the observed images are contaminated by some kind's noise, for example the random Gaussian noise and Poisson noise. In many cases, the images are blurred by the point spread function of the optical system and the relative motion between the sensors and the objects. So, the motion parameter estimation methods should be robust to the noise and the blurring in the degraded images. In many literatures, these two issues have not been discussed in depth. The key of our paper is given a new registration parameters estimation method which is robust to the Gaussian noise and the point spread function of the optical system.

In this paper, we present a new registration algorithm based on the Powell algorithm and Image's Gaussian pyramid decomposition. This new algorithm can provide very accuracy sub-pixel motion estimation of the translation parameters and the rotate parameter. We assume that there are global motion between the reference image and the float image.

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1 Sub-pixels motion parameters estimation

Image registration is an essential component in digital image processing fields such as GIS, remote sensing image applications [3-6]. The precision of registration is an important factor when we select one method to register the different images. In some cases, the control points are selected by the user to give the data about the transformation. However, this method is limited for the reason that the accuracy of registration has been descended. It is difficult to select the matching points in real applications. Then, automatic registration algorithm can be used to estimate the motion parameters and complete the task of image registration using the computer.

Some registration algorithms can provide integer pixel motion parameters estimation, but fail in the estimation of sub-pixel motion. In some applications fields, the sub-pixel parameters estimation is necessary such as image sequence super-resolution (SR) reconstruction and video compression. SR algorithms make use of the different details recorded in multiple images and combining them, generate a single higher quality image. Presence of sub-pixel level translations and their accurate estimation are essential since overlapping regions of images with translations measured in integer numbers basically have the same details.

There are some ways used for sub-pixel motion estimation. The mutual information (MI) is one of the sub-pixel registration algorithms which are used mostly in multimodal registration [7]. The MI, originating from the information theory, is a measure of statistical dependency between two data sets and it is particularly suitable for registration of images from different modalities [1]. Some modified algorithms based on the MI theory have been given in many literatures. However, for the reason that MI is based on the statistic information, when there are noise contained in the images the histogram would be influenced severely, then the MI of two images would not be reflect the accuracy cross entropy. The robust of the MI algorithms should be investigated further. On the other hand, the blurring phenomena maybe influence the calculation of entropy of images. Some effective robust to noise registration algorithm based on the MI methods should be given more attention in the future.

Correlation methods are mostly used registration ways in many cases which based on the shift property of Fourier transform. Many new algorithms in this field have been proposed [5-8]. The phase correlation surface based on the Fourier transform was proposed, and then the parabolic fitting was used to model this surface. The parabolic function and Gaussian function were used to fit the correlation surface. In [9], the phase correlation matrix was considered as a 2-dimensional signal which leads to an over-determined system of linear equations, and then accurate results can be obtained for sub-pixel registration in the Fourier domain. The singular value decomposition (SVD) of the phase correlation was used to estimate the sub-pixel translation independently along each dimension. The extension of this SVD method in multi-dimensional data was given. In [10], the linear approximation of point spread function was considered and then multi-resolution analysis was used to estimation the sub-pixel translation.

2 Proposed algorithm

In order to give an accuracy depict of image translation and rotate process, the shift values along x-direction and y-direction are denoted as Δx and Δy . The reference image and float image are denoted as $I_1(x, y)$ and $I_2(x, y)$. Let θ be the rotate angle between two images. An example of image transformation is given in Fig. 1.

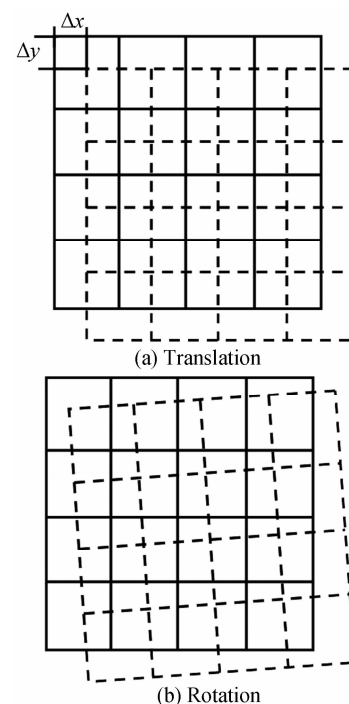


Fig. 1 Example of a sub-pixel translation and rotation

The transformation T can be formulated as an affine transform as following. Then image $I_2(x, y)$ can be

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} \Delta x \\ \Delta y \end{pmatrix} \quad (1)$$

written as the transformation of image $I_1(x, y)$.

$$I_2(x, y) = I_1[T(x, y)] \quad (2)$$

where operator T denotes the affine transform in eq. (1). To estimate the parameters $d = (\Delta x, \Delta y, \theta)^T$, we can try to find the following minimization problem

$$J(d) = \sum_{x,y} \{I_2(x, y) - I_1[T(x, y)]\}^2 \quad (3)$$

Usually, it is hard to solve this least square problem. In [9], the authors presented the so called Lucas algorithm to estimate the registration parameters. However, the disadvantage of this algorithm is that it is difficult to compute the Hessian matrix during the iteration steps. In [10], a modified algorithm based on the Lucas's algorithm was proposed in which the Hessian matrix is unnecessary to compute. The compute time has reduced compared with that of Lucas algorithm.

Object function $J(d)$ is quadric function of parameters vector d , but it is hard to give an obvious expression for variants. Therefore, we use the Powell search algorithm to find the parameters vector d by minimizing the object function. In Powell search algorithm, it is unnecessary to deduce the derivation of the object function. Now, we give a review for Powell direct search algorithm for the following positive definite quadric function.

$$f(x) = \frac{1}{2}x^T Ax + b^T x + c \quad (4)$$

Powell algorithm is a direct search method which does not use the gradient calculation of the object function. During the iteration process, the conjugate basis functions are constructed. Then the minimization of the object function is investigated using the basis functions. The algorithm is given as:

- 1) Initialize: let $k=0$ denote the iteration number, and $\xi_i = e_i$ denote the i -th unit vector, ($i=1, \dots, n$), x_0 denotes a initial solution of Eq. (4);
- 2) For $i=1, \dots, n$, compute

$$x_i = x_{i-1} + \alpha_i \xi_i, \text{ and } \alpha_i = \arg \min_a f(x_{i-1} + \alpha \xi_i),$$
- 3) Let $\xi = x_n - x_0$, compute

$$x_{n+1} = x_n + \alpha_{n+1} \xi, \text{ where } \alpha_{n+1} = \arg \min_a f(x_n + \alpha \xi);$$
- 4) If $k = n - 1$, stop, print $x^* = x_{n+1}$; else, turn to 5);
- 5) Let $\xi_i = \xi_{i+1}$, $\xi_n = \xi$, $x_0 = x_{n+1}$, turn to 3).

To accelerate the calculation and obtain an accuracy estimation of registration parameters, we use the multi-scale Gaussian pyramid decomposition approach to get the multi-level expression of the images. The l level Gaussian pyramid can be shown as following in Fig. 2.

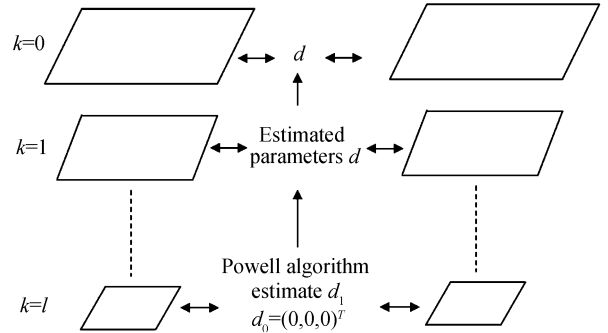


Fig. 2 Multi-level gaussian pyramid decomposition and parameter estimation

3 Numerical experiments

In this section, some numerical experiments are given to demonstrate the performance of this proposed registration algorithm. In the first experiment, the reference images are shifted and rotated artificially to get the object images. The image 'cameraman' in Matlab platform and a remote sensing SPOT image are used as the reference images. Firstly, these two images are shifted along horizontal and vertical directions, and then they are rotated by a small angle. The original images and the translated images are shown in Fig. 3. To evaluate the performance of the proposed algorithm, the frequency domain method proposed by Vandewalle^[10], Lucchese's method, Keren's method.



Fig. 3 Original images and transformed images

In Fig. 1, the original ‘Cameraman’, ‘SPOT’ images and the distorted images are shown. They are all shifted by sub-pixel parameters and then rotated by a little angle. From Tab. 1, we can see that our proposed algorithm could estimate the shift and rotation parameters accurately. Keren’s

method can also provide an approximate solution compared with other algorithms. We can obtain parameters from frequency domain method, but the results are not stable. The worst one is Lucchese’s method which provides bigger solution compared the true parameter values.

Tab. 1 Registration parameters estimated using different algorithms

	$(\Delta x, \Delta y), \theta$	Frequency	Lucchese	Keren	Proposed
Cameraman	(0.921 8, 0.405 7)	(0.888 3, 0.768 5)	(2.0, 1.25)	(0.988, 0.539 8)	(0.921 8, 0.405 7)
	0.410 3	0	1.949 0	0.342 6	0.410 3
	(0.738 2, 0.935 5)	(1.370 3, 1.552 9)	(3.25, 2.25)	(0.824, 1.127 1)	(0.738 2, 0.935 5)
	0.893 6	0	2.950 4	0.770 7	0.893 7
SPOT	(0.176 3, 0.916 9)	(0.151 2, 1.304 4)	(0, 1.0)	(0.18, 1.046 7)	(0.176 3, 0.916 9)
	0.057 9	-0.1	0.523 4	0.049 4	0.057 9
	(0.352 9, 0.138 9)	(21.2, 8.43)	(-91.2, 0.5)	(0.397 9, 0.112 0)	(0.352 9, 0.138 9)
	0.603 8	30	67.45	0.496 3	0.603 8
SPOT	(0.813 2, 0.202 8)	(1.50, 0.182 9)	(3.25, 0)	(0.808 9, 0.189)	(0.813 2, 0.202 8)
	0.272 2	0	0.758 5	0.223 0	0.272 2
	(0.009 9, 0.198 7)	(0.014 8, 0.824 5)	(0, -4.25)	(0.039 9, 0.191 2)	(0.009 9, 0.198 7)
	0.198 8	0	-0.671 8	0.158 4	0.198 8

Usually, the observed images are degraded by random noise. We want to know whether or not the registration parameters affected by the noise. We select cameraman image as the example only. Firstly, Gaussian zero mean noise is added into original image to model the observed image. The standard variation of noise is 0.01. Then, the image is shifted along x -dimensional and y -dimensional and rotated using random number generator. The images are shown in Fig. 4. The size of these images is 256×256 . From Fig. 4, we can see that the images have been degraded

severely by the random noise. The registration parameters estimated using different algorithms are shown in Tab. 2.

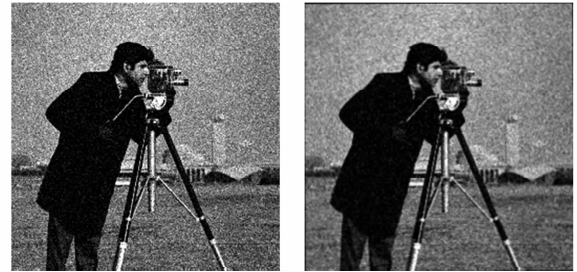


Fig. 4 Noise image and Transformed Image

Tab. 2 Estimated results when random noise contained in the images

	$(\Delta x, \Delta y), \theta$	Frequency	Lucchese	Keren	Proposed
Cameraman	(0.950 1, 0.486 0)	(0.971 2, 0.882 3)	(3.25, 0)	(1.006, 0.598 9)	(0.950 1, 0.486 0)
	0.456 5	0	2.242 1	0.371 5	0.456 5
	(0.231 1, 0.891 3)	(0.161 4, 1.266 3)	(0, 1.0)	(0.232, 0.996)	(0.231 1, 0.891 3)
	0.018 5	-0.1	0.408 5	0.013 8	0.018 5
Cameraman	(0.606 8, 0.762 1)	(1.167 7, 1.386 2)	(3.25, 0)	(0.694, 0.916 3)	(0.606 8, 0.762 1)
	0.821 4	0	3.006 9	0.665 6	0.821 4

From Tab. 2, we can see that the estimated results using our proposed algorithm are consistent with the true parameters, i. e. the random noise does not affect the estimated results. Because of the influence of the random noise, the frequency domain method and Lucchese method can not provide accuracy results. Keren’s method can give more accurate estimation results than frequency method and Lucchese method. This experiment demonstrate the robust of our proposed algorithm

for the noise degraded image case.

In the third experiment, the original cameraman image is blurred using point spread function (psf) of the optical system. We use Gaussian psf with standard deviation 0.5 and size 5×5 . Then, the image is shifted and rotated using random numbers. The size of these images is 128×128 . The original image and the observed image are shown in Fig. 5.



Fig. 5 Original image and blurred and translated image

Tab. 3 Estimated results from blurred and translated image

	$(\Delta x, \Delta y), \theta$	Frequency	Lucchese	Keren	Proposed
Cameraman	(0.444 7, 0.921 8)	(0.328 8, 1.435 5)	(-1.25, 1.0)	(0.476, 1.186 7)	(0.457 7, 1.080 4)
	0.405 7	-0.1	3.366 2	0.316 9	0.457 5
	(0.615 4, 0.738 2)	(0.712 6, 1.288 3)	(0,0)	(0.673 1, 1.01 9)	(0.603 2, 0.718 6)
	0.935 5	-0.1	4.795 1	0.769 6	0.936 5
	(0.791 9, 0.176 3)	(0.778 9, 0.639 5)	(-0.25, 1.25)	(0.875, 0.396)	(0.742 9, 0.718 6)
	0.916 9	-0.1	3.766 8	0.667 6	0.923 5

using the proposed method and the original true parameters is 0.1084. The MSE of Frequency method, Lucchese method and Keren method are 1.0554, 12.6077 and 0.1020, respectively.

Although our algorithm can provide accuracy estimation of registration parameters, however, the time consumed is the biggest compared with that of other algorithms. In Tab. 4, the time used is listed in the first experiment. Further research should be done to reduce the computing time in this algorithm.

Tab. 4 Times used to estimate the registration parameters

	Frequency	Lucchese	Keren	Proposed
Time	2.113 s	9.193 s	3.084 s	107.305 s

4 Conclusions and remarks

Image registration is a very important step in some applications, such as image fusion in remote sensing image processing and super-resolution reconstruction of image sequences. Accuracy estimation of translation and rotate parameters should be given. Here we present a new algorithm to estimate the registration parameters based on the Powell optimal algorithm and image's Gaussian pyramid decomposition. Numerical experiments results have shown that this new algorithm can provide accuracy estimation of the registration parameters. The main disadvantage of our algorithm is the tedious waiting time which maybe constrains its application in real worlds. In future,

During the process of blurring, the pixel value is the weighted average of its neighbor pixels (in Tab. 3). In this case, the registration parameters estimated algorithms have been affected. But, the results of our proposed algorithm are most clear to the original parameters, and another one is Keren's method. The mean-square-error (MSE) of between the estimated registration parameters

we would do some research in this topic in-depth to increase the calculation time.

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基于 Powell 算法的亚像元配准方法

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摘要: 基于 Powell 优化算法和图像的金字塔分解, 提出了一个新的亚像元运动参数估计方法. 用参考图像和待配准图像构造的非线性最小二乘公式作为 Powell 算法的目标函数, 并且以运动参数作为变量. 由细到粗的多分辨率分解为准确的亚像元参数估计提供了基础. 数值实验结果表明, 该算法可以用于准确的稳健的亚像元参数估计, 其不足之处在于运行时间比较长.

关键词: 图像配准; Powell 算法; Gaussian 金字塔; 超分辨率重建; 图像融合



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