A Controlled Quantum Teleportation Scheme of a Three-particle Unknown State in General Form*

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Abstract: A controlled teleportation scheme of a three-particle arbitrary state is proposed, in which a seven-particle entangled state is used as quantum channel. After projective measurement, the sender announces measurement results. Under the control of the controller, the receiver performs unitary operations on particles to reconstruct the original state. Using the controlled teleportation scheme, some controlled quantum communication protocols can be realized.

Key words: Quantum information, Controlled teleportation, Seven-particle entangled state

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0 Introduction

Quantum teleportation is a charming process of quantum information, which can make an unknown quantum state from one place to another without propagation of the physical object. In 1993, Bennett et al[1] first shown that a single particle unknown state can be teleported via an Einstein-Podolsky-Rosen (EPR) pair. Since then, quantum teleportation[2-14] is always an interesting topic due to its important applications in quantum computation and quantum communication. Recently, controlled quantum teleportation was studied by some researchers[15-25], in which the controllers are included and teleportation can be completed with the controllers′ agreement and cooperation.

Borras et al[26] found a kind of six-particle maximally entangled state (named as Borras state) using the numerical search procedure. It is not equal to six-particle GHZ state under stochastic local operations and classical communication. And it can be used to teleport an arbitrary three-particle state[27].

We can obtain a seven-particle entangled state with Borras state. Using it, we propose a controlled teleportation scheme of three-particle state in general form.

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1 Construction of the seven-particle entangled state

If the supervisor (Charlie) has a resource of Borras states

$$|\chi^{000}\rangle=\frac{1}{2\sqrt{2}}\sum_{\lambda=1}^{8}|\phi_\lambda\rangle_{A_1,A_2,A_3}|\mathrm{GHZ}_\lambda\rangle_{B_1,B_2,B_3} \quad (1)$$

where $|\phi_\lambda\rangle_{A_1,A_2,A_3}$ is one of three-particle product states $\{|000\rangle, |001\rangle, |010\rangle, |011\rangle, |100\rangle, |101\rangle, |110\rangle, |111\rangle\}$ and the corresponding state $|\mathrm{GHZ}_\lambda\rangle_{B_1,B_2,B_3}$, is one of three-particle GHZ states which is complete orthogonal state in three-dimensional Hilbert Space

$$\begin{aligned}
|\mathrm{GHZ}_1\rangle&=\frac{1}{\sqrt{2}}(|000\rangle+|111\rangle)_{B_1,B_2,B_3}\\
|\mathrm{GHZ}_2\rangle&=\frac{1}{\sqrt{2}}(|110\rangle-|001\rangle)_{B_1,B_2,B_3}\\
|\mathrm{GHZ}_3\rangle&=\frac{1}{\sqrt{2}}(|100\rangle-|011\rangle)_{B_1,B_2,B_3}\\
|\mathrm{GHZ}_4\rangle&=\frac{1}{\sqrt{2}}(|010\rangle+|101\rangle)_{B_1,B_2,B_3}\\
|\mathrm{GHZ}_5\rangle&=\frac{1}{\sqrt{2}}(|101\rangle-|010\rangle)_{B_1,B_2,B_3}\\
|\mathrm{GHZ}_6\rangle&=\frac{1}{\sqrt{2}}(-|100\rangle-|011\rangle)_{B_1,B_2,B_3}\\
|\mathrm{GHZ}_7\rangle&=\frac{1}{\sqrt{2}}(|110\rangle+|001\rangle)_{B_1,B_2,B_3}\\
|\mathrm{GHZ}_8\rangle&=\frac{1}{\sqrt{2}}(|000\rangle-|111\rangle)_{B_1,B_2,B_3}
\end{aligned} \quad (2)$$

Then Charlie introduces auxiliary particle C in state $\frac{1}{\sqrt{4}}(|0\rangle+|1\rangle)$ and makes a CNOT gate operation on particle C and one of the particle from the entangled states with particle C acts as the controlled particle and the selected particle acts as the target particle. Without loss of generality, we suppose particle A_1 as the target particle. As a result, Charlie can obtain the following seven-particle entangled state

$$|\varphi_7\rangle_{A_1,A_2,A_3,B_1,B_2,B_3,C}=$$

$$\frac{1}{8\sqrt{2}}\Big[\sum_{\lambda=1}^{8}|\phi_\lambda\rangle_{A_1,A_2,A_3}|\mathrm{GHZ}_\lambda\rangle_{B_1,B_2,B_3}|0\rangle_C+ \qquad (3)$$

$$\sum_{\lambda=1}^{8}|\phi_{(\lambda+4)(\mathrm{mod}8)}\rangle_{A_1,A_2,A_3}|\mathrm{GHZ}_\lambda\rangle_{B_1,B_2,B_3}|1\rangle_C\Big]$$

Where $(\lambda+4)(\mathrm{mod}8)=\begin{cases}\lambda+4 & \lambda\leqslant 4\\ \lambda-4 & \lambda>4\end{cases}$.

2 Controlled quantum teleportation of a three-particle arbitrary state

Charlie remains particle and transmits particles (A_1, A_2, A_3) and (B_1, B_2, B_3) particles to the sender (Alice) and the receiver (Bob) respectively. For insuring the control process of the teleportation, before he distributes the particles, Charlie performs a single particle unitary operation on one of the transmitted particles, for example particle B_1. After that, the seven-particle entangled state $|\varphi_7\rangle$ is changed into the following state

$$|\varphi_7'\rangle=U_{B_1}|\varphi_7\rangle \qquad (4)$$

where unitary operation U_{B_1} performed by Charlie before transmission is unknown to Alice and Bob. So the state $|\varphi_7'\rangle$ is unknown to them. Alice and Bob confirm Charlie that they have received transmitted particles through public channel.

Suppose Alice owns a three-particle arbitrary state to be teleported, which can be written as

$$|\xi\rangle_{a_1,a_2,a_3}=\sum_{i,j,k=0}^{1}\alpha_{ijk}|ijk\rangle \qquad (5)$$

where the coefficients satisfy the normalized condition $\sum_{i,j,k=0}^{1}|\alpha_{ijk}|^2=1$.

Alice, Bob and Charlie use the seven-particle entangled state as quantum channel to realize the controlled quantum teleportation of the above three-particle arbitrary state. The whole system state $|\Psi\rangle$ can be denoted as

$$|\Psi\rangle=|\xi\rangle_{a_1,a_2,a_3}\otimes|\varphi_7'\rangle_{A_1,A_2,A_3,B_1,B_2,B_3,C} \qquad (6)$$

Alice performs projective measurement on particles $(a_1, a_2, a_3, A_1, A_2, A_3)$ in the following bases[27]

$$|\chi^{nlm}\rangle=U_{a_1}^{n}\otimes U_{a_2}^{l}\otimes U_{a_3}^{m}|\chi^{000}\rangle_{a_1,a_2,a_3,A_1,A_2,A_3} \qquad (7)$$

where $U_{a_1}^{n}\otimes U_{a_2}^{l}\otimes U_{a_3}^{m}$ denotes the unitary operations U^n, U^l, U^m $(n, l, m=0, 1, 2, 3)$ on the former three particles (a_1, a_2, a_3), respectively. $U^0=I=|1\rangle\langle 1|-|0\rangle\langle 0|$ is the two-dimensional identity operation and $U^1=X=|1\rangle\langle 0|+|0\rangle\langle 1|$, $U^2=Z=|0\rangle\langle 0|-|1\rangle\langle 1|$, $U^3=XZ$. After measurements, the state of particles $(a_1, a_2, a_3, A_1, A_2, A_3)$ will be projected into one of the states denoted as Eq. (7). And the state of particles (B_1, B_2, B_3, C) can be expressed as

$$|\Phi\rangle_{B_1,B_2,B_3,C}=U_{B_1}(U_{B_1}^{n})\otimes(U_{B_2}^{l})\otimes$$

$$(U_{B_3}^{m})\frac{1}{\sqrt{2}}\Big(\sum_{i,j,k=0}^{1}\alpha_{ijk}|ijk\rangle_{B_1,B_2,B_3}|0\rangle_C+ \qquad (8)$$

$$\sum_{i,j,k=0}^{1}\alpha_{ijk}|\bar{i}\bar{j}\bar{k}\rangle_{B_1,B_2,B_3}|1\rangle_C\Big)$$

Here $\bar{i}$ is the counterpart of the binary number $\bar{i}$, i. e. $\bar{i}=1-i$.

If he agrees to help Bob recover the original state, Charlie should perform the measurement on the basis of $\{|0\rangle, \langle 1|\}$ and inform Bob of the measurement outcome and which unitary operation on particle B_1 he performed.

After receiving the measurement outcome from Alice and the classical message from Charlie, Bob performs the corresponding unitary operations on the particles (B_1, B_2, B_3) to reconstruct the original state on them. That is, if Charlie′s measurement outcome is $|0\rangle$, Bob performs the unitary operations $(U_{B_1}U_{B_1}^{n})\otimes U_{B_2}^{l}\otimes U_{B_3}^{m}$ on the particles (B_1, B_2, B_3); if Charlie′s measurement outcome is $|1\rangle$, Bob should perform the unitary operations $(U_{B_1}U_{B_1}^{n}U_{B_1}^{l})\otimes U_{B_2}^{l}\otimes U_{B_3}^{m}$.

After unitary operations, the three-particle state originally in Alice′ side is reconstructed on Bob′s particles successfully. The controlled teleportation process completes.

3 Summary

In this paper, we construct a seven-particle entangled state with Borras state. Using it, we propose a controlled teleportation scheme. In this scheme, the sender may teleport a three-particle state in general form to the receiver under the control of the supervisor. If the supervisor does not cooperate with the sender and the receiver, for example, the supervisor makes no measurement and publicizes false message, the receiver can not obtain the original state. Using this teleportation scheme, some quantum communication protocol may be realized.

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一种三粒子一般态的远程控制传送方案

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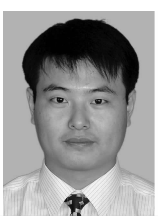
摘　要：基于七粒子纠缠态信道，提出一种三粒子一般态的远程控制传送方案．发送者进行投影测量后，发布测量结果．在控制者的控制下，接受者根据发送者的测量结果对所在处的粒子进行适当的幺正操作从而重构原始态．此方案可用来实现控制量子通信．

关键词：关键词：量子信息；量子控制传态；七粒子纠缠态



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