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Study of Geometrical Optics on Left-handed Material (Ⅱ)*

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Abstract Unique optical properties of the simple lens composed of left-handed material (LHM) are reasonedly studied here according to Geometrical Optics. Analytic calculation of the negative refraction at the spherical surface is conducted demonstrating that the usual optical formulas still hold true for the LHM lens. The inherent merits of the LHM lenses are discussed comparing to their counterparts made of right-handed material (RHM). Finally, single thin LHM lens with a complete correction of spherical aberration is verified and a practical example is presented accordingly.

Keywords Left-handed materials; Negative refractive index; LHM lens; Spherical aberration

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0 Introduction

Novel physical effects in dielectric media with both negative permittivity and negative permeability were first analyzed by Vesolago[1]. In recent years, there has been great interest on this subject due to the experimental demonstrations of negative index artificial materials[2] and the introduction of the perfect lens concept by Pendy[3]. It has also been proposed that photonic-gap material can behave as effective negative index material at optical frequencies[4] which makes the research on the properties and design of optical LHM elements crucial. Such as the focusing of a plane microwave by a planoconcave lens fabricated from a photonic crystal was demonstrated by Vodo[5] et al that obeys the standard laws with negative index of refraction. The aberration properties of the thin negative index lenses were also discussed by Schurig and Smith[6] who point out that negative index lenses offer clear performance advantages over their positive index lenses counterparts.

Inherently, the perfect lens doesn' t focus plane waves and only has a near-sight working distance equal to its thickness. However, many optical applications require the ability of lenses to focus radiation from the distant objects. This can be achieved with negative index media by using curved Surface. The capability of creating curved optical LHM elements with negative index of refraction materials has been demonstrated in[7]. In this paper, the unique properties of the simple spherical profile LHM lenses were further detailedly investigated. It is demonstrated that spherical surface and lenses with negative index of refraction still obey the standard optical formulas in the traditional geometrical optics and have many advantages over their positive index counterparts made of right-handed materials (RHMs) such as the stronger focusing power, the opener working space, the slimmer body and lighter weight. It is also verified that single thin LHM lens with special shape does provide a complete correction of spherical aberration while it is impossible for a normal RHM lens.

1 Negative refraction at the spherical surfaces of the LHM lens

Among the optical imaging systems, a lens consisting of two spherical surfaces is the most common as well as practical. In order to study the optical properties and design of a LHM lens, it is primary and compulsory to first investigate the behaviors of the negative refraction at a spherical refracting surface between LHM and RHM. As illustrated in Fig. 1, consider a spherical surface of radius of curvature r, separating a LHM ($n'<0$) from a RHM ($n>0$). A ray from object point M on the axis to image point M' via any point B on

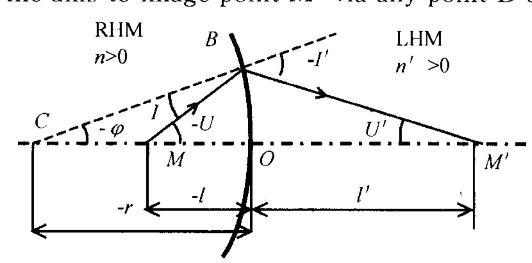

Fig. 1 Refraction of a ray at a single spherical refracting surface between LHM and RHM

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the refracting surface is plotted in accordance with the reversed Snell's law[1] which can be expressed as

$$n\sin I = -n'\sin(-I') \tag{1}$$

where $n' < 0$ and $I' < 0$ indicate the negative refraction. Here, the nomenclature and sign convention are following the definition in the reference[8]. From the geometry and variables involved in Fig. 1, meanwhile, supposing that $\overline{MB} = -u$ and $\overline{M'B} = u'$, it follows that

$$I = \varphi - U \tag{2}$$

and

$$I' = \varphi - U' \tag{3}$$

In the triangle ΔMBC and $\Delta M'BC$,

$$\sin U/r = \sin\varphi/(-u) \tag{4}$$

and

$$\sin U'/r = \sin\varphi/u' \tag{5}$$

From equation (1) through (5), one obtains

$$uu'n\cos I + urn' = uu'n'\cos I' + u'rn \tag{6}$$

In the paraxial condition, $\cos I \approx 1$, $\cos I' \approx 1$, and image distance $l' \approx u$, and object distance $l \approx u$ when B approaches vertex O. Accordingly, equation (6) can be simplified as

$$\frac{n'}{l'} - \frac{n}{l} = \frac{n'-n}{r} \triangleq \varphi = -\frac{n}{f} = \frac{n'}{f'} \tag{7}$$

where $\varphi = (n'-n)/r$ is surface refracting power and f and f' are the object and image focal lengths of the refracting surface. Expectably, it has the same form as the conjugate foci formula derived at a surface between two conventional media except that n' is negative now. Therefore it confirms that the usual optical formulas for conventional imaging calculation still hold true for LHM regardless of the sign of the refraction index in the paraxial condition. It is worth noting that (see Fig. 2) the

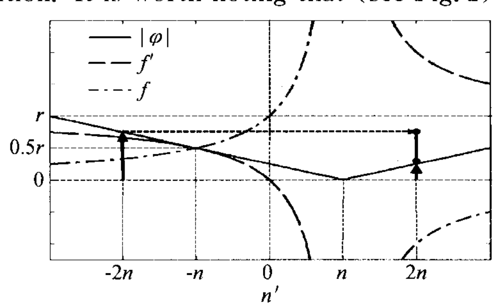

Fig. 2 Absolute value of surface refraction power $|\varphi|$, focal lengths f' and f as functions of the refractive index n'

peculiar RHM-LHM surface can provide higher absolute value of φ than an RHM-RHM one for equal absolute value of indices and its two focal lengths always satisfy

$$0 < f' < |r| \text{ and } |f| < |r|$$

for n' and n have opposite signs ($n'<0, n>0$). It also should be pointed out, for the same reason, that a RHM-LHM surface is always convergent if it is convex and always divergent if it is concave, while it is more complicated for an RHM-RHM surface[9].

The ray-tracing sketch of a typical simple LHM lens with a negative refractive index ($n<0$) placed in air is shown in Fig. 3. Suppose that the

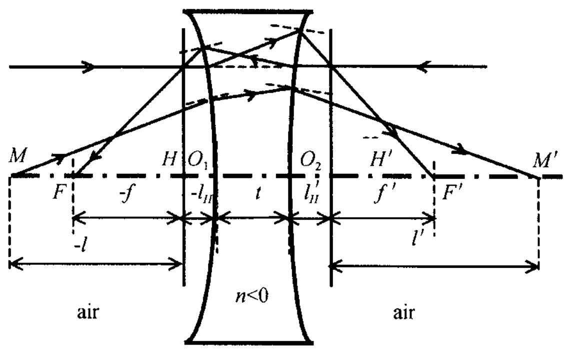

Fig. 3 Sketch and parameters of a simple LHM lens

radii of curvature of the first and second surfaces are r_1, r_2 and the thickness is $t = \overline{O_1O_2}$, where O_1 and O_2 are the front vertex and rear vertex of the lens. Usable formulas can be easily derived by combining the equation (7) for each refracting surface of the lens, such as the conjugate foci formula

$$\frac{1}{l'} - \frac{1}{l} = \frac{(n-1)[n(r_2-r_1)+(n-1)t]}{nr_1r_2} \tag{8}$$

the front focal length f and the back focal length f'

$$f' = -f = \frac{r_1r_2}{(n-1)[(r_2-r_1)+(1-1/n)t]} \tag{9}$$

and the positions of the first principal plane l_H and the second principal plane l'_H

$$\begin{cases} l_H = \dfrac{-tr_1}{n(r_2-r_1)+(n-1)t} \\ l'_H = \dfrac{-tr_2}{n(r_2-r_1)+(n-1)t} \end{cases} \tag{10}$$

Evidently, one founds that the optical properties of LHM lenses are influenced by their negative refractive indices.

2 Unique merits of the simple lens composed of LHM

In this section, the advantages of the LHM lenses over their RHM counterparts are discussed detailedly. The positions of the image focal points F' and the two principal planes H, H' of the LHM and RHM simple lenses relative to their bodies were shown in Fig. 4. Interestingly, there is an asymmetric relationship between the LHM and RHM lens profiles where a converging lens made from LHM should have a concave shape rather than a convex one for RHM lens. According to equation(9), one founds that the bi-concave LHM lens is always positive or converging regardless of its thickness and the absolute values of its radii of curvature. However, the sign of the image focal length for the bi-convex LHM lens depends on its thickness for the given radii of curvature i. e. if $0 \leqslant d < d_{-\infty}$, $f' < 0$, the lens is a diverging lens; if $d > d_{-\infty}$, $f' < 0$, the lens is a converging lens, where $d_{-\infty} = n(r_2 - r_1)/(1-n)$ is the singular thickness for equation (9). What is more, as we can see from Fig. 4, that for many types of the LHM lenses, the two principal planes always lie out of their bodies ($l'_H > 0$, $l_H < 0$). Therefore they are ready to provide full-opened working space which greatly facilitates the optical design whereas for most RHM lenses, some portion of the working space is occupied by their bodies and thus can't be utilized.

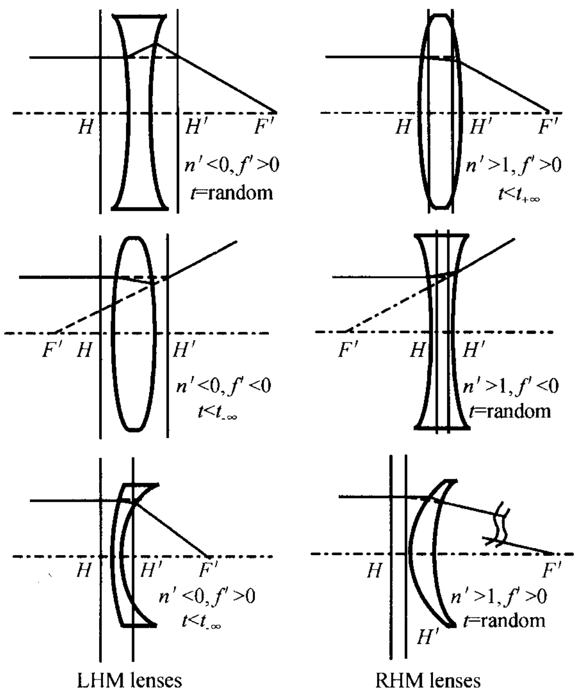

Fig. 4 The positions of the image focal points and the two principal planes of the LHM and RHM lenses

The back focal lengths f' as the functions of n are shown in Fig. 5 for the bi-concave lens with $r_2 = -r_1 = 10t$, and the bi-convex lens with $r_1 = -r_2 = 10t$. One is aware from Fig. 5 that a RHM lens with an index $n = 1$ (air lens) has no refractive power while an LHM lens with $n = -1$ has considerable refractive power. However, one more significant merit of a LHM lens with $n = -1$ is the fact that it is impedance matched to the surrounding media (air) with $Z = 1$, which means that none of the incident light energy is reflected at all.

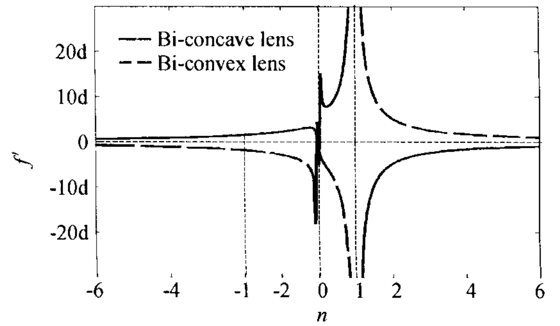

Fig. 5 The back focal length f' as the function of refractive index n for the bi-concave and bi-convex lenses

As implied in equation (9), if one compares two lenses of the same absolute value of refractive index $|n|$ but opposite in sign, a LHM lens has greater focusing power than a RHM lens. For instance, the absolute value of the focusing power for bi-concave lens with $r_2 = -r_1 = 10t$ is determined by

$$|\varphi| = \frac{1}{|f'|} \left| \frac{(n-1)(19n-1)}{n} \right| \frac{1}{100d} \tag{13}$$

where if $n=3$, $|\varphi| = 0.3733/d$, and if, $n=-3$, $|\varphi| = 0.7733/d$. This provides a benefit of LHM lens, compared to RHM lens, that for the same focal length a larger radius of curvature is allowable. In the practical applications where larger radius of curvature is desirable, it is worth noting that a LHM lens provides a radius of curvature that is 5 times larger than a RHM lens, for a plano-concave ($n = -1.5$) and a plano-convex ($n = 1.5$), each having the same focal length because $r = (n-1)f'$. This preferable fact eases facture and is propitious to reduced aberrations. Conversely, a LHM lens has a much shorter focal length as comparing to a RHM lens with the same radius of curvature. Therefore the LHM lens can be more compact and has less lateral amplification that is again in favor of reduced aberrations. Moreover, the LHM lenses are comparatively much lighter than the RHM lenses, due to their slimmer bodies with intrinsically concave shapes and newly developed honeycomb-like construction techniques[10], which is a significant advantage for the optical applications where light optical elements are favored.

3 Single thin LHM lens free of spherical aberration

Spherical aberration is an image imperfection that is due to the spherical lens shape. It is known that when both an object and its image are real, the spherical aberration of single positive index thin lens can't be zero with normal material ($n>1$) unless its surfaces are made aspherical[6]. Now, study the spherical aberration of a negative index thin lens made of LHM with aperture stop (AS) located at the lens. A thin LHM lens of negative refractive index ($n<0$), focal length f' and consisting of two spherical surfaces of radii of curvature r_1 and r_2 was shown in Fig. 6. Consider a

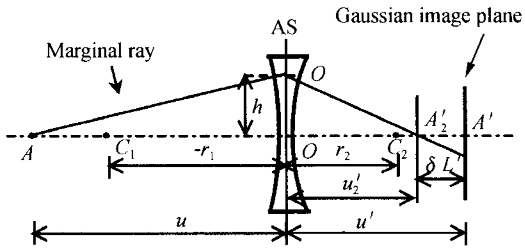

Fig. 6 The spherical aberration of a marginal ray in an axial bundle transmitting through a thin LHM lens

point object A located on the optical axis at a distance u from the lens. The lens forms the image of A at A' in the Gaussian image plane with the image distance u', given by the relation

$$\frac{1}{u}+\frac{1}{u'}=(n-1)\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{1}{f'} \tag{14}$$

The spherical aberration $\delta L'$ of the marginal ray $AQ\overline{A'}$ passing through a point Q in the plane of the aperture stop with a height h is given by[11]

$$\delta L'=A_2'-A'=-h^4[n^3/(n-1)+(3n+2)\cdot(n-1)\eta^2+(n+2)/(n-1)\sigma^2+4(n+1)\cdot\eta\sigma]/[32n(n-1)f'^3] \tag{15}$$

where η and σ are the position and shape factors of a thin lens, respectively, which are defined as

$$\eta=(2f'/u)-1=1-(2f'/u') \tag{16}$$

$$\sigma=(r_2+r_1)/(r_2-r_1) \tag{17}$$

It is evident from equation (14) and (17) that when both an object and its image are real, $-1\leqslant\eta\leqslant 1$.

By setting $\delta L'$ to zero in equation (15), one obtains the relation to terms of η, σ and n for a thin lens free of spherical aberration

$$n^3+(3n+2)(n-1)^2\eta^2+(n+2)\sigma^2+4(n^2-1)\eta\sigma=0 \tag{18}$$

It was shown in Fig. 7 that the variation of shape factors of thin lenses free of spherical aberration with its refractive index for the given position factors, $\eta=-1,+1,0$ corresponding to an object at infinity, the focal plane and a distance $2f'$ from the lens, respectively. The solutions of n for some given values of η and σ in equation (15) are listed detailedly in Tab. 1 where $\sigma=-1, 0, +1$ correspond to the plano-convave, equi-concave and convave-plano lenses, respectively. One can also find in Fig. 6 that thereis no solution for σ when $n>1$. Therefore, it is impossible with a single thin lens made of RHM ($n>1$) to provide a complete correction of spherical aberration.

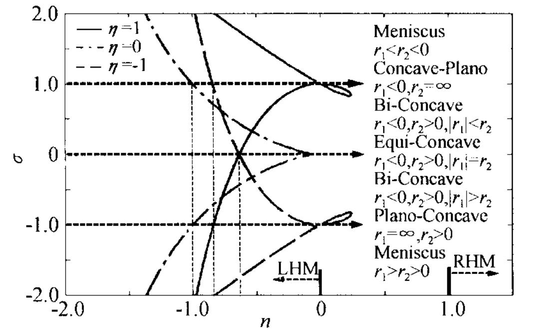

Fig. 7 Single thin LHM lenses with zero spherical aberration

Tab. 1 Refractive indices of serveral typical thin LHM lenses with zero spherical aberration composed of LHMs

n	$\eta=-1$	$\eta=0$	$\eta=1$
$\sigma=-1$	…	-1	-0.839
$\sigma=0$	-0.635	…	-0.635
$\sigma=1$	-0.839	-1	…

As a practical example, we determine the radii of curvature of the surfaces of a thin LHM lens of refractive indexfocusing a parallel beam of light at an imaging distance of 15 cm from it with free spherical aberration. By substituting $n=-1$ and $\eta=-1$ into equation (15), one gets $\sigma_{zero}=\sqrt{5}\approx\pm 2.236$. Plugging them into equation (17) gives

$$r_2/r_1=\frac{\sigma_{zero}+1}{\sigma_{zero}-1}=2.618 \text{ and } 0.3820 \tag{19}$$

Since $f'=15$ cm, $n=-1$, combining the equation (14) and (16) yields $r_1=-18.54$ cm, $r_2=-48.53$ cm and $r_1=48.53$ cm, $r_2=18.54$ cm, which are corresponding to two meniscus lenses possessing the following merits: complete correction of spherical aberration, no reflection of the incident light energy, low weight with slim bodies and their spherical surfaces easy for practical fabrication.

4 Conclusion

With the development of the new technologies in the fabrication of LHMs, relevant researches on the unique properties of LHMs are extensive and ubiquitous[12~14], where the brand-new theories are always proposed. Here, the optical properties of the simple negative index lenses which are composed of LHM were studied in view of Geometrical Optics. The analytic calculation about the negative refraction at the spherical surface between RHM and LHM demonstrates that the general optical formulas still hold true for LHM lenses which are inherently possessing many unique merits as compared with their counterparts composed of RHM. At the final section, single thin LHM lens with a complete correction of the spherical aberration was verified that is feasible for a certainty with a practical example presented accordingly. However, further experiments need to be performed in the future to verify the designs mentioned above. In deed, the emergence of LHM lenses expands the theory of optical design and inevitably opens the door to a new breed of optical devices and applications.

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关于左手性介质几何光学的研究（二）

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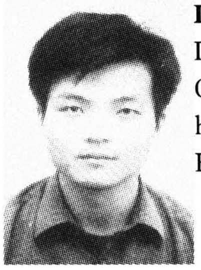
摘 要 利用几何光学方法研究了由左手性介质制成的简单透镜的光学特性．推理分析了左手性介质透镜处球形界面的负折射规律，并依此表明了常用光学公式仍适用于左手性介质．指出了左手性介质透镜相对于用传统介质制作的普通透镜所具备的固有优点，最后证实了利用单块左手性介质透镜即可完全消球差，并相应提供了零球差左手性介质透镜的设计实例．

关键词 左手性介质；负折射率；左手性介质透镜；球差



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