Modification of Coupled Equations for Cascaded CW Raman Fiber Lasers*

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Abstract A comprehensive and accurate numerical model is presented based on basic coupled equations to describe multi-Stokes Raman lasing performances in optical fibers. Though much work has been done on this, there are insufficiencies and shortcomings more or less in the literatures after analysis of them. These insufficiencies are also pointed out in this paper. The interactions between forward and backward traveling waves together with interactions between Stokes at different orders are all taken into account in this model. Effective core area for the interaction between multi-Stokes has also been obtained. The numerical solution of the differential system will lead in a natural way to calculate Stokes powers at the output end of the fiber. The proposed model is important for the design of CW cascaded Raman fiber lasers.

Keywords Cascaded Raman laser; Stimulated Raman scattering; Numerical modeling; Effective core area; Optical fiber laser

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0 Introduction

In recent years, CW (continuous wave) multi-Stokes Raman fiber lasers have attracted much interest because they can provide suitable pump sources for Raman amplifiers operated in optical communications[1~3]. For example, cascaded Raman fiber lasers with a silica single mode fiber pumped around 1μm can generate radiation at wavelength of 1. 24 μm for the 1. 3 μm Raman amplifier through three-fold Stokes conversion. It can also generate radiation at wavelength of the 1. 48 μm for remotely pumping erbium-doped fiber amplifiers or pumping Raman amplifiers in the 1. 55 μm spectral window by six-fold Stokes conversion[4~6]. Such a laser consists of three and six pairs of Bragg gratings for 1. 24 μm and 1. 48 μm, respectively, since germano-silicate fibers have a small Stokes frequency shift of $\sim 440\ \mathrm{cm}^{-1}$. However, for a phosphosilicate fiber the first and the second Stokes orders at 1. 24 μm and 1. 48 μm can be obtained[7~9]. Regardless of what kind of fiber, optimization of the power /quantum conversion efficiency in multi-Stokes Raman fiber lasers requires numerical modeling, since there exists complex nonlinear, multiple-boundary power transfers between the different Stokes orders. The first numerical model of Raman fiber laser was presented in 1979 by Auyeung and Yariv[10], only considering the first Stokes wave. Several other theoretical models have been presented after. In ref. [11], effective core area was associated with the interaction between the pump and the ith Stokes mode. In ref. [12], a numerical model was implemented to describe 1064 nm pumped Raman fiber laser at 1240 nm. In ref. [13], the single pass evolution of the pump and Stokes beams in a fiber cavity was considered and the Raman gain coefficient g_R was unchangeable for any the Stokes order. In ref. [14], effective core area was the constant for all the Stokes order. An elaborated coupled-equation analysis of Raman fiber laser was described in ref. [15], but there exist some shortcomings, which will be discussed in the next section. In this paper, we present a comprehensive, accurate numerical model based on nonlinear-coupled equations for a multi-Stokes Raman fiber laser. In addition, we obtain a general expression for the effective core area for the interaction between multi-Stokes in contrast to the model referenced in ref. [11].

1 Theoretical model

The basic coupled equations for a CW Raman laser is

$$\begin{aligned}\frac{\mathrm{d}I_s}{\mathrm{d}z}&=g_R I_p I_s-\alpha_s I_s\\ \frac{\mathrm{d}I_p}{\mathrm{d}z}&=-\frac{\omega_p}{\omega_s}g_R I_s I_p-\alpha_p I_p\end{aligned}\tag{1}$$

where α_s and α_p represent the fiber loss at the signal

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and pump frequencies ω_s and ω_p, respectively[16], I_s and I_p represent the signal intensity and pump intensity, respectively, and g_R is the Raman gain coefficient.

For a two orders cascaded Raman laser, the second-order Stokes wave is generated pumped by the first-order Stokes wave. That is, the mutual interaction exists between the pump wave, the first-order Stokes wave and the second-order Stokes wave. Similarly, in the n orders Raman fiber laser, the kth-order Stokes interacts simultaneously with the $(k-1)$th and $(k+1)$th-order waves traveling in both directions. That is, cascaded Raman generation can be viewed as an iteration of fundamental stimulated Raman scattering (SRS) processes in which each generated Stokes wave acts as a pump to produce the next one. So, by taking into account the coupling between both co-propagating and counter-propagating waves, the evolution of the pump and Stokes intensity along the fiber length can be described, in steady state, by a set of ordinary nonlinear differential equations as follows

$$\begin{aligned}
&\frac{dI_p^{\pm}}{dz}=\mp\alpha_p I_p^{\pm}\mp\frac{\nu_p}{\nu_{s1}}g_R^{0,1}(I_{s1}^{+}+I_{s1}^{-})I_p^{\pm}\\
&\qquad\vdots\\
&\frac{dI_{sk}^{\pm}}{dz}=\mp\alpha_{sk}I_{sk}^{\pm}\mp\frac{\nu_{sk}}{\nu_{sk+1}}g_R^{k,k+1}(I_{sk+1}^{+}+I_{sk+1}^{-})I_{sk}^{\pm}\pm\\
&\qquad g_R^{k-1,k}(I_{sk-1}^{+}+I_{sk-1}^{-})I_{sk}^{\pm},k=1,\cdots,n\\
&\qquad\vdots\\
&\frac{dI_{sn}^{\pm}}{dz}=\mp\alpha_{sn}I_{sn}^{\pm}\pm g_R^{n-1,n}(I_{sn-1}^{+}+I_{sn-1}^{-})I_{sn}^{\pm}
\end{aligned}\tag{2}$$

In the above system, I_{sk} represents the kth-order Stokes intensity (+ stands for forward and-for backward propagating), ν_{sk} its frequency and $g_R^{k-1,k}$ its Raman gain, which is related to the dopant concentration of the Raman fiber and the two frequencies ν_{k-1} and ν_k[16]. The zero-order of above parameters represent the pump. Here spontaneous Raman scattering which is often correct in most cases is neglected.

In general, the intensity term I_i with the appropriate subscripts in (1) and (2) varies with respect to an (r, θ, z) cylindrical coordinate scheme. However, since the weakly guiding transverse mode LP_{01} is axially symmetric, it is independent of θ. In addition, it is assumed that the radial dependence of the intensity terms remains constant as the waves propagate along the fiber. That is, the depletion and growth in intensity take place only in the z-direction. Thus, we have

$$I_i(r,\theta,z)=f_i(r,\theta)I_i(z)\tag{3}$$

where $f(r,\theta)$ refers to the radial variation in intensity and is dimensionless, $I(z)$ refers to the axial variation in intensity and has the dimension of power/area.

In practice, power is often used instead of intensity. If consider the variations of pump power P_p and signal power P_{sk} along the fiber, it gets

$$\begin{aligned}
&P_p(z)=\int_0^{2\pi}d\theta\int_0^{\infty}I_p(r,\theta,z)rdr=\\
&\qquad I_p(z)\cdot\int_0^{2\pi}d\theta\int_0^{\infty}f_p(r,\theta)rdr=I_p(z)A_p\\
&P_{sk}(z)=\int_0^{2\pi}d\theta\int_0^{\infty}I_{sk}(r,\theta,z)rdr=\\
&\qquad I_{sk}(z)\cdot\int_0^{2\pi}d\theta\int_0^{\infty}f_{sk}(r,\theta)rdr=I_{sk}(z)A_{sk}
\end{aligned}\tag{4}$$

Define $\Gamma_{sk-1,sk}$ as

$$\Gamma_{sk-1,sk}=\int_0^{2\pi}d\theta\int_0^{\infty}f_{sk-1}(r,\theta)f_{sk}(r,\theta)rdr,k=1,\cdots,n\tag{5}$$

Now, integrate (2) with respect to r and θ to obtain coupled equations in terms of power with respect to length

$$\begin{aligned}
&\frac{dP_p^{\pm}}{dz}=\mp\alpha_p P_p^{\pm}\mp\frac{\nu_p}{\nu_{s1}}\frac{\Gamma_{p,s1}}{A_pA_{s1}}g_R^{0,1}(P_{s1}^{+}+P_{s1}^{-})P_p^{\pm}\\
&\qquad\vdots\\
&\frac{dP_{sk}^{\pm}}{dz}=\mp\alpha_{sk}P_{sk}^{\pm}\mp\frac{\nu_{sk}}{\nu_{sk+1}}\frac{\Gamma_{sk,sk+1}}{A_{sk}A_{sk+1}}g_R^{k,k+1}(P_{sk+1}^{+}+\\
&P_{sk+1}^{-})P_{sk}^{\pm}\pm\frac{\Gamma_{sk-1,sk}}{A_{sk-1}A_{sk}}g_R^{k-1,k}(P_{sk-1}^{+}+P_{sk-1}^{-})P_{sk}^{\pm}\\
&\qquad\vdots\\
&\frac{dP_{sn}^{\pm}}{dz}=\mp\alpha_{sn}P_{sn}^{\pm}\pm\frac{\Gamma_{sn-1,sn}}{A_{sn-1}A_{sn}}g_R^{n-1,n}(P_{sn-1}^{+}+\\
&P_{sn-1}^{-})P_{sn}^{\pm}\quad(k=1,\cdots,n)
\end{aligned}\tag{6}$$

where P_{sk} represents the kth-order Stokes power.

Following Urquhart[11], define effective core area for the interaction between the $(k-1)$th-order Stokes wave and kth-order Stokes wave as

$$\begin{aligned}
&A_{eff}^{k-1,k}=\frac{A_{sk-1}A_{sk}}{\Gamma_{sk-1,sk}}=\\
&\frac{\int_0^{2\pi}d\theta\int_0^{\infty}f_{sk-1}(r,\theta)rdr\cdot\int_0^{2\pi}d\theta\int_0^{\infty}f_{sk}(r,\theta)rdr}{\int_0^{2\pi}d\theta\int_0^{\infty}f_{sk-1}(r,\theta)f_{sk}(r,\theta)rdr},k=1,\cdots,n
\end{aligned}\tag{7}$$

Then, (6) can be simplified to

$$\begin{aligned}
&\frac{dP_p^{\pm}}{dz}=\mp\alpha_p P_p^{\pm}\mp\frac{\nu_p}{\nu_{s1}}\frac{g_R^{0,1}}{A_{eff}^{0,1}}(P_{s1}^{+}+P_{s1}^{-})P_p^{\pm}\\
&\qquad\vdots\\
&\frac{dP_{sk}^{\pm}}{dz}=\mp\alpha_{sk}P_{sk}^{\pm}\mp\frac{\nu_{sk}}{\nu_{sk+1}}\frac{g_R^{k,k+1}}{A_{eff}^{k,k+1}}(P_{sk+1}^{+}+P_{sk+1}^{-})\cdot\\
&P_{sk}^{\pm}\pm\frac{g_R^{k-1,k}}{A_{eff}^{k-1,k}}(P_{sk-1}^{+}+P_{sk-1}^{-})P_{sk}^{\pm},k=1,\cdots,n\\
&\qquad\vdots\\
&\frac{dP_{sn}^{\pm}}{dz}=\mp\alpha_{sn}P_{sn}^{\pm}\pm\frac{g_R^{n-1,n}}{A_{eff}^{n-1,n}}(P_{sn-1}^{+}+P_{sn-1}^{-})P_{sn}^{\pm}
\end{aligned}\tag{8}$$

This set of equations is very similar to Eq. (1) in ref. [15]. However, note that there are mistakes in Eq. (1) of ref. [15], i. e., the superscripts of two important parameters A_{eff} and g_R are incorrect. For example, when $k=1$, g_R^{k-1} in Eq. (1) of ref. [15] becomes g_R^0, which is meaningless in practice.

2 The effective core area

As for the effective core area A_{eff}, a typical method for obtaining it is to follow the procedure described in ref. [11], cited in ref. [15]. However, the model used for the stimulated Raman scattering in ref. [11] is different from the model here. Specifically, in ref. [11], there is no mutual coupling of the Stokes modes, and A_{eff} dependents on the pump and the kth Stokes wave. In our model, $A_{eff}^{k-1,k}$ is associated with the kth-order Stokes wave and its pump light, i. e. the $(k-1)$th-order Stokes wave in the fiber.

Now derive the effective core area $A_{eff}^{k-1,k}$ in our model. If the radial variation of intensity is that of the weakly guiding analysis, one must resort to numerical integration. However, for single-mode fibers, the fundamental mode field distribution can be well approximated by a Gaussian function

$$\Psi_i(r)=\exp\left(-r^2/\omega_i^2\right) \tag{9}$$

where ω_i represents the spot size of the mode field pattern, i. e., the half width at $1/e^2$ maximum for the ith beam. Note that ω_i can be obtained by fitting the Gaussian intensity distribution to the exact distribution as deduced from the resolution of the eigenvalue problem for weakly guided modes[17]. Then, $f(r,\theta)$ has the form

$$f_i(r,\theta)=\exp\left(-2r^2/\omega_i^2\right) \tag{10}$$

From (10) and (7), the effective core area $A_{eff}^{k-1,k}$ can be shown to be

$$A_{eff}^{k-1,k}=\frac{A_{sk-1}A_{sk}}{\Gamma_{sk-1,sk}}=\frac{\pi}{2}(\omega_{sk-1}^2+\omega_{sk}^2) \tag{11}$$

Following Marcuse[18], one has the following empirical expression for ω

$$\frac{\omega}{a}\approx(0.65+\frac{1.619}{V^{3/2}}+\frac{2.879}{V^6}) \tag{12}$$

where a is the core radius and V is the normalized waveguide parameter. At each wavelength, the guided modes of the fiber have a different V-value and consequently a different spatial distribution. Here, it is assumed that the generated Stokes beams are fairly far from the cutoff wavelength. If the pump wavelength is below the cutoff, within the linear polarization (LP) approximation we can assume that the pump power is distributed on two transversal LP modes.

The system (8) can be solved using a standard Runge-Kutta integration method or so-called shooting method with boundary conditions, or a collocation software for boundary-value ordinary-differential equations, which are mentioned in some papers, for example, in ref. [14] and ref. [15], and thorough discussion are not given here.

3 Conclusions

It has presented a comprehensive, accurate numerical model for optimizing the power/quantum conversion efficiency of multi-Stokes Raman fiber lasers. This model takes into account all interactions between forward and backward traveling waves. A general expression has been obtained for the effective core area, which is critical for describing the interaction between the kth-order Stokes wave and the $(k-1)$th-order Stokes wave.

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连续级联喇曼光纤激光器耦合波方程的修正

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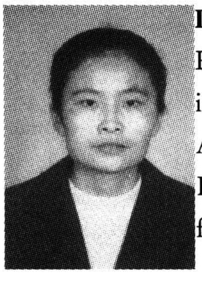
摘　要　从最基本的耦合波方程出发，考虑光波正反向传输的情况下，推导出了一个全面的、准确的关于连续级联喇曼光纤激光器的理论模型，描述了各级 Stokes 光波功率沿光纤长度的变化．指出了相关一些文献中存在的不足，并且给出了多级斯托克斯光波相互作用的有效作用面积的表达式．此模型有助于喇曼光纤激光器的设计．

关键词　级联喇曼激光器；受激喇曼散射；数值模型；有效作用面积；光纤激光器



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