Change of Total Refractive Index in Finite V-shaped Quantum Well*

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Abstract The change of refractive index in finite V-shaped quantum well is investigated. The first- and third-harmonic generation coefficient is obtained by using Compact-density-matrix approach and iterative method, and the numerical results are presented for GaAs/AlGaAs finite V-shaped quantum well. The results show that the change of total refractive index will be large with decreasing incident intensity.

Keywords Optical refractive index; Finite V-shaped quantum well; Harmonic generation
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0 Introduction

Since the end of 1980s, basic investigations on nonlinear optical properties[1~6,14~16] of semiconductor quantum wells, quantum wires, quantum dots, superlattices and macrostructures have attracted much attention in theoretical and applied physics, because of their relevance for studying practical applications and as a probe for the electronic structure of microscopic media. The research results have shown that decrease in dimensionality of the system for semiconductor can lead to a dramatically enhancement of nonlinearities[7]. With the quantum confinement of a carrier in a semiconductor, the discrete energy levels are formed. These energy level tailored by modifying the shape of a well using different compositions profiles and external applied potentials. The quantum size effect drastically changes the electronic and optical properties of the well material.

Nonlinear refractive index change, one of nonlinear optical properties, has been calculated in a single quantum well, poly quantum wells, quantum dots in the past. In 1987, Tohya Hiroshima[8] presented the nonlinear refractive index change in $GaAs/Al_xGa_{1-x}As$ quantum well in the electric field. The field effect on the refractive index in $GaAs/Al_xGa_{1-x}As$ quantum well by Yasuo Kan[9]. In 1991, Kelin J. Kuhn[10] presented free carrier induced change of the refractive index in $GaAs/Al_xGa_{1-x}As$ quantum well. They all obtained very large refractive index change. Modifying the shape of a quantum well can change the system wavefunction and the energy level, which both would drastically change optical properties. In this paper, it only concentrates on V-shaped quantum well and the study of nonlinear refractive index change.

1 Theory

Here, it is considered that a finite V-shaped quantum well as following

$$V(z)=\begin{cases} 0 & (z<-L) \\ -V_0+\frac{V_0}{L}|z| & ((\frac{V_0+E}{V_0})L\leqslant z\leqslant L) \\ 0 & (Z>L) \end{cases} \tag{1}$$

where $V(z)$ is the confining potential, z represents the growth of the well, z axis is perpendicular to the well layer, L is the width of VQW, V_0 is a parameter in describing confining potential.

It was supposed an electron confined in such VQW. In the effective mass approximation, the Hamiltonian of the electron in VQW is

$$H=-\frac{\hbar^2}{2m_e^*}\nabla+V(z) \tag{2}$$

where m_e^* is the effective mass of the electron, $\hbar$ is Planck' constant. So the Schrödinger equation can be written as following

$$[-\frac{\hbar^2}{2m_e^*}\nabla+V(z)]\Psi(z)=E\Psi(z) \tag{3}$$

The solvation of Eq. (3) and its energy level are given by

$$\Psi_1(\xi)=C_1\sqrt{|\xi|}\{J_{1/3}(2|\xi|^{3/2}/3)+J_{-1/3}(2|\xi|^{3/2}/3)\} \quad (0\leqslant z\leqslant(\frac{V_0+E}{V_0})L)$$

$$\Psi_2(\xi)=C_2\sqrt{|\xi|}\{I_{1/3}(2|\xi|^{3/2}/3)+K_{1/3}(2|\xi|^{3/2}/3)\} \quad ((\frac{V_0+E}{V_0})L\leqslant z\leqslant L)$$

$$\Psi_3(z)=De^{-\beta z} \quad (L\leqslant z)$$

$$\Psi_4(z)=De^{\beta z} \quad (z\geqslant -L) \tag{4}$$

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where $\xi=\sqrt[3]{\frac{2m_e^* L^2}{\hbar^2 V_0^2}}\,(E+V_0-\frac{V_0}{L}z)$ and J, I, K represent BesselJ function, BesselI function and BesselK function respectively, $\beta=\sqrt{\frac{-2m_e^* E}{\hbar}}$, C_1, C_2 and D is normalization constant

$$E_n=-V_0+V_0\{\frac{3\pi\hbar}{4L\sqrt{2m_e^* V_0}}(n+\frac{1}{2})\}^{2/3} \quad (5)$$

where n means the number of the energy level of the VQW.

In the below, it will derive the expression of linear and nonlinear Optical index change in VQW. Let us suppose VQW with an optical field with frequency ω which is incident with a polarization vector normal to VQW. The incident electronic field is defined as

$$E(t)=\tilde{E}e^{i\omega t}+\tilde{E}e^{-i\omega t} \quad (6)$$

Let the symbol ρ denote the one-electron density matrix operator for this regime. The evolution of one-electron density matrix obeys the following time-dependent Schrödinger equation

$$\frac{\partial\rho_{ij}}{\partial t}=(i\hbar)^{-1}[H_0-qzE(t),\rho]_{ij}-\Gamma_{ij}(\rho-\rho^{(0)})_{ij} \quad (7)$$

where, $\rho^{(0)}$ is the unperturbed density matrix, Γ is the relaxation time. For simplicity, we assume that all of the off-diagonal elements of Γ are same and $\Gamma_{ij}=1/\Gamma_1$, and all of the diagonal elements of Γ are adopted the other same value too, $\Gamma_{ij}=1/\Gamma_2$. Equation is usually solved using iterative method[11]

$$\rho(t)=\sum_n\rho^{(n)}(t) \quad (8)$$

With

$$\frac{\partial\rho_{ij}^{(n+1)}}{\partial t}=\frac{1}{\hbar}\{[H_0,\rho^{(n+1)}]_{ij}-i\hbar\Gamma_{ij}\rho_{ij}^{(n+1)}\}-\frac{1}{i\hbar}[er,\rho^{(n)}]_{ij}E(t) \quad (9)$$

In this text, it only concentrate on electronic transition between the ground state and the first excited state. The electronic polarization $P(t)$ of VQW can be expanded as equation(9), it is be limit to considering the first three orders

$$P(t)=\varepsilon_0\chi_\omega^{(1)}\tilde{E}e^{-i\omega t}+\varepsilon_0\chi_{2\omega}^{(2)}\tilde{E}e^{-2i\omega t}+\varepsilon_0\chi_{3\omega}^{(3)}\tilde{E}e^{-3i\omega t}+c.c. \quad (10)$$

where it's limited to considering the first three orders, ε_0 is the vacuum permittivity, $\chi^{(1)}$, $\chi_{2\omega}^{(2)}$, $\chi_{3\omega}^{(3)}$ are the linear, second-harmonic generation and third-harmonic generation coefficients, respectively. As V-shaped potential is symmetrical, $\chi_{2\omega}^{(2)}$ will be zero. So it only need calculate linear and third-harmonic generation coefficients $\chi^{(1)}$, $\chi_{3\omega}^{(3)}$. By using density matrix method, it can find the expression of linear and third-harmonic generation coefficient $\chi^{(1)}$, $\chi_{3\omega}^{(3)}$. So it lays emphasis on the calculations of the change of the first and third order optical refractive index change. They obey the following equation[12]

$$\frac{\Delta n(\omega)}{n_r}=\mathrm{Re}(\frac{\chi(\omega)}{2n_r}) \quad (11)$$

Finally, the expressions of optical refractive index change was obtained.

$$\frac{\Delta n^{(1)}(\omega)}{n_r}=\frac{\rho_s I}{2\varepsilon_0 n_r}|M_{10}|^2[\frac{\hbar\omega_{10}-\hbar\omega}{(\hbar\omega_{10}-\hbar\omega)^2+(\hbar\Gamma_{01})^2}] \quad (12)$$

$$\frac{\Delta n^{(3)}(\omega,I)}{n_r}=-\frac{uc}{4n_r^3\varepsilon_0}|M_{10}|^2\{\rho_s I/[(\hbar\omega_{10}-\hbar\omega)^2+(\hbar\Gamma_{01})^2]^2\}[4(\hbar\omega_{10}-\hbar\omega)|M_{10}|^2-\frac{(M_{11}-M_{00})^2}{(\hbar\omega_{10})^2+(\hbar\Gamma_{01})^2}\{(\hbar\omega_{10}-\hbar\omega)[\hbar\omega_{10}(\hbar\omega_{10}-\hbar\omega)-(\hbar\Gamma_{10})^2]-(\hbar\Gamma_{01})^2(2\hbar\omega_{10}-\hbar\omega\}] \quad (13)$$

where $\omega_{10}=\frac{E_1-E_0}{\hbar}$ is Bohr frequency, $M_{ij}=\langle\Psi_j|qz|\Psi_i\rangle$ is the dipole matrix $(i,j=0,1)$, ρ_s is the electron density in the VQW. I is the optical power unit area, n_r is the refractive index without nonlinear optical effect, u is the permeability of the system and c is the light speed in vacuum.

2 Results and discussions

The parameters used in numerical work are adopted as[13]: $m_e^*=0.067m_0$, $n_r=3.2$, $\varepsilon_0=8.85\times10^{12}\,\mathrm{F\cdot m^{-1}}$, $V_0=0.1$ eV, $\rho_s=5\times10^{22}\,\mathrm{m}^3$.

In Fig. 1, it plots the values of the linear, third-order and total refractive index change versus the phonton energy $h\nu$ for incident intensity $I=1$ mW/cm^3. From this figure it was see that the third-order refractive index change $\Delta n^{(3)}$ can not be ignored, comparing with the linear refractive index change $\Delta n^{(1)}$. On the other hand, the value of $\Delta n^{(3)}$ has a opposite sign to the value of $\Delta n^{(1)}$, so it creates the value of Δn_{total} decreasing. Therefore the third-order

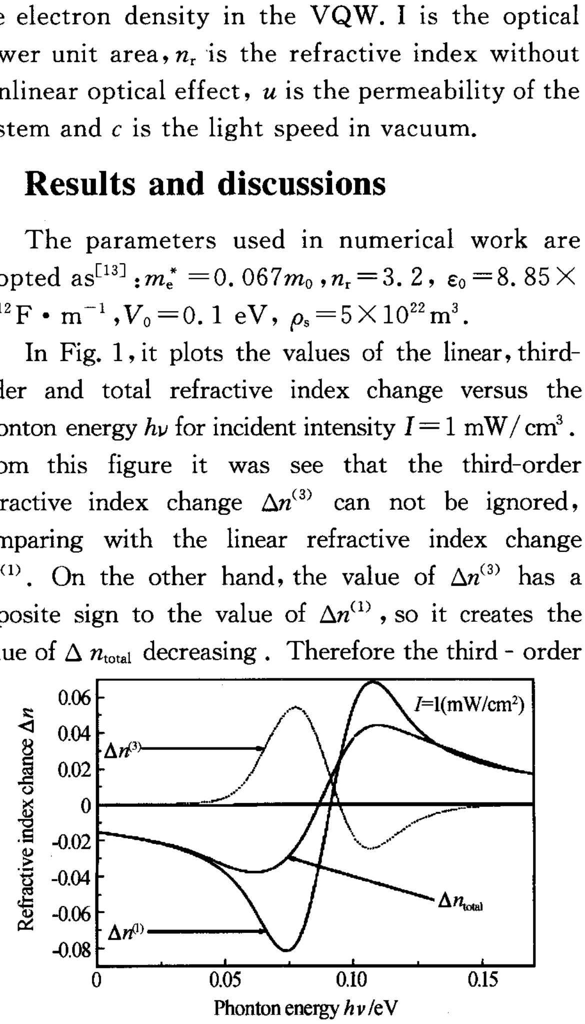

Fig. 1 The linear, third-order and total refractive index change in finite V-shaped quantum well versus the phonton energy $h\nu$ for incident intensity $I=1$ mW/cm^3

nonlinear refractive index change has a marked influence on the change of refractive index.

In Fig. 2, the linear, the third-order and total refractive index change are plotted as functions of phonton energy for the other big incident intensity $I=3$ mW/cm^3. From Fig. 3, it was seen that linear refractive index change is the same as above in Fig . 1 , after increasing incident intensity. It means that linear index change is not related to incident intensity, which it' s also can be obtained in equation (12). However, the third-order refractive index change makes a large change and it even surpasses the peak value of linear refractive index change when the incident optical intensity exceeds certain value.

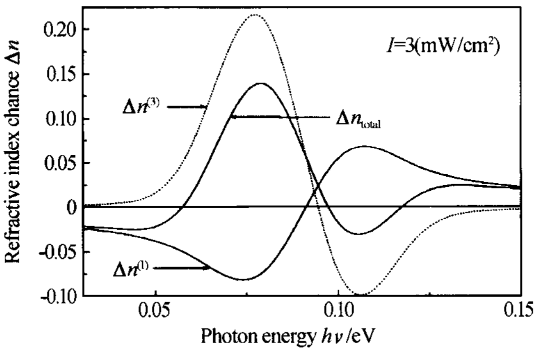

Fig. 2 The linear, third-order and total refractive index change in finite V-shaped quantum well versus the phonton energy $h\nu$ for incident intensity $I=3$ mW/cm^3

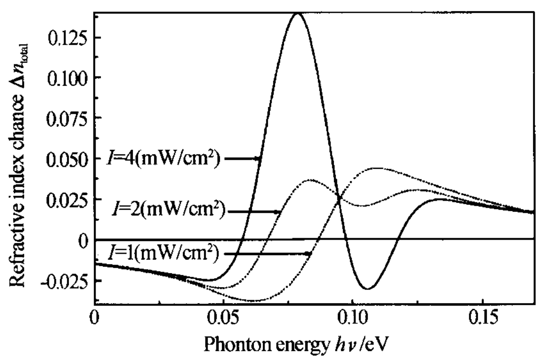

Fig. 3 The total refractive index change in finite V-shaped quantum well versus the phonton energy $h\nu$ for incident intensity $I=1$ mW/cm^3, 2 mW/cm^3, 4 mW/cm^3

In Fig. 3, it's plotted the value of total refractive index change versus the phonton energy $h\nu$ for incident intensity $I = 1$ mW/cm^3, 2 mW/cm^3, 4 mW/cm^3, respectively. When increasing incident intensity, the total refractive index change will be smaller concomitantly. So if we want to give a larger change of refractive index change in practice, a relatively weaker incident optical intensity should be adopted. It was also seen that the total refractive index change become large when the incident optical is equal to the value of 4 mW/cm^3. It also means adopting the value 4 mW/cm^3 have exceed the saturated value.

3 Conclusion

It' s present a simple and straightforward study of linear, nonlinear and total refractive index change in finite V-shaped quantum well by density matrix method. The main results show that the incident optical intensity effects on the nonlinear refractive index change, however the linear refractive index change is not related to the incident optical intensity. Furthermore the total refractive index change will become larger with increasing the incident optical intensity. Theoretical study may provide a power proof on experiment applies as regards practical applications on electro-optical devices.

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有限深V型势阱中总折射率的改变

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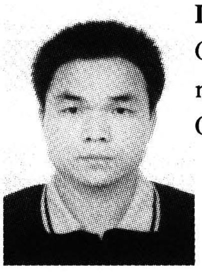
摘 要 研究了有限深V型势阱折射率改变，并且利用量子力学中的密度矩阵算符理论和迭代法导出了一次，三次谐波极化率系数．最后，以GaAs有限深V型势阱为例作了数值计算．数值结果表明，减少入射光强度，或增加电子浓度使总折射率改变变大．

关键词 有限V型势阱；光折射率；谐波产生



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