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Study of Geometrical Optics on Left-handed Material (I) *

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Abstract Properties of light propagating are studied at the interface between right-handed material (RHM) and left-handed material (LHM) according to Fermat's principle, and the least-time principle is demonstrated unsuitable for the case anymore. The shape equations of single LHM refractive surface are derived on basis of perfect imaging condition. Defects of a LHM slab lens are also pointed out with an improved compound lens is recommended intentionally .

Keywords Left-handed materials; Fermat' s principle; Least-time principle; Perfect imaging condition; Slab lens

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0 Introduction

Recently, there have been many studies about meta-materials that have simultaneously negative electrical permittivity and magnetic permeability. These materials, called left-handed materials (LHMs), theoretically discussed by Veselago[1], have unique electrodynamic properties such as the reversal of Snell' s law, the Doppler effect, the Cherenkov radiation and negative refractive index *n*. LH materials are often made by arrays of metallic wires and arrays of split-ring resonators[2] or planar transmission lines periodically loaded with series capacitors and shunt inductors[3]. Pendry has proposed the intriguing possibility that LHM slab might overcome known problems with common lenses to achieve a ' perfect ' lens that would focus the entire spectra[4]. Although the concept of a perfect lens is a result of an ideal theoretical model, the sub-wavelength resolution, which is deteriorated by other factors including losses, spatial dispersion and others which can be indeed much better than that of a conventional lens.

Materials with negative refractive index have not yet been found or demonstrated using existing materials or compounds. Composite materials, however, can be engineered to have the unique property. The reflection and refraction law still hold true in LHM. Fresnel formula at the interface between LHM and RHM has been extensively investigated by Yang et al[5]. The sub- wavelength imaging quality of a superlens is studied numerically by Nicholas Fang and Xiang Zhang[6]. In this paper, further studies on LHM are investigated in view of Geometrical Optics. The research we pursuing is to reveal how negative refractive index leads to a variety of surprising new phenomena and the possibility of applications in the optical field.

1 Fermat's principle at the interface between RHM and LHM

Fermat' s principle states that the passage of a light ray between two points is the path that has the extreme optical length, which means that the light spreads from one point of space to another on the most short, long or equal path. In mostly common case, the optical length between points *A* and *B* can be formulated by the integral equation,

$$L=\int_{A}^{B} n \cdot \mathrm{d}l \qquad (1)$$

where L is the optical length, n is the refractive index along the way of propagation, A and B are two fixed points. From Fermat' s principle, the variation of optical length, δL, is mathematically equal to zero in deed.

$$\delta L=\delta\int_{A}^{B} n \cdot \mathrm{d}l=0 \qquad (2)$$

In the familiar case that light passes through the planar interface between two right-handed media, as depicted in Fig. 1 (a), the light ray follows the path (AO_2B) of least time that obeys the Snell's law which is known widely as the least-time principle. However, it is correct for the precondition that refractive indices n_1, n_2, of the two media are both positive and the same as the sign of each part' s optical length. If one of the mediums is LHM with a negative index, the least-time principle doesn't hold true anymore. This will be demonstrated in the subsequence.

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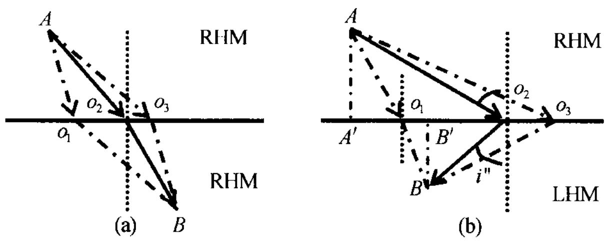

Fig. 1 Passage of rays through a planar boundary between two RHMs (a) or RHM and LHM(b)

As illustrated vividly with Fig. 1 (b), the light propagating through the interface between RHM and LHM follows the route AO_2B. Indeed, supposing for example that $AA'=a$, $BB'=b$ then optical length from the fixed point A to B can be calculated as follows

$$L=\frac{n_1 a}{\cos i}+\frac{n_2 b}{\cos i''} \tag{3}$$

and given that

$$A'B'=a\cdot\tan i+b\cdot\tan i''=l \tag{4}$$

where i is the incident angle, i'' is the refractive angle, and l is the settled horizontal distance from point A to B. From Fermat's principle, get

$$(\mathrm{d}L/\mathrm{d}i)=0 \tag{5}$$

Substituting equation (3) into the upper equation (5) yields (after equating the terms) that

$$\frac{n_1 a\sin i}{\cos^2 i}=-\frac{n_2 b\sin i''}{\cos i''} \tag{6}$$

and the differential form of equation (4) can be easily obtained as

$$\frac{\mathrm{d}i''}{\mathrm{d}i}=-\frac{a\cdot\cos^2 i''}{b\cdot\cos^2 i} \tag{7}$$

Combining the expressions (6) and (7) , one obtains

$$n_1\sin i=n_2\sin i'' \tag{8}$$

It demonstrates Snell′s law formula is still hold true in systems containing optical parts made by LHMs. Notice that $n_1>0$, $n_2<0$, which means that $i''<0$, Thus the incident and refractive rays locate on the same side of the normal line, whereas in the traditional Geometrical Optics they always lie oppositely to the normal line as shown in Fig. 1 (a). Then how is the optical length of the path that light exactly propagates by different from its neighbouring ones? With a further supposition that $n_1=1$, $n_2=-1$ and $b=0.5a$, $l=a$, one can calculate, from Fig. 1 (b), that the optical length written out as a function of the incident angle i is

$$L=a\left\{\frac{1}{\cos i}-\frac{1}{2\cos\left[\arctan\left(2-2\tan i\right)\right]}\right\} \tag{9}$$

The complicated relation between L and i can be elucidated by the curve in the way of numerical calculation as plotted in Fig. 2 . Looking into the curve by attention , it ′ s found that the horizontal

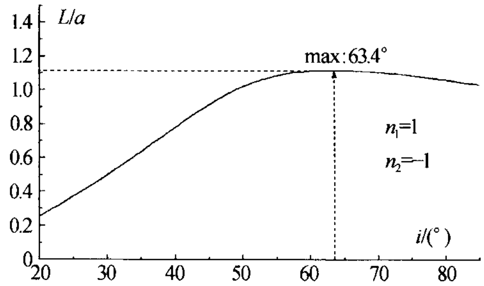

Fig. 2 Relative optical length L/a versus the incident angle i for the RHM-LHM case

coordinate of the vertex, 63. 4°, corresponds to the angle of incidence that has the maximum value. By substituting the acquired incident angle $i=63.4°$ into the simplified equation (2), that yields $\tan i+0.5\tan i''=1$, one gets $i''=-63.4°$ which validates the correctness of Snell′ s law. As mentioned above, the optical length of the path beam actually propagates through the boundary between RHM and LHM is the longest comparing to others in close vicinity. On the contrary, it is known well that the optical length of actual path for the RHM-RHM boundary case is the shortest way. One also can figure out effortlessly from Fig. 1 (b) that the time consumed for the actual path AO_2B is in between that of other ones (e. g. AO_1B, AO_2B) where the Fermat′ s least-time principle doesn′ t suit any longer. In fact, the correct formula for reckoning the consumed time is as follows

$$t=\frac{L'}{c}=\frac{\int_A^B |n|\,\mathrm{d}l}{c} \tag{10}$$

So Fermat′ s least-time principle is no longer applicable to system containing optical parts made by LHM.

2 Perfect imaging condition for the single LHM camber surface

When light rays from the source point A in air strike the surface of, say, LHM, they penetrate through the arbitrary boundary and congregate into an imaging point A'. If one want to make a perfect image of A at A', as shown in Fig. 3, the perfect

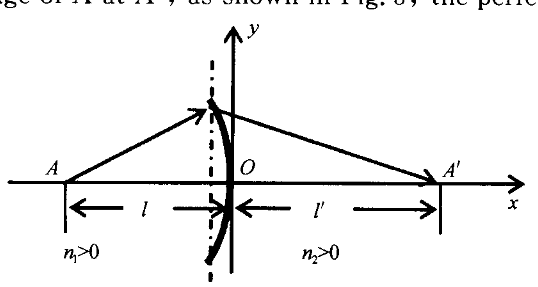

Fig. 3 Passage of light rays through the single camber surface with RHM in left and LHM in right

imaging condition that optical lengths of all rays' path from A to A' are equal to each other should be satisfied[7], which can be formulated as

$$AA' = n_1 \cdot AH + n_2 \cdot HA' = \text{Costant} \qquad (11)$$

By Setting the origin at the vertex of camber surface and x axis parallelingto AA' construct the Cartesian coordinate and making the assumption that $AO = l$, $OA' = l'$, one obtains the analytical form of equation (11) that can be written as

$$n_1 \sqrt{(l+x)^2+y^2} + n_2 \sqrt{(l'-x)+y^2} = n_1 l + n_2 l'$$

or

$$n_2[l' - \sqrt{(l'-x)^2+y^2}] + n_1[l - \sqrt{(l+x)^2+y^2}] = 0 \qquad (12)$$

Commonly this is a quartic equation with an ovate shape when plotted in curve. The camber surface that makes the beams' paths of equal optical length between A and A' is acquired by turning around the ovate curve on x axis. In view of probable application, then, to analyse two peculiar cases which are mostly applied in practice.

1) $n_1 = -n_2$, $l = l'$. By substituting them into equation (12), one finds, after uncomplex calculation, that the surface turns to a planar one through the origin described as $x=0$ (see in Fig. 4 (a)). Thus, single plane may be used for real imaging and a slab made by LH material can be a 'perfect' lens[4].

2) $n_1 = -n_2$, $l = kl'$, $(k>0)$, where k is the proportional constant. From equation (12), one gets

$$\left[\left(\frac{k+1}{k-1}\right)^2 - 1\right]x^2 - 2\left[\left(\frac{k+1}{k-1}\right) - 1\right]lx - y^2 = 0 \qquad (13)$$

which illuminates that surface of equal optical length is the left branch of a hyperbolic one passing through the origin (right branch is omitted for the unfeasible reason). Generatrix of the hyperbolic surface is portrayed visually in Fig. 4 (b) where $k=2$.

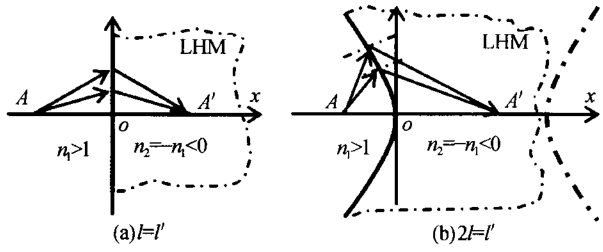

Fig. 4 Single surface imaging with LHM in right areas

In Fig. 4 (a), points A and A' are symmetric to the planar boundary that is similar to the imaging property of a mirror. As a matter of fact in all optical parts made purely by RHM, the planar mirror is the only one that can make a perfect image, that is, it can eliminate all kinds of aberrations. Unfortunately, the image it comes into being is a virtual one that extremely confines its applicable fields. However, the emergence of LHM fetches up the demerit absolutely for a 'LHM mirror' achieves a real image. Contrary to the traditional opinion, a concave lens made by LHM is able to converge light beams as elucidated in Fig. 4 (b). All this offers another brand-new way to make lenses and lets the choice of optical parts in system design more abundant.

3 Defects and improvement of the 'perfect' slab lens

The concept of a perfect lens was first introduced by Pendry[4], who suggested the modish idea that a slab of a lossless LHM (after the precedent work from Veselago), shown in Fig. 5, can be used for creating a perfect image of a point source as mentioned previously in section 2.

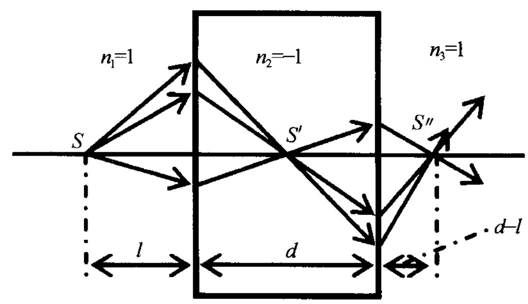

Fig. 5 Imaging diagram of a LHM slab

According to the same absolute value for the two refractive indices, but opposite in sign, on both sides of the interface, light rays emitting from the source point S with different incident angles have the same optical length, zero. Therefore, the LHM slab meets the requirement that a perfect imaging needed. However, there are some defects with the LHM slab except for the inherent dispersion in view of practical applications. Firstly, the position of two imaging points with one inner and another outer of the slab implies that the distance between S and S'' is twice as long as the thickness of slab. When it is used for long-distance imaging, the LHM slab used should be considerable thick that might be unrealisable. Secondly, though the optical lengths of different light rays are equal to each other with the zero value, the time they spent varies according to the incident angles: the acuter the angle is the more time consumed. Thus, when the objective is an ultra short pulse of light or moves in high speed, the form of focused imaging will be distorted in the time domain where waveform of the pulse detected is widened. This shortcoming is not naturally possessed in usual lenses made by RHM, for the equal optical length means the same time spent.

For fixed point-to-point imaging application, the scheme to solve the problems listed above is represented in Fig. 6.

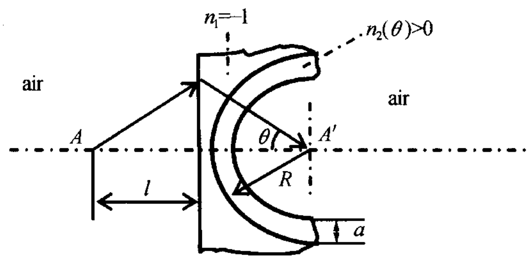

Fig. 6 Sketch for an improved compound lens including a plano-concave LHM lens and a crust-like RHM lens

The compound lens shown in Fig. 6 consists of two lenses. Left one is a plano-concave lens with a negative refractive index, $n_1 = -1$, and radius of the spherical surface is R. Right part is a crust-like lens with varied positive refractive index as a function of the angle, θ, and the thickness of the crust is a. By Supposing that the objective distance is l, and setting the spherical centre of the compound lens at the image point A', one can quickly write out equation (14) to calculate the time used.

$$\frac{2l-a}{c}+\frac{n_2(0)a}{c}=\frac{2l/\cos\theta-a}{c}+\frac{n_2(\theta)a}{c} \tag{14}$$

Thus, the refractive index of right lens is

$$n_2(\theta)=n_2(0)+\frac{2l}{a}\left(1-\frac{1}{\cos\theta}\right) \tag{15}$$

This compound lens can make all light rays with equal time spent when propagating from A to A' and so as to eliminate the distortion in time domain clearly. Further more, the lens with a part made by LHM restores not only the phase of propagating waves but also the amplitude of evanescent states, which means that better resolution is achieved.

For thin lenses, Geometrical Optics gives the result that the focal length f is related to the lens's radius of curvature, R, by $f=R/(n-1)$. A material with $n=+1$ does not refract optical fields, whereas a material with $n=-1$ does. The result is that negative-index lenses can be more compact, with a host of other benefits[8]. As a matter of fact, by grinding the lens to specific convex or concave angles, these Left-handed materials combined with traditional materials should be able to serve all lens needs with easy-to-make planar surfaces. Furthermore, the spherical aberration of a slab lens for given thickness d and incident angle i can be formulated as

$$\delta L'=\frac{n^2-1}{2n^3}\cdot d\cdot i^2 \tag{16}$$

where n is the slab's refractive index. Thus the spherical aberration is determined by n and a LHM slab lens has negative value of it while for a RHM slab lens it is positive. Then one can take the easy way to completely eliminate spherical aberration by combining the RHM and LHM slabs to a compound one as shown in Fig. 7, which can also let the image be focused in where one likes by altering the thickness of the right slab.

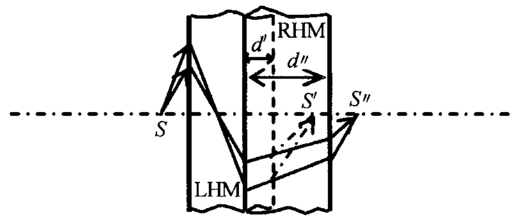

Fig. 7 A compound slab lens made by LHM and RHM

4 Conclusion

In fact, new materials with negative indices are rewriting the laws of optics and could lead to a perfect lens with a resolution not limited by wavelength. Here, Format's principle in Geometrical optics is studied in detail at the interface between RHM and LHM showing that the least-time principle doesn't serve the turn anymore. The ovate equations of single LHM refractive surface are derived from the perfect imaging condition showing the probably shapes for LHM lenses. Some defects of the 'perfect' LHM slab lens are pointed out and the solution is recommended with a compound lens. Optical experiments using arrays of nanowires are already demonstrating that the concept of a negative refractive index could be realized in practical systems. With the development of technologies in fabricating LHM, studies of Geometrical Optics about LHM is crucial in order to design new kinds of optical parts. Extensive researches on LHM[9~11] are needed in view of optical applications and what can be surely point out is that the notion of negative refraction often leads to possibilities that go against intuition. In future papers, the aberrations of compound lenses made by LHM to advance the design of LHM optical lenses will be presented in earnest.

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关于左手性介质几何光学的研究(一)

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摘　要　根据几何光学中的费马原理和完善成像原理研究了光线经过正负折射率界面时的传播特性，推导了单负折射率完善成像的曲面方程，讨论了单块负折射率透镜成像特性的一些缺陷及改进方案，依此指出了最短时间原理的适用范围的局限性和利用左手性介质制作光学器件的优越性，为进一步研究左手性介质的光学特性和相关光学器件设计提供了理论基础.

关键词　左手性材料；费马原理；最短时间原理；完善成像条件；平板透镜

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