差频瑞利增强极化拍频研究*

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摘 要 基于马尔可夫随机场模型的理论给出了差频瑞利增强极化拍(REPB)的解析解，并且研究了三种马尔可夫随机场模型下场关联效应和瑞利型共振增强效应对拍频信号的影响，指出了振幅涨落和位相涨落在时域和频域的不同作用．差频 REPB 信号展现出了时域对称特性，信号在三种马尔可夫随机场模型下的差别随着带宽的增大逐渐消失．

关键词 差频瑞利增强极化拍；场关联效应；位相涨落；振幅涨落

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0 引言

噪音光作为一种可用于探测原子和分子的动力学过程独特技术手段，兼有传统的频域窄带光谱和时域飞秒超短脉冲光谱的特点[1~3]．描述激光场的噪音特性可以用很多不同的随机模型，它们都有相同的二阶场关联函数，不同仅存在于高阶场关联函数．当激光场足够强以至于可发生多光子的相互作用时，从二阶相关函数获得的激光光谱线型已不足以描述光场相互作用的特征，所以必须采用高阶相关函数来描述．本文采用马尔可夫随机模型来描述激光场，原子对马尔可夫随机光场的响应现在已被广泛的了解[4,5]．马尔可夫随机光场因造成涨落的原因不同而分为混沌场模型，布朗相位扩散场模型和实高斯场模型，它们有不同的四阶相关函数．

瑞利型增强极化是一种普遍存在于光克尔介质中的三阶非线性过程，对它的研究将有助于揭示发生在液体物质内部的超快动力学过程．本文基于马尔可夫随机场模型在理论上首次给出了研究了瑞利增强极化的差频瑞利增强极化拍(REPB)，首次给出了拍频信号的解析解，并且比较了三种马尔可夫随机场模型下场关联效应和瑞利型共振增强效应对拍频信号的影响，并详细报道了研究结果．

1 基本原理

瑞利型共振增强极化拍频是一种三阶非线性极化现象．如图 1，孪生光束 1 和 2 由相同的中心频率成分 ω_1 和 ω_2 组成，且它们之间有一个小夹角，光束 3 的频率为 ω_3，它沿光束 1 的相反方向传播．在光克尔介质中(无热栅效应)，光束 1 和 2 在介质中的非线性相互作用会感生非共振静态栅 G_1 和 G_2．光束 3 被这两个静态栅衍射，产生的两个四波混频信号(NDFWM)的频率均为 ω_3，它们沿几乎与光束 2 的相反方向前进．另外，光束 3 和 2 在介质中会感生两个大角度动态栅，如果使两束光频率失谐量 $\Delta=\omega_3-\omega_1$ 远小于 $\Delta'=\omega_3-\omega_2$ ($\Delta'\ll\Delta$ 且 $\Delta\approx0$)，则可忽略光束2的ω_2分量与光束3形成的栅的影响．而光束 2 的 ω_1 分量与光束 3 在介质中感生的动态栅对光束 1 中 ω_1 分量的衍射就会增强由 G_1 产生的 NDFWM 信号，将这个增强过程称为瑞利增强非简并四波混频(RENDFWM)．最终的 REPB 信号由这个 RENDFWM 信号和 G_2 产生的 NDFWM 信号干涉形成，其方向几乎与光束 2 反向．

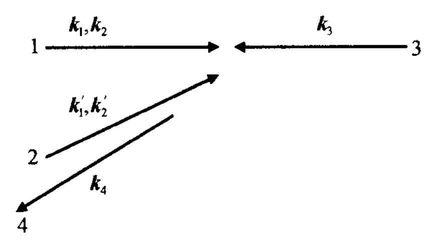

图 1 混频光路

Fig. 1 Diagram of the geometry of RENDFWM

激光场的复数光场可以写为

$$E_{p1}=E_1(\boldsymbol{r},t)+E_2(\boldsymbol{r},t)=A_1(\boldsymbol{r},t)\exp(-\mathrm{i}\omega_1 t)+A_2(\boldsymbol{r},t)\exp(-\mathrm{i}\omega_2 t)=\varepsilon_1 u_1(t)\exp[\mathrm{i}(\boldsymbol{k}_1\cdot\boldsymbol{r}-\omega_1 t)]+\varepsilon_2 u_2(t)\exp[\mathrm{i}(\boldsymbol{k}_2\cdot\boldsymbol{r}-\omega_2 t)] \quad (1)$$

$$E_{p2}=E_1'(\boldsymbol{r},t)+E_2'(\boldsymbol{r},t)=A_1'(\boldsymbol{r},t)\exp(-\mathrm{i}\omega_1 t)+A_2'(\boldsymbol{r},t)\exp(-\mathrm{i}\omega_2 t)=\varepsilon_1' u_1(t-\tau)\exp[\mathrm{i}(\boldsymbol{k}_1'\cdot\boldsymbol{r}-\omega_1 t+\omega_1\tau)]+\varepsilon_2' u_2(t-\tau)\exp[\mathrm{i}(\boldsymbol{k}_2'\cdot\boldsymbol{r}-\omega_2 t+\omega_2\tau)] \quad (2)$$

式中 $\varepsilon_i,\boldsymbol{k}_i(\varepsilon_i',\boldsymbol{k}_i')$分别是第 i 个光束的常量场振幅和波矢，$u_i(t)$是包含了位相和振幅起伏的无量纲因子，是 t 的复遍历随机函数，它服从混沌场的高斯统计分布．τ 是光束光 1 相对于光束 2 的时间延迟量．

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另外,光束 3 的复电场可以写为

$$E_3(\boldsymbol{r},t)=A_3(\boldsymbol{r},t)\exp(-\mathrm{i}\omega_3 t)=\varepsilon_3 u_3(t)\cdot \exp[\mathrm{i}(\boldsymbol{k}_3\cdot\boldsymbol{r}-\omega_3 t)] \tag{3}$$

由光束 1 和 2 感生的静态栅的序参量满足如下方程

$$\frac{\mathrm{d}Q_1}{\mathrm{d}t}+\gamma Q_1=\chi\gamma E_1(\boldsymbol{r},t)[E_1'(\boldsymbol{r},t)]^* \tag{4}$$

$$\frac{\mathrm{d}Q_2}{\mathrm{d}t}+\gamma Q_2=\chi\gamma E_2(\boldsymbol{r},t)[E_2'(\boldsymbol{r},t)]^* \tag{5}$$

式中 γ 和 χ 是两栅的弛豫速率和非线性极化率. 而光束 2 和 3 干涉形成的大角度动态栅的序参量满足方程

$$\frac{\mathrm{d}Q_3}{\mathrm{d}t}+\gamma Q_3=\chi\gamma[E_1'(\boldsymbol{r},t)]^* E_3(\boldsymbol{r},t) \tag{6}$$

所以各栅对应的非线性极化为

$$P_1=Q_1(\boldsymbol{r},t)E_3(\boldsymbol{r},t)=S_1(\boldsymbol{r})\int_0^\infty u_1(t-t')\cdot u_1^*(t-t'-\tau)u_3(t)\exp(-\gamma t')\mathrm{d}t' \tag{7}$$

$$P_2=Q_2(\boldsymbol{r},t)E_3(\boldsymbol{r},t)=S_2(\boldsymbol{r})\int_0^\infty u_2(t-t')\cdot u_2^*(t-t'-\tau)u_3(t)\exp(-\gamma t')\mathrm{d}t' \tag{8}$$

$$P_3=Q_3(\boldsymbol{r},t)E_1(\boldsymbol{r},t)=S_1(\boldsymbol{r})\int_0^\infty u_1^*(t-t'-\tau)\cdot u_3(t-t')u_1(t)\exp[-(\gamma-\mathrm{i}\Delta)t']\mathrm{d}t' \tag{9}$$

式中

$$S_1(\boldsymbol{r})=\chi\gamma\varepsilon_1(\varepsilon_1')^*\varepsilon_3\exp\{\mathrm{i}[(\boldsymbol{k}_1-\boldsymbol{k}_1'+\boldsymbol{k}_3)\cdot\boldsymbol{r}-\omega_3 t-\omega_1\tau]\}$$

$$S_2(\boldsymbol{r})=\chi\gamma\varepsilon_2(\varepsilon_2')^*\varepsilon_3\exp\{\mathrm{i}[(\boldsymbol{k}_2-\boldsymbol{k}_2'+\boldsymbol{k}_3)\cdot\boldsymbol{r}-\omega_3 t-\omega_2\tau]\}$$

式中三阶非线性极化 P_1, P_3 和 P_2 有相同的频率 ω_3, P_1+P_3 和 P_2 分别对应 RENDFWM 过程和 NDFWM 过程,其波矢分别为 $\boldsymbol{k}_1-\boldsymbol{k}_1'+\boldsymbol{k}_3$ 和 $\boldsymbol{k}_2-\boldsymbol{k}_2'+\boldsymbol{k}_3$.

2 理论计算

在整个随机过程中 NDFWM 信号正比于 $P^{(3)}$ 绝对值平方的平均值,即 $\langle|P^{(3)}|^2\rangle$[6],而总的三阶极化强度为 $P^{(3)}=P_1+P_2+P_3$,所以

$$I(\Delta,\tau)\propto\langle|P^{(3)}|^2\rangle=\langle P^{(3)}(P^{(3)})^*\rangle=\langle(P_1+P_2+P_3)[(P_1)^*+(P_2)^*+(P_3)^*\rangle$$

不同的马尔可夫随机场模型只会影响四阶场关联函数,对二阶无影响[7,8]. 如果激光光源具有洛伦兹线型,则有[5]

$$\langle u_i(t_1)u_i^*(t_2)\rangle=\exp(-\alpha_i|t_1-t_2|) \tag{10}$$

式(10)中 $\alpha_i=\frac{1}{2}\delta\omega_i$, $\delta\omega_i$ 为 ω_i 的激光线宽.

2.1 混沌场模型

混沌场通常被用来描述振幅和相位都发生涨落的多模激光源. 假定激光源是混沌场,此时 $u_i(t)$ 服从高斯统计,其四阶相干函数满足

$$\langle u_i(t_1)u_i(t_2)u_i^*(t_3)u_i^*(t_4)\rangle=\langle u_i(t_1)u_i^*(t_3)\rangle\cdot\langle u_i(t_2)u_i^*(t_4)\rangle+\langle u_i(t_1)u_i^*(t_4)\rangle\langle u_i(t_2)u_i^*(t_3)\rangle \tag{11}$$

经过运算,得到

1) $\tau>0$ 时

$$I(\Delta,\tau)\propto\chi^2\{B+\eta^2[(1+\frac{2\gamma_a\gamma^2}{(\gamma_a^2+\Delta^2)\gamma}+\frac{\gamma_a}{(\Delta^2+\gamma_a^2)(\gamma+\alpha_1)})\exp(-2\alpha_1|\tau|)+\gamma^2(\frac{\gamma_a}{(\gamma_a^2+\Delta^2)\gamma}+\frac{2(\gamma\gamma_a+\gamma_a^2-\Delta^2)}{(\gamma_a^2+\Delta^2)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{2(\gamma+\gamma_a)}{(\gamma+2\alpha_1)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{1}{\gamma(\gamma+2\alpha_1)}]+\eta(A+A^*+\frac{A}{\gamma_a-i\Delta}+\frac{A^*}{\gamma_a+i\Delta})\cdot\mathrm{e}^{-(\alpha_1+\alpha_2)|\tau|}\} \tag{12}$$

当 α_1, $\alpha_2\ll\gamma$ 时,式(12)化简为

$$I(\Delta,\tau)\propto\chi^2\{1+\exp(-2\alpha_2|\tau|)+\eta^2[\frac{(4\gamma^2+\Delta^2)}{(\gamma^2+\Delta^2)}\exp(-2\alpha_1|\tau|)+\frac{(4\gamma^2+\Delta^2)}{(\gamma^2+\Delta^2)}]+\eta(A+A^*+\frac{A}{\gamma-\mathrm{i}\Delta}+\frac{A^*}{\gamma+\mathrm{i}\Delta})\mathrm{e}^{-(\alpha_1+\alpha_2)|\tau|}\} \tag{12a}$$

2) $\tau<0$ 时

$$I(\Delta,\tau)\propto\chi^2\{B+\eta^2\gamma^2[\frac{1}{\gamma^2}+\frac{2\gamma_b}{\gamma(\gamma_b^2+\Delta^2)}+\frac{1}{(\gamma_b+\mathrm{i}\Delta)(\gamma_c-\mathrm{i}\Delta)}+\frac{-\gamma_b+\gamma_c}{2(\gamma_b+\mathrm{i}\Delta)(\gamma_c+\mathrm{i}\Delta)(\gamma-\alpha_1)}]\cdot\exp(-2\alpha_1|\tau|)+\gamma^2\eta^2[\frac{\gamma_a}{(\gamma_a^2+\Delta^2)\gamma}+\frac{2(\gamma\gamma_a+\gamma_a^2-\Delta^2)}{(\gamma_a^2+\Delta^2)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{2(\gamma+\gamma_a)}{(\gamma+2\alpha_1)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{1}{\gamma(\gamma+2\alpha_1)}]+\eta^2\gamma^2[\frac{-2\gamma_a\gamma_c+2\Delta^2}{(\gamma_b^2+\Delta^2)(\gamma_c^2+\Delta^2)}-\frac{-2\gamma-\gamma_b+\gamma_c+2\alpha_1}{2(\gamma_b+\mathrm{i}\Delta)(\gamma_c+i\Delta)(\gamma-\alpha_1)}+\frac{\gamma_a}{(\gamma_a^2+\Delta^2)(\gamma+\alpha_1)}]\cdot\exp(-2\gamma|\tau|)+\eta^2\gamma^2[\frac{(\gamma_b-\gamma_a)(\gamma+\gamma_c+\mathrm{i}\Delta)\exp-(\gamma_a+\mathrm{i}\Delta)|\tau|}{\gamma(\gamma_a+\mathrm{i}\Delta)(\gamma_b+\mathrm{i}\Delta)(\gamma_c+\mathrm{i}\Delta)}+\frac{(\gamma_b-\gamma_a)(\gamma+\gamma_c-\mathrm{i}\Delta)\exp-(\gamma_a-\mathrm{i}\Delta)|\tau|}{\gamma(\gamma_a-\mathrm{i}\Delta)(\gamma_b-\mathrm{i}\Delta)(\gamma_c-\mathrm{i}\Delta)}]+\eta(A+A^*+\frac{A}{\gamma_b-\mathrm{i}\Delta}+\frac{A^*}{\gamma_b+\mathrm{i}\Delta}-\frac{2\gamma\alpha_1\mathrm{e}^{-(\gamma_b-\mathrm{i}\Delta)|\tau|}A}{(\gamma_b-\mathrm{i}\Delta)(\gamma_a-\mathrm{i}\Delta)}-\frac{2\gamma\alpha_1\mathrm{e}^{-(\gamma_b+\mathrm{i}\Delta)|\tau|}A^*}{(\gamma_b+\mathrm{i}\Delta)(\gamma_a+\mathrm{i}\Delta)})\mathrm{e}^{-(\alpha_1+\alpha_2)|\tau|}\} \tag{13}$$

式中，$\eta=\varepsilon_1'^*\varepsilon_1/\varepsilon_2'^*\varepsilon_2(\varepsilon_1\approx\varepsilon_{1'}',\varepsilon_2\approx\varepsilon_2')$；$\Delta\boldsymbol{k}=(\boldsymbol{k}_1-\boldsymbol{k}_1')-(\boldsymbol{k}_2-\boldsymbol{k}_2')$；$\gamma_a=\alpha_1+\alpha_3+\gamma$；$\gamma_b=-\alpha_1+\alpha_3+\gamma$；$\gamma_c=-\alpha_1-\alpha_3+\gamma$；$B=\mathrm{e}^{-2\alpha_2|\tau|}+\frac{\gamma}{\gamma+2\alpha_2}$；$A=\mathrm{e}^{\mathrm{i}\Delta\boldsymbol{k}\cdot\boldsymbol{r}-\mathrm{i}(\omega_1-\omega_2)\tau}=\mathrm{e}^{\mathrm{i}\theta}$. 差频 REPB 通常在 $\tau>0$ 与 $\tau<0$ 时是不同的．但是，当$|\tau|\to\infty$时，式(12)与式(13)相等．因为当$|\tau|\to\infty$时，光束 1 与 2 完全不相干，故 τ 的正负不会影响信号．REPB 的特点由物质响应(共振项)与光场响应(非共振项)的相互作用确定，所以式(12)与式(13)不仅反映了孪生光场的性质，也包含了分子振动的特点．$\tau>0$ 时，REPB 的时域变化主要反映激光场的特性，$\tau<0$ 时则反映分子振动特性．差频 REPB 信号随 τ 的变化展现出以 $\omega_2-\omega_1$ 为频率的调制，同时以 $\alpha_1+\alpha_2$ 为速率衰减．由式(12)与式(13)还可看出拍频信号不仅有时域振荡，而且在空间中沿 $\Delta\boldsymbol{k}$ 方向(几乎垂直于信号传播方向)也有周期为 $2\pi/\Delta k$ 的振荡．其中 $\Delta k\approx2\pi|\lambda_1-\lambda_3|\theta/\lambda_3\lambda_1$，$\theta$ 是光束 1 与 2 的夹角．通常极化拍模型都假定孪生光束是平面波，所以分别沿 $\boldsymbol{k}_1-\boldsymbol{k}_1'+\boldsymbol{k}_3$ 和 $\boldsymbol{k}_2-\boldsymbol{k}_2'+\boldsymbol{k}_3$ 方向的 RENFWM 信号与 NFWM 信号也是平面波，它们的传播方向略有不同，其干涉会导致拍信号在空域的振荡．

2.2 布朗相位扩散模型

本文用相散模型来计算拍频信号的强度. 相散模型通常用来描述有稳定振幅的激光源．此模型假定激光场的振幅是一个常量，而位相则做随机涨落. 如果激光光源具有洛伦兹线型，则此模型下的四阶相关函数满足

$$\langle u_i(t_1)u_i(t_2)u_i^*(t_3)u_i^*(t_4)\rangle=\exp\left[-\alpha_i(|t_1-t_3|+|t_1-t_4|+|t_2-t_3|+|t_2-t_4|)\right]\cdot\exp\left[\alpha_i(|t_1-t_2|+|t_3-t_4|)\right]\tag{14}$$

将式(10)，(14)代入式 $I(\Delta,\tau)\propto\langle|P^{(3)}|^2\rangle$，得到

1) $\tau>0$ 时

$$I(\Delta,\tau)\propto\chi^2\{B_1+\eta^2\gamma^2[(\frac{\gamma_a}{(\gamma_a^2+\Delta^2)\gamma}+\frac{2(\gamma\gamma_a+\gamma_a^2-\Delta^2)}{(\gamma_a^2+\Delta^2)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{2(\gamma+\gamma_a)}{(\gamma+2\alpha_1)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{1}{\gamma(\gamma+2\alpha_1)})+(\frac{4\alpha_1(\gamma+\gamma_a)}{\gamma(\gamma+2\alpha_1)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{2\alpha_1}{\gamma^2(\gamma+2\alpha_1)})e^{-(\gamma+2\alpha_1)|\tau|}]+\eta[(A+A^*+\frac{A}{\gamma_a-\mathrm{i}\Delta}+\frac{A^*}{\gamma_a+\mathrm{i}\Delta})\mathrm{e}^{-(\alpha_1+\alpha_2)|\tau|}]\}\tag{15}$$

当 $\alpha_1,\alpha_2\ll\gamma$ 时，式(15)化简为

$$I(\Delta,\tau)\propto\chi^2\{1+\eta^2\frac{(4\gamma^2+\Delta^2)}{(\gamma^2+\Delta^2)}+\eta(A+A^*+\frac{A}{\gamma-\mathrm{i}\Delta}+\frac{A^*}{\gamma+\mathrm{i}\Delta})\mathrm{e}^{-(\alpha_1+\alpha_2)|\tau|}\}\tag{15a}$$

2) $\tau<0$ 时

$$\begin{aligned}I(\Delta,\tau)\propto\chi^2\{&B_1+\gamma^2\eta^2[\frac{\gamma_a}{(\gamma_a^2+\Delta^2)\gamma}+\frac{2(\gamma\gamma_a+\gamma_a^2-\Delta^2)}{(\gamma_a^2+\Delta^2)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{2(\gamma+\gamma_a)}{(\gamma+2\alpha_1)((\gamma+\gamma_a)^2+\Delta^2)}+\\&\frac{1}{\gamma(\gamma+2\alpha_1)}]+\eta^2\gamma^2[-\frac{2\alpha_1\exp-(\gamma_a+\mathrm{i}\Delta)|\tau|}{\gamma(\gamma_a+\mathrm{i}\Delta)(\gamma_a+\mathrm{i}\Delta+2\alpha_1)}-\frac{2\alpha_1\exp-(\gamma_a-\mathrm{i}\Delta)|\tau|}{\gamma(\gamma_a-\mathrm{i}\Delta)(\gamma_a-\mathrm{i}\Delta+2\alpha_1)}]+\\&\eta^2[\frac{4\alpha_1\gamma\gamma_b}{(\gamma+2\alpha_1)(\gamma_b^2+\Delta^2)}+\frac{2\alpha_1}{(\gamma+2\alpha_1)})\mathrm{e}^{-(\gamma+2\alpha_1)|\tau|}+\eta^2\gamma^2[\frac{4\alpha_1^2\exp-(3\gamma-2\mathrm{i}\Delta+2\alpha_3+4\alpha_1)|\tau|}{(\gamma_a-\mathrm{i}\Delta)(\gamma+\gamma_a-\mathrm{i}\Delta)(2\alpha_1+\gamma_a-\mathrm{i}\Delta)}+\\&\frac{4\alpha_1^2\exp-(3\gamma+2\mathrm{i}\Delta+2\alpha_3+4\alpha_1)|\tau|}{(\gamma_a+\mathrm{i}\Delta)(\gamma+\gamma_a+\mathrm{i}\Delta)(2\alpha_1+\gamma_a+\mathrm{i}\Delta)}]+\eta^2\gamma^2[-\frac{8\alpha_1^2\exp-(\gamma+\gamma_a-\mathrm{i}\Delta)|\tau|}{\gamma(\gamma_b-\mathrm{i}\Delta)(\gamma+\gamma_a-\mathrm{i}\Delta)(2\alpha_1+\gamma_a-\mathrm{i}\Delta)}-\\&\frac{8\alpha_1^2\exp-(\gamma+\gamma_a+\mathrm{i}\Delta)|\tau|}{\gamma(\gamma_b+\mathrm{i}\Delta)(\gamma+\gamma_a+\mathrm{i}\Delta)(2\alpha_1+\gamma_a+\mathrm{i}\Delta)}]+\eta(A+A^*+\frac{A}{\gamma_b-\mathrm{i}\Delta}+\frac{A^*}{\gamma_b+\mathrm{i}\Delta}-\frac{2\gamma\alpha_1\mathrm{e}^{-(\gamma_b-\mathrm{i}\Delta)|\tau|}A}{(\gamma_b-\mathrm{i}\Delta)(\gamma_a-\mathrm{i}\Delta)}-\\&\frac{2\gamma\alpha_1\mathrm{e}^{-(\gamma_b+\mathrm{i}\Delta)|\tau|}A^*}{(\gamma_b+\mathrm{i}\Delta)(\gamma_a+\mathrm{i}\Delta)})\mathrm{e}^{-(\alpha_1+\alpha_2)|\tau|}\end{aligned}\tag{16}$$

式中，$B_1=\frac{\gamma}{\gamma+2\alpha_2}+\frac{2\alpha_2}{\gamma+2\alpha_2}\mathrm{e}^{-(2\alpha_2+\gamma)|\tau|}$．式(15)表明 $\tau>0$ 时信号反映激光场的性质，$\tau<0$ 时，式(16)主要由分子振动特性确定．

2.3 实高斯模型

实高斯模型是指只有振幅涨落且振幅为实数的场．其四阶相关函数满足为

$$\langle u_i(t_1)u_i(t_2)u_i(t_3)u_i(t_4)\rangle=\langle u_i(t_1)u_i(t_3)\rangle\langle u_i(t_2)u_i(t_4)\rangle+\langle u_i(t_1)u_i(t_4)\rangle\langle u_i(t_2)u_i(t_3)\rangle+\langle u_i(t_1)u_i(t_2)\rangle\langle u_i(t_3)u_i(t_4)\rangle\tag{17}$$

将式(17)代入 $I(\Delta,\tau)\propto\langle|P^{(3)}|^2\rangle$，得到

1) $\tau>0$ 时

$$I(\Delta,\tau)\propto\chi^2\{B_2+\eta^2[(2+\frac{2\gamma_a\gamma^2}{(\gamma_a^2+\Delta^2)(\gamma+\alpha_1)}+\frac{4\gamma\gamma_a}{(\Delta^2+\gamma_a^2)})\exp(-2\alpha_1|\tau|)+\gamma^2(\frac{\gamma_a}{(\gamma_a^2+\Delta^2)\gamma}+ \frac{2(\gamma\gamma_a+\gamma_a^2-\Delta^2)}{(\gamma_a^2+\Delta^2)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{2(\gamma+\gamma_a)}{(\gamma+2\alpha_1)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{1}{\gamma(\gamma+2\alpha_1)})+(\frac{2\gamma^2(\gamma+\gamma_a)}{(\gamma+2\alpha_1((\gamma+\gamma_a)^2+\Delta^2)}- \frac{2\alpha_1}{(\gamma+2\alpha_1)}-\frac{2\gamma(\gamma+\gamma_a)}{(\gamma+\gamma_a)^2+\Delta^2})e^{-(\gamma+2\alpha_1)|\tau|}]+\eta[(A+A^*+\frac{A}{\gamma_a-i\Delta}+\frac{A^*}{\gamma_a+i\Delta})e^{-(\alpha_1+\alpha_2)|\tau|}]\} \tag{18}$$

当 $\alpha_1,\alpha_2\ll\gamma$ 时，式(18)化简为

$$I(\Delta,\tau)\propto\chi^2\{1+2\exp(-2\alpha_2|\tau|)+\eta^2[2\frac{(4\gamma^2+\Delta^2)}{(\gamma^2+\Delta^2)}\exp(-2\alpha_1|\tau|)+\frac{(4\gamma^2+\Delta^2)}{(\gamma^2+\Delta^2)}]+ \eta(A+A^*+\frac{A}{\gamma-i\Delta}+\frac{A^*}{\gamma+i\Delta})e^{-(\alpha_1+\alpha_2)|\tau|}\} \tag{18a}$$

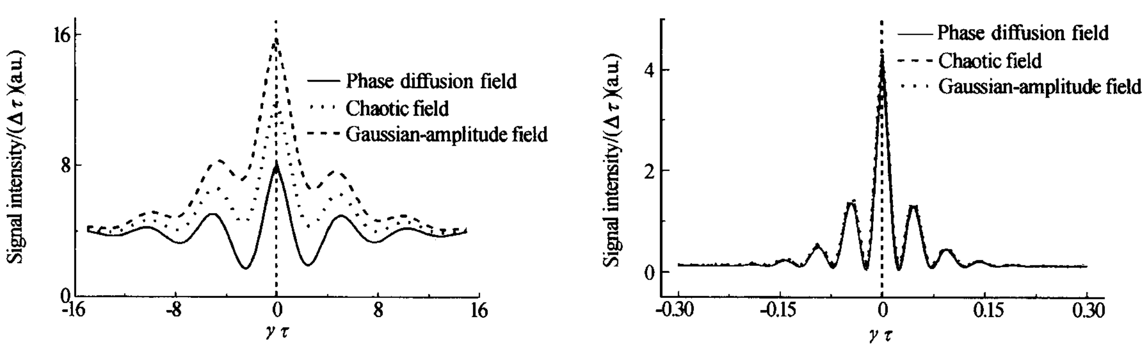
2) $\tau<0$ 时

$$I(\Delta,\tau)\propto\chi^2\{B_2+\eta^2[2+\frac{4\gamma\gamma_b}{(\gamma_b^2+\Delta^2)}+\frac{2\gamma_b\gamma^2}{(\gamma_b^2+\Delta^2)(\gamma-\alpha_1)}]\exp(-2\alpha_1|\tau|)+\gamma^2\eta^2[\frac{\gamma_a}{(\gamma_a^2+\Delta^2)\gamma}+ \frac{2(\gamma\gamma_a+\gamma_a^2-\Delta^2)}{(\gamma_a^2+\Delta^2)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{2(\gamma+\gamma_a)}{(\gamma+2\alpha_1)((\gamma+\gamma_a)^2+\Delta^2)}+\frac{1}{\gamma(\gamma+2\alpha_1)}]+ \eta^2\gamma^2[-\frac{4(\gamma^2+\Delta^2-(\alpha_1+\alpha_3)^2)}{(\gamma_a^2+\Delta^2)(\gamma_c^2+\Delta^2)}+\frac{2\gamma_c}{(\gamma_c^2+\Delta^2)(\gamma-\alpha_1)}+\frac{2\gamma_a}{(\gamma_a^2+\Delta^2)(\gamma+\alpha_1)}]\exp(-2\gamma|\tau|)+ \eta^2\gamma^2[-\frac{2(\gamma+\gamma_c-i\Delta)\alpha_1}{\gamma(\gamma_a+i\Delta)(\gamma_b+i\Delta)(\gamma_c-i\Delta)}-\frac{2(3\gamma-2\alpha_1)\alpha_1}{\gamma^2(\gamma_a+i\Delta)(\gamma_b+i\Delta)(\gamma_c-i\Delta)(\gamma+\gamma_b+i\Delta)}]\exp-(\gamma_a+ i\Delta)|\tau|+\eta^2\gamma^2[-\frac{2(\gamma+\gamma_c+i\Delta)\alpha_1}{\gamma(\gamma_a-i\Delta)(\gamma_b-i\Delta)(\gamma_c+i\Delta)}-\frac{2(3\gamma-2\alpha_1)\alpha_1}{\gamma^2(\gamma_a-i\Delta)(\gamma_b-i\Delta)(\gamma_c+i\Delta)(\gamma+\gamma_b-i\Delta)}]\cdot \exp-(\gamma_a-i\Delta)|\tau|+\eta^2(\frac{2\gamma^2\gamma_b}{(\gamma+2\alpha_1)(\gamma_b^2+\Delta^2)}-\frac{2\alpha_1}{(\gamma+2\alpha_1)}-\frac{2\gamma\gamma_b}{\gamma_b^2+\Delta^2})e^{-(\gamma+2\alpha_1)|\tau|}+ \eta^2\gamma^2[\frac{4\alpha_1^2\exp-(3\gamma-2i\Delta+2\alpha_3)|\tau|}{(\gamma_a-i\Delta)(\gamma_b-i\Delta)(\gamma+\gamma_a-i\Delta)(\gamma+\gamma_b-i\Delta)}+\frac{4\alpha_1^2\exp-(3\gamma+2i\Delta+2\alpha_3)|\tau|}{(\gamma_a+i\Delta)(\gamma_b+i\Delta)(\gamma+\gamma_a+i\Delta)(\gamma+\gamma_b+i\Delta)}]+ \eta(A+A^*+\frac{A}{\gamma_b-i\Delta}+\frac{A^*}{\gamma_b+i\Delta}-\frac{2\gamma\alpha_1e^{-(\gamma_b-i\Delta)|\tau|}A}{(\gamma_b-i\Delta)(\gamma_a-i\Delta)}-\frac{2\gamma\alpha_1e^{-(\gamma_b+i\Delta)|\tau|}A^*}{(\gamma_b+i\Delta)(\gamma_a+i\Delta)})e^{-(\alpha_1+\alpha_2)|\tau|} \tag{19}$$

式中 $B_2=2e^{-2\alpha_2|\tau|}+\frac{\gamma}{\gamma+2\alpha_2}+(-1+\frac{\gamma}{\gamma+2\alpha_2})e^{-(2\alpha_2+\gamma)|\tau|}$.

3 讨论

通过数值模拟分别做出三种马尔可夫随机场模型下信号光强度 $\eta=1,r=0,\frac{\Delta}{\alpha_1}=0,\frac{\alpha_3}{\gamma}=0.1$ 在时域(图 2(a),(b))和 $\eta=1,r=0$ 在频域的变化关系，见图 3，图 3(a)为 $\gamma\tau=0$ 时的曲线．

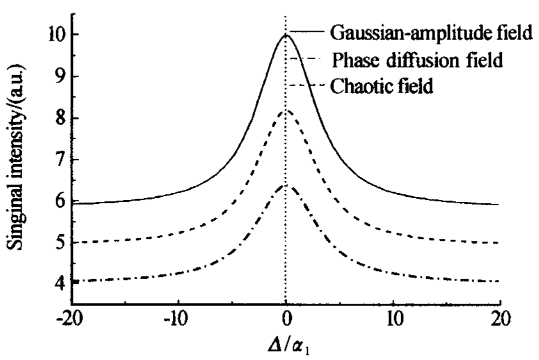


(a) $\frac{\alpha_1}{\gamma}=\frac{\alpha_2}{\gamma}=0.12$　　(b) $\frac{\alpha_1}{\gamma}=\frac{\alpha_2}{\gamma}=13$

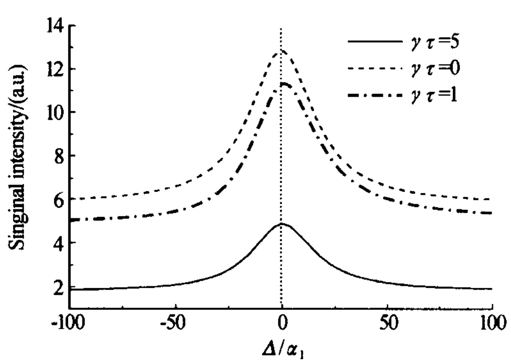
图 2　REPB 随 $\gamma\tau$ 变化

Fig. 2　Curve of the REPB veries with $\gamma\tau$

REPB信号在时域的变化主要反映场关联效应对信号的影响，所以信号有时域对称特点. 信号的解析解显示信号是以 $\omega_2-\omega_1$ 为频率，以 $\alpha_1+\alpha_2$ 为衰减速率的阻尼调制. 这是因为当 $|\tau|$ 小于激光相干时间 τ_c 时，$G_1(G_2)$ 会产生稳定的干涉条纹，随着 τ 的变化，源自 G_1 与 G_2 的衍射信号产生相长或相消干涉形成拍振荡，同时随着 $|\tau|$ 的增大，光束1和2中的 $\omega_1(\omega_2)$ 分量的相干叠加越来越小导致衍射信号减弱形成拍信号衰减. 当 $|\tau|$ 增至远大于 τ_c 时，光束1和2中的 $\omega_1(\omega_2)$ 分量以各自的特定时间常量 $\alpha_1^{-1}(\alpha_2^{-1})$ 运动，信号只是各入射光的非相干加和，故此区域信号趋于一常量. 当 $\alpha_1,\alpha_2\ll\gamma$ 时，比较式(12a)，(15a)，(18a)，三种模型的区别源于非振荡项. 实高斯模型的非振荡项最大，且有衰减因子 $\exp(-2\alpha_1|\tau|)$，相散模型的非振荡项与 $|\tau|$ 无关，如图2(a)，实高斯模型下的调制信号随 $|\tau|$ 增大呈明显的衰减，而相散模型下的调制信号则围绕一个定值作阻尼振荡. 这是因为此时激光脉宽远大于栅的弛豫时间，所以栅的变化总能跟上光场的位相涨落，故位相涨落对非振荡信号没有影响. 同时由于脉冲持续时间较长，脉冲的重叠区域很大，所以振幅的变化对信号的影响增强. 当 $\alpha_1,\alpha_2\ll\gamma$ 时(图2(b))，三种模型的信号没有明显的区别，这是因为两栅弛豫时间很长导致序参量的积分效应冲刷了栅的作用，



(a) $\frac{\alpha_1}{\gamma}=\frac{\alpha_2}{\gamma}=0.6,\frac{\alpha_3}{\gamma}=0.5$



(b) $\frac{\alpha_1}{\gamma}=\frac{\alpha_2}{\gamma}=0.06,\frac{\alpha_3}{\gamma}=0.05$

图3 REPB随 Δ/α_1 变化

Fig. 3 Curve of the REPB veries with Δ/α_1

使衍射信号变弱，极大的降低了振幅涨落在信号中的作用. 同时由于激光脉宽很窄，所以谱线具有很高的时间分辨率. 图3(a)，(b)是信号在频域的变化曲线，它主要反映瑞利型共振增强效应对信号的影响. 由图3(a)可看出三种模型下的信号有很大不同，振幅涨落为信号贡献了比较大的非共振背底. 图3(b)是混沌模型下的不同延迟时得到的频域变化曲线. 在 $\gamma\tau=1$ 时，信号峰值有小的漂移，这是因为此时的场关联效应对信号的主导作用强于共振效应，所以峰值不会出现在 $\Delta=0$ 处，而当 $\tau=5$ 时，场关联效应减弱，共振效应主导信号，峰值又移回 $\Delta=0$ 处.

4 结论

本文基于马尔可夫随机场理论研究了瑞利型增强四波混频极化拍频，给出了拍信号的解析解及模拟信号谱，分析了场关联效应和共振增强效应对信号的影响，指出了振幅涨落和位相涨落在时域和频域的不同作用. 研究还发现差频REPB信号展现出了时域对称特性，信号在三种马尔可夫随机场模型下的区别对带宽的变化很敏感，随着带宽的增大逐渐消失.

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Difference-Frequency Rayleigh-Enhanced Polarization Beats

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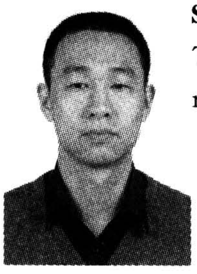
Abstract The theoretical analysis based on three Markovian stochastic models provides a closed-form solution for the difference-frequency Rayleigh-enhanced polarization beats. The subtle Markovian field correlation effects have been investigated. The different roles of the amplitude fluctuations and the phase fluctuations have been pointed out both in time- and frequency-domains. The difference-frequency REPB signal shows symmetric temporal behavior, and differences among three Markovian stochastic fields are quite sensitive to the change of bandwidth, and as the bandwidth increases, the difference becomes obscure.

Keywords Difference-frequency Rayleigh-enhanced polarization beats field correlation effects phase fluctuations amplitude fluctuations



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