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# Entanglement Translation and Quantum Teleportation of the Single-photon Entangled State*

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**Abstract** A method of generating the single-photon entangled states of a single photon state and a vacuum state through utilizing optical beam-splitter and single-photon source is described. A setup for realizing entanglement translation of the single-photon entangled state is designed. A proposal is presented for teleporting an unknown single-photon entangled state. In this proposed, the two single-photon entangled states is utilized as the quantum channel. The process of the entanglement translation and quantum teleportation are achieved by using the 50/50 symmetric beam splitters and the photon detectors with the help of classical information.

**Keywords** Quantum information; Quantum entanglement; Entanglement translation; Quantum teleportation

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## 0 Introduction

In recent years, entanglement has generated much interest in the quantum information processing such as quantum teleportation[1], superdense coding[2], quantum key distribution[3], telecoloning[4] and quantum positioning and clock synchronization[5]. Hence, quantum entanglement has been viewed as an essential resource for quantum information processing, many research articles on quantum entanglement and its application in quantum information processing have been published[6~9].

There are different types of entanglement for the light fields. In the Caltech teleportation experiment[10], a unknown coherent state was transmitted from a sender to a receiver, the state used for realizing the teleportation was a two-mode squeezed state. In the proposal of teleportation presented by S. J. van Enk et al[11], the transported state was an arbitrary coherent superposition state of two coherent states and the quantum channel was an entangled coherent state. In the recent papers[12~14], we have researched using the entangled squeezed vacuum states for realizing quantum information processing.

The recent developments in experimental techniques for generating and manipulating the single photon[15] have made quantum information processing utilizing single particle entanglement feasible, here, single particle entanglement refers to entanglement of a single particle state and vacuum state. Jae-Weon Lee et al[16] proposed a quantum cryptography scheme based on entanglement between a single particle state and a vacuum state. Inspired by their idea, a scheme is proposed for translating entanglement of single-photon entangled state and teleporting an arbitrary single-photon entangled state of a single photon state and a vacuum state through using the two single-photon entanglement state as quantum channel.

## 1 Generation of the entangled state

The experimental setup consists of a single photon source (S) and a lossless 50/50 beam-splitter $BS_1$, which can generate a single photon entangled state.

Let $\hat{a}_1$ and $\hat{a}_2$ denote the bosonic annihilation operators of the two light beams entering the two input ports of the beam-splitter and $\hat{b}_1$ and $\hat{b}_2$ denote the bosonic annihilation operators of the two light beams leaving the two output ports of the 50/50 beam-splitter, respectively. The boundary conditions at the surface of the beam-splitter lead to the mode transformation[16]

$$\begin{pmatrix} \hat{b}_1 \\ \hat{b}_2 \end{pmatrix} = \begin{pmatrix} \sqrt{R} & \sqrt{1-R} \\ -\sqrt{1-R} & \sqrt{R} \end{pmatrix} \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} \tag{1}$$

where $R$ is the reflectively of the beam-splitter. If setting $R = 1/2$, the beam-splitter is of 50/50. Then we obtain

$$\begin{aligned} \hat{b}_1 &= \frac{1}{\sqrt{2}}\hat{a}_1 + \frac{1}{\sqrt{2}}\hat{a}_2 \\ \hat{b}_2 &= -\frac{1}{\sqrt{2}}\hat{a}_1 + \frac{1}{\sqrt{2}}\hat{a}_2 \end{aligned} \tag{2}$$

It's assumed that the two input light beams of

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the beam-splitter $BS_1$ are in a single photon state and a vacuum state

$$|\Psi|\rangle_{12}=|1\rangle_1|0\rangle_2=\hat{a}_1^+|0\rangle_1|0\rangle_2 \tag{3}$$

After interaction with the beam-splitter, the output state emerging from the beam-splitter $BS_1$ is given by

$$|\Psi'\rangle_{12}=\frac{1}{\sqrt{2}}(\hat{b}_1^+-\hat{b}_2^+)|0\rangle_1|0\rangle_2= \frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_2-|0\rangle_1|1\rangle_2) \tag{4}$$

where subscripts 1 and 2 refer to two modes of the photons, $|1\rangle$ and $|0\rangle$ are the single photon state and the vacuum state, respectively. The state given in Eq. (4) represents a single-photon entangled state, its entanglement entropy is

$$S(\rho^{(1)})=-\mathrm{tr}(\rho^{(1)}\log\rho^{(1)})=1 \tag{5}$$

So, it is a maximally entangled state.

## 2 Entanglement translation

The method for preparing the single-photon entangled state has been discussed. Next, the scheme of entanglement translation is discussed. Assume there are three remote partners (Alice, Bob, Clara) who want to establish a connection. First, Alice shared the quantum channel $|\Psi\rangle_{13}$ with Bob and quantum channel $|\Psi\rangle_{24}$ with Clara, where

$$\begin{aligned}|\Psi\rangle_{13}&=\frac{1}{\sqrt{2}}(|1\rangle_1|0\rangle_3-|0\rangle_1|1\rangle_3)\\ |\Psi\rangle_{24}&=\frac{1}{\sqrt{2}}(|1\rangle_2|0\rangle_4-|0\rangle_2|1\rangle_4)\end{aligned} \tag{6}$$

Then the initial state of the whole system consisting of system 1, 2, 3 and 4 is given by

$$|\Psi\rangle_{1234}=\frac{1}{2}(|1\rangle_1|1\rangle_2|0\rangle_3|0\rangle_4-|1\rangle_1|0\rangle_2|0\rangle_3|1\rangle_4- |0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4+|0\rangle_1|0\rangle_2|1\rangle_3|1\rangle_4) \tag{7}$$

Note that at this time the modes 1 and 2 are located at the Alice's location, mode 3 is at Bob's location and mode 4 is at Clara's location.

Now Bob wants to establish a direct connection with Clara for realizing quantum communication between her and Clara. Then Bob sends a request through a classical channel to Alice. Alice, here as an operator, accepts this request and lets her modes 1 and 2 enter the input ports of the beam-splitter $BS_2$. After interacting with the beam-splitter, it's seen from Eq. (2) that the output state of the whole system becomes

$$\begin{aligned}|\Psi'\rangle_{1234}=\frac{1}{\sqrt{8}}(&|2\rangle_1|0\rangle_2|0\rangle_3|0\rangle_4-|0\rangle_1|2\rangle_2|0\rangle_3|0\rangle_4-\\ &|1\rangle_1|0\rangle_2|0\rangle_3|1\rangle_4+|0\rangle_1|1\rangle_2|0\rangle_3|1\rangle_4-\\ &|1\rangle_1|0\rangle_2|1\rangle_3|0\rangle_4-|0\rangle_1|1\rangle_2|1\rangle_3|0\rangle_4+\\ &\sqrt{2}|0\rangle_1|0\rangle_2|1\rangle_1|1\rangle_4)\end{aligned} \tag{8}$$

Subsequently Alice makes the two-mode photon number measurement on modes 1 and 2 at her side and sends the classical information to Bob and Clara to finish the entanglement translation process. When zero photon is detected at 1 mode and one photon is detected at 2 mode simultaneity, the Bob's and Clara's state collapses into

$$|\Psi''\rangle_{34}=\frac{1}{\sqrt{2}}(|0\rangle_3|1\rangle_4-|1\rangle_3|0\rangle_4) \tag{9}$$

where it has been considered normalization factor. If one photon is detected at 1 mode and zero photon is detected at 2 mode simultaneity, the Bob's and Clara's state collapses into

$$|\Psi''\rangle_{34}=\frac{1}{\sqrt{2}}(|0\rangle_3|1\rangle_4+|1\rangle_3|0\rangle_4) \tag{10}$$

Then a quantum channel between Bob and Clara has been established.

## 3 Quantum teleportation

Quantum teleportation is a process in which an unknown quantum state is transmitted from a system at the sender's (Alice's) location to another system at the receiver's (Bob's) location through a quantum channel with the help of some classical information.

Now it begins with a description of our teleportation scheme. It's assumed that Alice wants to teleport an completely unknown entanglement state of a single photon state and a vacuum state $|\varphi\rangle_{12}$ to Bob, where

$$|\varphi\rangle_{12}=\alpha|1\rangle_1|0\rangle_2+\beta|0\rangle_1|1\rangle_2 \tag{11}$$

$\alpha$ and $\beta$ are any complex numbers. The normalization term leads to

$$|\alpha|^2+|\beta|^2=1 \tag{12}$$

The two other single-photon entangled state they share and take as quantum channels are

$$\begin{aligned}|\Psi\rangle_{35}&=\frac{1}{\sqrt{2}}(|1\rangle_3|0\rangle_5-|0\rangle_3|1\rangle_5)\\ |\Psi\rangle_{46}&=\frac{1}{\sqrt{2}}(|1\rangle_4|0\rangle_6-|0\rangle_4|1\rangle_6)\end{aligned} \tag{13}$$

Then the total initial state is

$$|\Psi\rangle=|\varphi\rangle_{12}\otimes|\Psi|\rangle_{35}\otimes|\Psi|\rangle_{46} \tag{14}$$

where modes 1, 2, 3, and 4 are at Alice's side while modes 5 and 6 are at Bob's side.

Alice lets modes 1 and 2 enter the input ports of the beam-splitter $BS_3$. Subsequently her performs two photon number measurements for the two light beams leaving the two output ports of the beam-splitter $BS_3$. When the result of measurement is one photon at 0 mode and zero photon at 1 mode, the state of the system becomes (the unnormalized state)

$$|\Psi'\rangle=-\alpha|0\rangle_3|1\rangle_4|1\rangle_5|0\rangle_6-\beta|1\rangle_3|0\rangle_4|0\rangle_5|1\rangle_6+\alpha|0\rangle_3|0\rangle_4|1\rangle_5|1\rangle_6+\beta|1\rangle_3|1\rangle_4|0\rangle_5|0\rangle_6 \quad (15)$$

When the result of measurement is zero photon at 0 mode and one photon at 1 mode, the state of the system becomes

$$|\Psi''\rangle=\alpha|0\rangle_3|1\rangle_4|1\rangle_5|0\rangle_6-\beta|1\rangle_3|0\rangle_4|0\rangle_5|1\rangle_6-\alpha|0\rangle_3|0\rangle_4|1\rangle_5|1\rangle_6+\beta|1\rangle_3|1\rangle_4|0\rangle_5|0\rangle_6 \quad (16)$$

Alice lets modes 3 and 4 of state $|\Psi'\rangle$ enter the input ports of the beam-splitter $BS_4$. Subsequently her performs two photon number measurements for the two light beams leaving the two output ports of the beam-splitter $BS_4$. After Alice measures one photon in mode 3 and zero photon in mode 4, the state of the system collapses into the state

$$|\Phi\rangle_{56}=\alpha|1\rangle_5|0\rangle_6+\beta|0\rangle_5|1\rangle_6 \quad (17)$$

Following the same steps as before, Alice lets modes 3 and 4 of state $|\Psi''\rangle$ enter the input ports of the beam-splitter $\mathrm{BS}_5$ and performs photons measurement. If the result of measurement is one photon at 3 mode and zero photon at 4 mode, the Bob′ s state collapses into $|\Phi\rangle_{56}$. Then the unknown quantum state $|\Phi\rangle_{12}$ is destroyed in Alice′s place and a perfect replica is created in a remote Bob′ s site, the teleportation works perfectly.

## 4 Summary

On the basis of the function of the beam-splitter and single-photon detecting technology, we described a method to prepare the single-photon entangled states. A scheme for translating entanglement among three remote partners was proposed. A proposal for teleporting an unknown entangled state of the single photon state and the vacuum state was accomplished. In the teleportation scheme, the quantum channel used by the both sides of communication is two single photon entangled state. Owing to recent developments in experimental techniques for generating and manipulating the single-photon, these schemes should be feasible in theory and practice.

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# 单光子纠缠态的纠缠转移和量子隐形传态

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**摘　要**　使用光学分束器和单光子源，利用单光子态和真空态制备出了纠缠单光子态. 利用光学分束器作用和单光子探测，实现了三个通讯伙伴之间的纠缠转移. 提出了一个关于纠缠单光子态的量子隐形传态方案. 在这个方案中，被传送的是一个未知的单光子纠缠态. 通讯双方使用的量子信道是两个单光子纠缠态. 通过使用分束器作用和对输出态进行光子测量以及在经典信息的帮助下，纠缠转移和量子隐形传态的过程被完成.

**关键词**　量子信息；量子纠缠；纠缠转移；量子隐形传态

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