

基于 3D-VTGM-SBPM 平行定向耦合器模拟研究

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摘要 提出基于变量变换伽辽金法的三维半矢量束传播法(3D-VTGM-SBPM),可直接应用于三维光波导器件的光波传输模拟.采用适当的变量变换,将三维半矢量 BPM 基本方程归结为一阶常微分方程组,避免了非物理反射且能反映模场的偏振特性,成功的分析了基于脊形光波导的平行定向耦合器的光波传输特性.

关键词 光波导器件;变量变换伽辽金法;三维束传播法;半矢量;平行定向耦合器

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0 引言

光通信的迅猛发展推动了市场对光波导器件的需求,基于脊形光波导的平行定向耦合器是构建全光网的(AON)的重要器件,模拟分析其光波传输特性有助于优化其结构.束传输法(BPM)是一种灵活且具有良好拓展性的技术,已成为光波导数值分析的主要方法^[1],且已经出现了有限元法 BPM^[2]和有限差分法 BPM^[3]等常用数值计算方法.但这些方法往往方程繁杂,且要处理复杂的边界问题^[4].伽辽金法(GM)^[5,6]将电场或磁场展开成一组正交完备的基函数的叠加,其矩阵为对称型矩阵,计算效率较高.变量变换伽辽金法(VTGM)^[7]通过适当的正切(余切)函数变换,将无限平面映射成为单位平面,使该单位平面边界上的电磁场自然为零,避免了边界截断,提高了计算准确度.近年来,GM 和 VTGM 被广泛用于求解光波导本征值^[8,9]问题,但基于 VTGM 的束传输法多采用二维形式,基于 VTGM 的三维束传输法(BPM)仅有少量文献报道,且只有标量形式^[10].本文提出基于变量变换伽辽金法的三维半矢量束传播法(3D-VTGM-SBPM),将三维 BPM 基本方程归结为一阶常微分方程组,可直接模拟三维光波导器件的光波传输模拟,且能反映模场的偏振特性,是模拟分析光波导器件的较好方法.

1 3D-VTGM-SBPM

在慢变包络近似下,由麦克斯韦方程组可得光纤和光波导中单色光的三维半矢量 BPM 基本方程

$$2jk_0 \bar{n} \frac{\partial E_x}{\partial z} = \frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + k_0^2 (n^2 - \bar{n}^2) E_x +$$

$$\frac{\partial}{\partial x} \left[E_x \frac{\partial \ln(n^2)}{\partial x} \right] \quad (1)$$

$$2jk_0 \bar{n} \frac{\partial E_y}{\partial z} = \frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + k_0^2 (n^2 - \bar{n}^2) E_y +$$

$$\frac{\partial}{\partial y} \left[E_y \frac{\partial \ln(n^2)}{\partial y} \right] \quad (2)$$

式中, \bar{n} 称为参考折射率,引入如下变量变换

$$x = \sigma_x \tan [\pi(u-1/2)] \quad (3)$$

$$y = \sigma_y \tan [\pi(v-1/2)] \quad (4)$$

则将无限大平面映射成为单位平面,式(1)和

(2)分别变为

$$2j \bar{n} k_0 \frac{\partial E_x}{\partial z} = \left(\frac{du}{dx} \right)^2 \cdot \frac{\partial^2 E_x}{\partial u^2} + \frac{d^2 u}{dx^2} \cdot \frac{\partial E_x}{\partial u} + \left(\frac{dv}{dy} \right)^2 \cdot \frac{\partial^2 E_x}{\partial v^2} + \frac{d^2 v}{dy^2} \cdot \frac{\partial E_x}{\partial v} + k_0^2 (n^2 - \bar{n}^2) E_x + 2 \left(\frac{du}{dx} \right)^2 \cdot \frac{\partial}{\partial u} \left(E_x \frac{\partial \ln(n)}{\partial u} \right) + 2 \frac{d^2 u}{dx^2} E_x \frac{\partial \ln(n)}{\partial u} \quad (5)$$

$$2j \bar{n} k_0 \frac{\partial E_y}{\partial z} = \left(\frac{du}{dx} \right)^2 \cdot \frac{\partial^2 E_y}{\partial u^2} + \frac{d^2 u}{dx^2} \cdot \frac{\partial E_y}{\partial u} + \left(\frac{dv}{dy} \right)^2 \cdot \frac{\partial^2 E_y}{\partial v^2} + \frac{d^2 v}{dy^2} \cdot \frac{\partial E_y}{\partial v} + k_0^2 (n^2 - \bar{n}^2) E_y + 2 \left(\frac{dv}{dy} \right)^2 \cdot \frac{\partial}{\partial v} \left(E_y \frac{\partial \ln(n)}{\partial v} \right) + 2 \frac{d^2 v}{dy^2} E_y \frac{\partial \ln(n)}{\partial v} \quad (6)$$

通过变换,边界上的电磁场自然为零,避免了非物理反射.将电场在有限区间 $0 \leq u \leq 1, 0 \leq v \leq 1$ 展开成正弦级数,且为正交完全集,即

$$E_x = \sum_{k=1}^{N_x} C_k^x \Psi_k(u, v) = \sum_{p=1}^{N_x} \sum_{q=1}^{N_y} C_{pq}^x \Psi_{pq}(u, v) \quad (7)$$

$$E_y = \sum_{k=1}^{N_x} C_k^y \Psi_k(u, v) = \sum_{p=1}^{N_x} \sum_{q=1}^{N_y} C_{pq}^y \Psi_{pq}(u, v) \quad (8)$$

其中基函数为

$$\Psi_{pq} = \sqrt{2} \sin(p\pi u) \cdot \sqrt{2} \sin(q\pi v) = 2 \sin(p\pi u) \cdot \sin(q\pi v) \quad (9)$$

N_x, N_y 分别为 u, v 方向的级数项数. p, q 分别由式(10)、(11)决定

$$p = 1 + (k-1) \operatorname{div} N_y \quad (10)$$

$$q=1+(k-1)\text{mod}N_y \quad (11)$$

其中 div 和 mod 分别表示取整和求余. 将式(7)和(8)代入式(5)和(6), 并应用迦辽金法, 得

$$j \sum_{p=1q=1}^{N_x} \sum_{p'q'=1}^{N_y} R_{p'q'pq}^x \cdot \frac{dC_{pq}^x}{dz} = \sum_{p=1q=1}^{N_x} \sum_{p'q'=1}^{N_y} S_{p'q'pq}^x \cdot C_{pq}^x \quad (12)$$

$$j \sum_{p=1q=1}^{N_x} \sum_{p'q'=1}^{N_y} R_{p'q'pq}^y \cdot \frac{dC_{pq}^y}{dz} = \sum_{p=1q=1}^{N_x} \sum_{p'q'=1}^{N_y} S_{p'q'pq}^y \cdot C_{pq}^y \quad (13)$$

写成矩阵形式为

$$S^x C^x = R^x \frac{dC^x}{dz} \quad (14)$$

$$S^y C^y = R^y \frac{dC^y}{dz} \quad (15)$$

其中的矩阵元由式(16)~(19)确定

$$S_{p'q'pq}^x = \sum_{i=1}^4 I_i + k_0^2 \int_0^1 \Psi_{p'q'} n^2(u, v, z) \Psi_{pq} dudv + I_5 + I_6 - k_0^2 \bar{n}^2 \delta_{p'p} \delta_{q'q} \quad (16)$$

$$R_{p'q'pq}^x = 2jk_0 \bar{n} \int_0^1 \Psi_{p'q'} \Psi_{pq} dudv = 2jk_0 \bar{n} \delta_{p'p} \delta_{q'q} \quad (17)$$

$$S_{p'q'pq}^y = \sum_{i=1}^4 I_i + k_0^2 \int_0^1 \Psi_{p'q'} n^2(u, v, z) \Psi_{pq} dudv + I_7 + I_8 - k_0^2 \bar{n}^2 \delta_{p'p} \delta_{q'q} \quad (18)$$

$$R_{p'q'pq}^y = 2jk_0 \bar{n} \int_0^1 \Psi_{p'q'} \Psi_{pq} dudv = 2jk_0 \bar{n} \delta_{p'p} \delta_{q'q} \quad (19)$$

式中 $I_m (m=1\sim 8)$ 为 8 个二重积分, 可参考文献[7] 写出其解析表达式. 式(14)和式(15)是一组常微分方程, 在给定 $E(x, y, z=0)$ 后, 便可以求出 $z>0$ 区域的电场分布. 另外, 如令式(14)和式(15)中的 $d/dz=0$, 则可直接约化为本征值方程, 用于求解本征值问题.

2 数值结果与讨论

利用上述方法对图 1 的平行定向耦合器进行分析和模拟, 其参量分别为: $\omega=3 \mu\text{m}$, $d=1 \mu\text{m}$, $n_c=1$, $n_i=3.44$, $n_s=3.36$, $h=1 \mu\text{m}$, $t=0.9 \mu\text{m}$, $\lambda=1.55 \mu\text{m}$. 为使式(14)和式(15)快速收敛, 并保证场形不弯曲变形, 参照文献[7,9]选取 $\sigma_x=\omega+d/2$, $\sigma_y=h/2$.

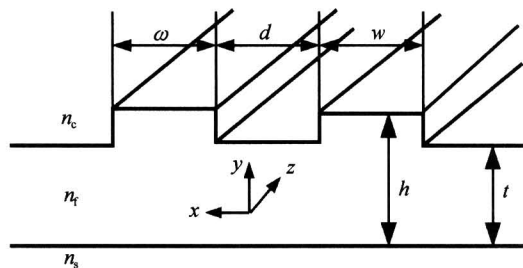


图 1 基于脊形光波导的平行对称定向耦合器
Fig. 1 Parallel symmetrical directional coupler based on optical rib waveguides

图 2 给出定向耦合器承载的电场 TE 波的偶模和奇模的模场分布; 图 3 给出定向耦合器承载的电

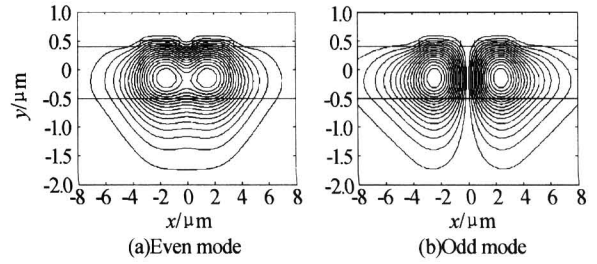


图 2 定向耦合器承载的电场 TE 模分布
Fig. 2 Electric TE mode field distributions supported by the directional coupler

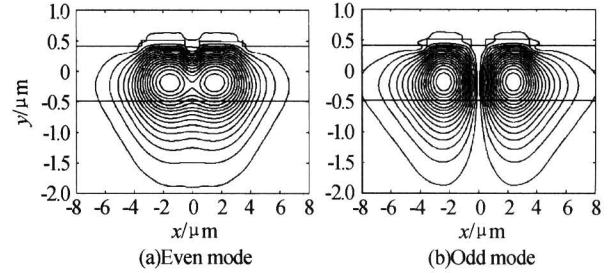


图 3 定向耦合器承载的电场 TM 模分布
Fig. 3 Electric TM mode field distributions supported by the directional coupler

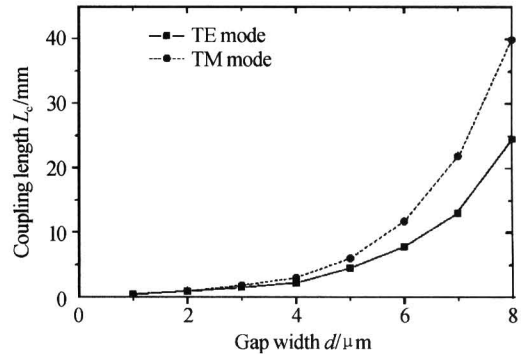


图 4 $1.55 \mu\text{m}$ 的光波在定向耦合器中传输时耦合长度随波导间距的变化
Fig. 4 Coupling length as a function of the gap width at a wavelength of $1.55 \mu\text{m}$ for the directional coupler

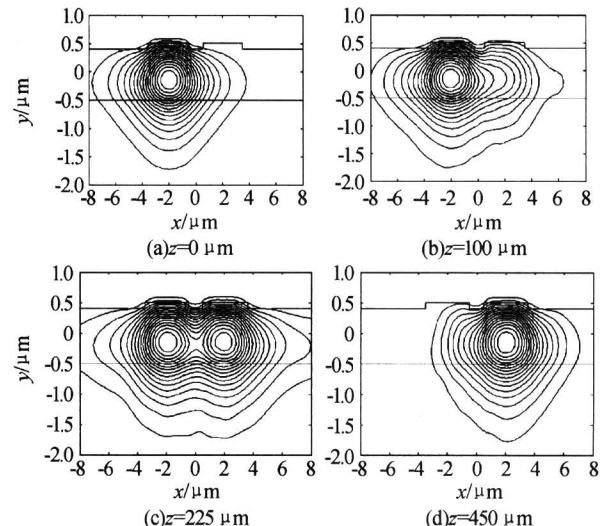


图 5 脊形光波导 TE 波基模在定向耦合器中的传输演变
Fig. 5 The fundamental TE mode developing in the directional coupler

场 TM 波偶模和奇模的模场分布. 在图 3 中看到了 TM 模的畸变, 证明半矢量方法更能反映物理实质. 图 4 是耦合长度 L_c 随耦合波导间距 d 的变化关系, 由图中可以看出, L_c 随波导间距增大近似指数增长, 这和文献[11]的结果吻合较好. 用左通道中的电场 TE 波基模进行激励, 得到它在定向耦合器中传输演变情况如图 5. 在 $z=450 \mu\text{m}$ 处实现了交叉态, 说明其耦合长度为 $450 \mu\text{m}$, 和文献[10,11]的结果吻合较好. TM 波在定向耦合器中的传输演变情况与此类似.

3 结论

本文提出了基于 VTGM 的三维半矢量束传播法 (3D-VTGM-SBPM) 的数理模型, 用以模拟分析基于脊形光波导的平行定向耦合器的光波传输特性. 对比了定向耦合器承载的电场 TE 波和 TM 波的偶模和奇模的模场分布, 看到了 TM 模的畸变, 证明半矢量方法更能反映物理实质. 用 TE 波基模进行激励, 传输 $450 \mu\text{m}$ 出现交叉态, 所得结果和其他已发表的结果非常接近. 3D-VTGM-SBPM 将光波传输问题归结为一阶常微分方程组, 导出矩阵小, 计算效率高, 将无限域问题转换成有限域问题, 避免了非物理反射, 提高了计算准确度, 还可直接约化为本征值方程用于求解本征值问题. 因采用半矢量方法, 既能反映光场的偏振特性, 又具有和标量方法相当的计算效率, 非常适合于分析光波导器件.

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Analysis of Parallel Directional Coupler Using 3D-Semi-vectorial BPM Based on the Variable Transformed Galerkin Method

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Abstract A novel three-dimensional semi-vectorial beam propagation method based on the variable transformed galerkin method(3D-VTGM-SBPM) is proposed for directly modeling optical waveguides, and is successfully applied to simulate parallel directional coupler. By adopting proper variable transformation, the basic three-dimensional semi-vectorial BPM equations is reduced to a first-order normal differential equation system. The artificial boundary condition is avoided, and the polarization characteristic of the mode field is displayed.

Keywords Optical waveguide devices; Variable transformed galerkin method(VTGM); Three-dimensional beam propagation method(3D-BPM); Semi-vector; Parallel directional coupler



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