Electron- Phonon Interaction on the Optical Kerr Effect in Cylindrical Quantum Wires*

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Abstract The optical Kerr effect in cylindrical quantum wires is investigated with density- matrix approach, most emphasis is devoted to the electron- phonon interactions on the optical Kerr effect. With characteristic parameters pertaining to GaAs quantum wire, the numerical results are presented. It is shown that, the optical Kerr effect increases as the wire radius R_0 decreases; the optical Kerr considering the electron-LO-phonon interactions is over twenty percent greater than the one without considering the electron- phonon interactions; the smaller the wire radius R_0, the sharper the peak will be, and the larger the peak intensity will be; when the wire radius R_0 is greater than 40 nm, the peak will disappear gradually.

Keywords Optical Kerr effect; Quantum wires; Electron-phonon interaction

CLCN O431.2 **Document Code** A

0 Introduction

Nano-structures enable many new phenomena to be discovered and provide potential device applications in future laser technology and the optical modulation technique. The electron-phonon interaction in these confined systems is one of the important aspects in determining their properties in physical processes, such as in the transport process or the electron relaxation process. Phonon modes in quantum wells and quantum wires with cylindrical and rectangular shapes have been studied by various methods such as dielectric continuum approaches[1~5]. For the case of quantum dots, Klein, et al. have derived the polar optical phonon modes in spherical quantum dots surrounded by other materials[6,7]. Theoretically, most of the studies on quantum dots are focused on spherical quantum dots, and only some of them are on rectangular and cylindrical quantum dots or on quantum disks. In fact, whereas the microcrystallites are approximately spherical, quantum dots are better described by thin disks or cylinders[8~10]. In the theoretical research on disk-shaped quantum structures, Kayanuma has studied Wannier excitons for infinite potential barriers by using the variational method[11]. Le Goff and Stebe have also researched the same problem for cylindrical quantum dots with finite potential[12]. Peeters and Scheifert have investigated the energy levels of a two-electron quantum disk with finite potential under magnetic field. Chen et al. have analyzed the influence of the bulk longitudinal-optical (LO) phonon and the surface optical (SO) phonon on the energy of an electron in a disk-like quantum dot[13], but they did not treat the electron-phonon interactions on the optical Kerr effect. In this paper, we will study the influence of electron-phonon interactions on the optical Kerr effect in cylindrical quantum wires. The optical Kerr effect is a third- order nonlinear optical phenomenon, which consists in an intensity-dependent variation of the complex optical index[14,15]. Indeed, the third-order nonlinear susceptibility of the low-dimensional semiconductors is greatly enhanced in comparison with the bulk semiconductors[16~25].

1 Theory

Let′s consider a polar semiconductor cylindrical quantum wire with radius R_0. The outer potential is approximately infinite, i. e.

$$V(r)=\begin{cases}\infty, r\geqslant R_0\\ 0, r<R_0\end{cases} \tag{1}$$

Then the Hamiltonian of the electron interacting with the longitudinal optical phonon can be represented by[22]

$$H=H_e+H_{LO}+H_{e-LO} \tag{2}$$

Where H_e is the Hamiltonian of the electron, which is given by

$$H_e=-\frac{\hbar^2}{2m_e^*}\nabla^2+V(r) \tag{3}$$

where m_e^* is the effective mass of the electron. The second term of Eq. (2) is the phonon Hamiltonian

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*Supported by the National Natural Science Fundation of China (No. 60478010), Science and Technology Committee of Guangdong Province (No. 2005B10301014, 2004B10301014, and 04105406), and Science and Technology Committee of Guangzhou (No. 2024 and 2004J1-C0226)

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Received date: 2004- 11- 29

which is given by

$$H_{\mathrm{LO}}=\sum_{mkl_z}\hbar\,\omega_{\mathrm{LO}}a_{ml}^{+}(k_z)a_{ml}(k_z) \tag{4}$$

where $a_{ml}^{+}(k_z)$ and $a_{ml}(k_z)$ is the creation and annihilation operator for the LO- phonon, respectively; ω_{LO} is the dispersionless frequency of the bulk longitudinal optical phonons. The last term of Eq. (2) is the electron- phonon interaction Hamiltonian which is given by[22]

$$H_{\mathrm{e-LO}}=-\sum_{mlk_z}[F_{\mathrm{LO}}^{ml}(k_z)J_m(\frac{\chi_{ml}r}{R_0})\mathrm{e}^{-\mathrm{i}m\varphi}\cdot \mathrm{e}^{-\mathrm{i}k_z z}a_{ml}^{+}+c.c] \tag{5}$$

with

$$|F_{\mathrm{LO}}^{ml}(\mathrm{k}_z)|^2=\frac{2\mathrm{e}^2\ \hbar\,\omega_{\mathrm{LO}}}{L_z J_{|m|+1}^{2}(\chi_{ml})(\chi_{ml}^{2}+R_0^2k_z^2)}\cdot(\frac{1}{\varepsilon_\infty}-\frac{1}{\varepsilon_0}) \tag{6}$$

where χ_{ml} is the l th zero of the Bessel function $J_m(x)$, L_z is the length of the quantum wire. ε_0 and ε_∞ is the static and high- frequency dielectric constants, respectively.

By using the perturbation theory, the first-order perturbation wave function $|\Psi_i^{(1)}\rangle$ is represented by

$$|\Psi_i^{(1)}\rangle=\sum_{j\neq i}\frac{\langle\Psi_j|H_{\mathrm{e-LO}}|\Psi_i\rangle}{E_i-E_j-\hbar\,\omega_{\mathrm{LO}}}|\Psi_j\rangle \tag{7}$$

Therefore, up to first order, the total wave function $|\Psi_i\rangle$ is

$$|\Psi_i\rangle=|\phi_i\rangle+|\Psi_i^{(1)}\rangle \tag{8}$$

After tedious and complicated calculations[16~21], the analytic expression of the optical Kerr effect $\chi^{(3)}(-\omega;\omega,-\omega,\omega)$ is found which is given by

$$\begin{aligned}\chi^{(3)}(-\omega;\omega,-\omega,\omega)=&\frac{5\mathrm{e}^4\sigma_s}{3\varepsilon_0\ \hbar^3}\mu_{03}\mu_{12}\mu_{21}\mu_{30}\times\\ &[\frac{1}{(\omega_{30}+\omega+\mathrm{i}\Gamma_{30})(\omega_{20}+2\omega+\mathrm{i}\Gamma_{20})(\omega_{10}+\omega+\mathrm{i}\Gamma_{10})}+\\ &\frac{1}{(\omega_{30}+\omega+\mathrm{i}\Gamma_{30})(\omega_{02}-2\omega-\mathrm{i}\Gamma_{02})(\omega_{10}-\omega-\mathrm{i}\Gamma_{10})}+\\ &\frac{1}{(\omega_{12}+\omega+\mathrm{i}\Gamma_{12})(\omega_{13}+2\omega+\mathrm{i}\Gamma_{13})(\omega_{23}+\omega+\mathrm{i}\Gamma_{23})}+\\ &\frac{1}{(\omega_{12}+\omega+\mathrm{i}\Gamma_{12})(\omega_{31}-2\omega-\mathrm{i}\Gamma_{31})(\omega_{32}-\omega-\mathrm{i}\Gamma_{32})}+\\ &\frac{1}{(\omega_{21}+\omega+\mathrm{i}\Gamma_{21})(\omega_{23}+2\omega+\mathrm{i}\Gamma_{23})(\omega_{13}+\omega+\mathrm{i}\Gamma_{13})}+\\ &\frac{1}{(\omega_{21}+\omega+\mathrm{i}\Gamma_{21})(\omega_{32}-2\omega-\mathrm{i}\Gamma_{32})(\omega_{31}-\omega-\mathrm{i}\Gamma_{31})}+\\ &\frac{1}{(\omega_{03}+\omega+\mathrm{i}\Gamma_{03})(\omega_{01}+2\omega+\mathrm{i}\Gamma_{01})(\omega_{02}+\omega+\mathrm{i}\Gamma_{02})}+\\ &\frac{1}{(\omega_{03}+\omega+\mathrm{i}\Gamma_{03})(\omega_{10}-2\omega-\mathrm{i}\Gamma_{10})(\omega_{20}-\omega-\mathrm{i}\Gamma_{20})}]\end{aligned} \tag{9}$$

where Γ_{ij} is the relaxation rate, $\omega_{ij}=(E_i-E_j)/\hbar$ is the Bohr frequency, $\mu_{ij}=|\langle\Psi_j|r|\Psi_i\rangle|$.

2 Results and discussions

Now the optical Kerr effect of the GaAs quantum wires shall be calculate. The pertinent parameters used in our numerical work include $m_e^*=0.067m_0$ (m_0 is the mass of a free electron), $\varepsilon_0=13.18$, $\varepsilon_\infty=10.89$, $\alpha=0.068$, $\hbar\,\omega_{\mathrm{LO}}=36.25$ meV, the lattice constant $a=5.6533$Å and $\sigma_s=5\times10^{24}$ m^{-3}. For simplicity, it is chosen $\hbar\Gamma_{10}=\hbar\Gamma_{01}=\hbar\Gamma_{21}=\hbar\Gamma_{12}=\hbar\Gamma_{32}=\hbar\Gamma_{23}=\hbar\Gamma$ meV, $\hbar\Gamma_{20}=\hbar\Gamma_{02}=\hbar\Gamma_{31}=\hbar\Gamma_{13}=\hbar\Gamma/2$ meV, $\hbar\Gamma_{30}=\hbar\Gamma_{03}=\hbar\Gamma/3$ meV .

From Fig. 1, it′s seen that the smaller the radius R_0 of the quantum wire, the larger the geometrical factor $\mu_{03}\mu_{12}\mu_{21}\mu_{30}$. It also shows that the geometrical factor $\mu_{03}\mu_{12}\mu_{21}\mu_{30}$ decreases quickly with increasing the quantum wire radius R_0 and the curves flatten at large wire radius R_0. The contribution of LO- phonon mode is considered. It is obvious that the geometrical factor $\mu_{03}\mu_{12}\mu_{21}\mu_{30}$ considering the electron- LO- phonon interactions is over twenty percent greater than the one without considering the electron- phonon interactions. The reason for this is that the LO- phonon in ionic crystals involves the relative motion of positive and negative ions. This depends on polarization and interacts strongly with electromagnetic waves. Therefore, the LO- phonon has a marked influence on the optical Kerr effect.

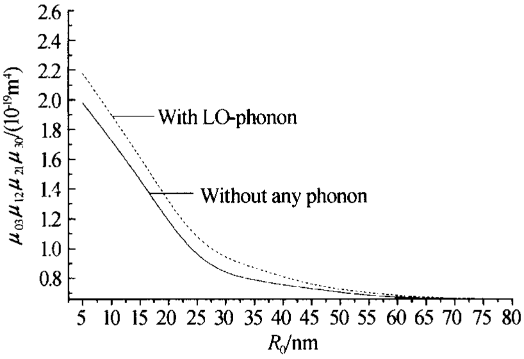

Fig. 1　The geometrical factor $\mu_{03}\mu_{12}\mu_{21}\mu_{30}$ of the quantum wire as a function of the quantum wire radius R_0 for two different cases: (1) without any phonon (solid line), and (2) with LO- phonon (dashed line)

In Fig. 2, the electron- LO- phonon interactions is also considered. It reveals once again that the optical Kerr effect $|\chi^{(3)}(-\omega;\omega,-\omega,\omega|$ increases when the wire radius R_0 decreases. A very important feature is that the smaller the wire radius R_0, the sharper the peak will be, and the larger the peak intensity will be . When the wire radius $R_0 > 40$ nm, the peak will disappear gradually.

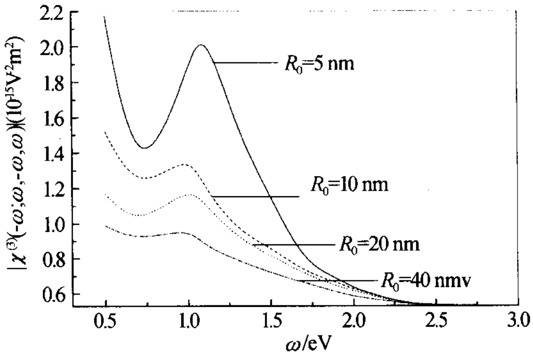

Fig. 2 The optical Kerr effect $|\chi^{(3)}(-\omega;\omega,-\omega,\omega|$ of the quantum wire as a function of the photon frequency ω for four different values of quantum wire radius R_0: (1) $R_0=5$ nm (solid line), (2) $R_0=10$ nm (dashed line), (3) $R_0=20$ nm(dotted line), and (4) $R_0=40$ nm (dot-dashed line)

3 Conclusion

The optical Kerr effect of the cylindrical quantum wire is studied. The numerical results show that, (1) the optical Kerr effect increases as the wire radius R_0 decreases; (2) theoptical Kerr considering the electron- LO- phonon interactions is over twenty percent greater than the one without considering the electron- phonon interactions; (3) the smaller the wire radius R_0, the sharper the peak will be, and the larger the peak intensity will be; (4) when the wire radius R_0 is greater than 40nm, the peak will disappear gradually.

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电子-声子相互作用对柱形量子线中光学克尔效应的影响

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收稿日期：2004-11-29

摘　要　本文利用密度矩阵方法研究了柱形量子线中的光学克尔效应，重点讨论了电子-声子相互作用对其的影响，并以 GaAs 量子线为例进行了数值计算。结果显示，随着量子线半径 R_0 的减小，光学克尔效应会逐渐增强；考虑了电子-声子相互作用时的光学克尔效应比未考虑电子-声子相互作用时的大 20% 多；量子线半径 R_0 越小，峰越尖锐，峰值越强；当量子线半径 R_0 大于 40 nm 时，峰会逐渐消失.

关键词　光学克尔效应；量子线；电子-声子相互作用

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