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Nonlinear Feedback Control in Single-Mode Laser Haken-Lorenz Chaotic System*

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Abstract A variable nonlinear feedback (VNF) method was proposed to control chaotic systems. Control principles and the technique to select the feedback coefficients were introduced. This control method was theoretically studied with a single-mode laser Haken-Lorenz system. The numerical simulation results showed that the maximum Lyapunov exponent of the system can be changed from positive values into negative ones by appropriately selecting feedback coefficients, and the orbits of the system can be changed from the chaotic attractor into the periodic orbits in phase space, the numbers of periodic orbits are $2^n \times 3^m p$ (n, m are integers). Comparison between the VNF method and the linear feedback method has been made which shows that the former is speeder, simpler and more efficient.

Keywords Nonlinear optics; Chaos control; Nonlinear feedback; Haken-Lorenz system; Periodic orbit; Lyapunov exponent

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0 Introduction

Since the parameter parturbation method (OGY) was initially proposed by Ott, Grebogi and Yorke, the unstable fixed point of a chaotic system was stabilized, chaos control has become a hotspot in the research[1~14]. Chaos control can make various of chaotic systems attain the following targets: The first is that when an effective stable control is added, the one of immense unstable periodic orbits embedded in chaotic system can be stabilized; the second is to control chaotic system to stay on some periodic orbits to have; the third is to realize periodic synchronization, chaotic synchronization and super-chaotic synchronization of two or more the same dynamic systems; the fourth is to eliminate the multiple chaotic attractor of a complicated chaotic system to realize the chaos control into a single chaotic attractor; the fifth is to establish an unti-control of chaos to produce chaos, that is, to make a system chaotic or to strengthen chaos of a system.

Since chaos control is widely used in information communications, many kinds of control methods have been proposed, such as occasional proportional feedback method[15], self-controlling feedback method[16], delayed feedback method[17], external force feedback method[18] and so on. Most of these methods fulfill control use linear feedbacks. Though these methods are practical and economical, they can't perform the best control, they even can't realize the control, in strong nonlinear systems, or under the best control conditions. In this paper, A variable nonlinear feedback (VNF) method is proposed to control chaotic systems. Control principles and the technique to select the feedback coefficients are introduced. This control method is theoretically studied with a single-mode laser Haken-Lorenz system. The numerical simulation results show that the maximum Lyapunov exponent of the system can be changed from positive values into negative ones by appropriately selecting feedback coefficients, and the orbits of the system can be changed from the chaotic attractor into the periodic orbits in phase space, the numbers of periodic orbits are $2^n \times 3^m p$ (n, m are integers). Comparison between the VNF method and the linear feedback method has been made which shows that the former is speeder, simpler and more efficient.

1 Control principle

Suppose the dynamic equations of an N-dimensional nonlinear continuous system can be described as follows

$$\frac{dx_i}{dt} = f_i(\{x_i\}, \{\mu_l\})$$
$$(i=1,2,\cdots N \quad l=1,2,\cdots m) \tag{1}$$

where $f_i(f_1, f_2 \cdots f_N)$ are nonlinear functions. $\{x_i\}$ system variables, and $\{\mu_l\}$ system parameters.

Now, a nonlinear feedback control is applied to the phase space constructed by the system functions

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f_i (f_1, f_2 ⋯ f_N) in the nonlinear continuous dynamic equations (1), and the feedback terms F_i are described as

$$F_i = -\sum_{j=2}^{n} k_{ij}(x_i - x_{ir})^j \quad (n \geqslant 2) \tag{2}$$

Then, the nonlinear continuous dynamic equations (1) become

$$\frac{\mathrm{d}x_i}{\mathrm{d}t} = f_i(\{x_i\}, \{\mu_l\}) - \sum_{j=2}^{n} k_{ij}(x_i - x_{ir})^j \tag{3}$$

where k_{ij} are feedback coefficients, x_{ir} are feedback parameters, which can be chosen to take the values of zero, or the solutions of the dynamic equations, or other nonzero values, according to the dynamic equations considered and the controlling targets in order to control the unstable periodic orbits and the other target orbits that are needed.

When applying the nonlinear feedback control to the chaotic system in which linear feedback control fails disabled, square feedback terms are preferred, that is

$$k_{i2} \neq 0, k_{i3} = k_{i4} = \cdots k_{in} = 0 \tag{4}$$

If the control does not take effect, the feedback terms of high powers can be considered. This is the variable nonlinear feedback (VNF) method.

Suppose x'_i is a perturbation in the neighborhood of x_{i0} which are the stable solutions of Equation(1), i. e.,

$$x'_i = x_i - x_{i0} \tag{5}$$

when the control is added, the linearized equations of the perturbation in the neighborhood of x_{i0} can be described as follows

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{bmatrix} x'_1 \\ \cdots \\ x'_i \\ \cdots \\ x'_N \end{bmatrix} = \begin{bmatrix} [(\frac{\mathrm{d}f_1}{\mathrm{d}x_1}) - 2k_{12}(x_1 - x_{1r})]_{\{x_i\}_0,\{a_l\}_0} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & [(\frac{\mathrm{d}f_i}{\mathrm{d}x_i}) - 2k_{i2}(x_i - x_{ir})]_{\{x_i\}_0,\{a_l\}_0} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & [(\frac{\mathrm{d}f_N}{\mathrm{d}x_N}) - 2k_{N2}(x_N - x_{Nr})]_{\{x_i\}_0,\{a_l\}_0} \end{bmatrix} \begin{bmatrix} x'_1 \\ \cdots \\ x'_i \\ \cdots \\ x'_N \end{bmatrix} \tag{6}$$

The $N \times N$ matrix above is called Jacobi matrix T,

$$T = \begin{bmatrix} [(\frac{\mathrm{d}f_1}{\mathrm{d}x_1}) - 2k_{12}(x_1 - x_{1r})]_{\{x_i\}_0,\{a_l\}_0} & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & [(\frac{\mathrm{d}f_i}{\mathrm{d}x_i}) - 2k_{i2}(x_i - x_{ir})]_{\{x_i\}_0,\{a_l\}_0} & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & [(\frac{\mathrm{d}f_N}{\mathrm{d}x_N}) - 2k_{N2}(x_N - x_{Nr})]_{\{x_i\}_0,\{a_l\}_0} \end{bmatrix} \tag{7}$$

The eigenvalue equation of the matrix is

$$\begin{vmatrix} [(\frac{\mathrm{d}f_1}{\mathrm{d}x_1}) - 2k_{12}(x_1 - x_{1r})]_{\{x_i\}_0,\{a_l\}_0} - \omega & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & [(\frac{\mathrm{d}f_i}{\mathrm{d}x_i}) - 2k_{i2}(x_i - x_{ir})]_{\{x_i\}_0,\{a_l\}_0} - \omega & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & [(\frac{\mathrm{d}f_N}{\mathrm{d}x_N}) - 2k_{N2}(x_N - x_{Nr})]_{\{x_i\}_0,\{a_l\}_0} - \omega \end{vmatrix} = 0 \tag{8}$$

According to this equations and the definition of Lyapunov exponent[19] which is the basic quantity to describe the chaotic system

$$\lambda=\lim_{t\to\infty}\frac{1}{t}\ln\left|\frac{x'(t)}{x'(0)}\right| \tag{9}$$

The maximum Lyapunov exponent can be changed from positive to negative values by adjusting the feedback coefficients k_{ij} to make the controlled system change from chaotic states to periodic orbits.

2 Control results

The dynamic equations of a single-mode laser Haken-Lorenz system can be written as follows[20]

$$\begin{cases}\dfrac{dx}{dt}=\sigma(y-x)\\ \dfrac{dy}{dt}=(\mu-z)x-y\\ \dfrac{dz}{dt}=xy-bz\end{cases} \tag{10}$$

When the system parameters $\sigma=10,\mu=30,b=8/3$, the system is in chaotic state. The steady state solution (x_0,y_0,z_0) of the Haken-Lorenz system is given by $\frac{dx}{dt}=\frac{dy}{dt}=\frac{dz}{dt}=0$, that is

$$\begin{cases}\sigma(y_0-x_0)=0\\ (\mu-z_0)x_0-y_0=0\\ x_0y_0-bz_0=0\end{cases} \tag{11}$$

The nonzero solution (x_0,y_0,z_0) is $(\pm\sqrt{b(\mu-1)},\pm\sqrt{b(\mu-1)},\mu-1)$.

Suppose x',y',z' is a perturbation at the point (x_0,y_0,z_0). Substituting it into the expression above, the linearized equation(10) turns to

$$\frac{d}{dt}\begin{bmatrix}x'\\y'\\z'\end{bmatrix}=\begin{bmatrix}-\sigma & \sigma & 0\\ \mu-z_0 & -1 & -x_0\\ y_0 & x_0 & -b\end{bmatrix}\begin{bmatrix}x'\\y'\\z'\end{bmatrix} \tag{12}$$

Hence, the eigenvalue equation is

$$\begin{vmatrix}-\sigma-\omega & \sigma & 0\\ \mu-z_0 & -1-\omega & -x_0\\ y_0 & x_0 & -b-\omega\end{vmatrix}=0 \tag{13}$$

That is

$$\omega^3+c_2\omega^2+c_1\omega+c_0=0 \tag{14}$$

where $c_2=\sigma+b+1$

$c_1=\sigma+b(\sigma+1)-\sigma(\mu-z_0)+x_0^2$

$c_0=b\sigma+\sigma x_0y_0-b\sigma(\mu-z_0)+\sigma x_0^2$

According to Routh-Hurwith condition, only when $c_2>0,c_0>0,c_2c_1-c_0>0$, the real part of the eigenvalue is negative, and the steady state solution (x_0,y_0,z_0) is stable. However, if $\sigma=10,\mu=30,b=8/3$,

$$c_2>0,c_0>0,c_2c_1-c_0<0 \tag{15}$$

So the real part of the eigenvalue is positive, and the steady state solution (x_0,y_0,z_0) is unstable.

Applying nonlinear feedback to the Haken-Lorenz system, equation (10), and only taking square feedback term $F_i=-k_{i2}(x_i-x_{ir})^2$, Equation(10) changes into

$$\begin{cases}\dfrac{dx}{dt}=\sigma(y-x)-k_{12}(x-x_r)^2\\ \dfrac{dy}{dt}=(\mu-z)x-y-k_{22}(y-y_r)^2\\ \dfrac{dz}{dt}=xy-bz-k_{32}(z-z_r)^2\end{cases} \tag{16}$$

In order to get the most obvious control results with a minimum cost, put $x_r=y_r=z_r=0$, and the feedback coefficients $k_{12}=k_{22}=k_{32}=k$. Using the similar steps as(11)～(15), we obtain the eigenvalue equation of the linearized perturbation equations of Equation(16) as follows

$$\begin{vmatrix}-\sigma-2kx_0-\omega' & \sigma & 0\\ \mu-z_0 & -1-2ky_0-\omega' & -x_0\\ y_0 & x_0 & -b-2kz_0-\omega'\end{vmatrix}=0 \tag{17}$$

That is

$$\omega'^3+c'_2\omega'^2+c'_1\omega'+c'_0=0 \tag{18}$$

where $c'_2=\sigma+b+1+2k(x_0+y_0+z_0)$

$c'_1=(\sigma+2kx_0)(1+2ky_0)+(\sigma+1+2kx_0+2ky_0)(b+2kz_0)-\sigma(\mu-z_0)+x_0^2$

$c'_0=(\sigma+2kx_0)(1+2ky_0)(b+2kz_0)+\sigma x_0y_0-\sigma(\mu-z_0)(b+2kz_0)+x_0^2(\sigma+2kx_0)$

By adjusting the feedback coefficients k, so as to fulfill the Routh-Hurwith condition $c'_2>0,c'_0>0,c'_2c'_1-c'_0>0$ and at the same time, which makes the maximum Lyapunov exponent of the Haken-Lorenz system less than zero, the system can be stably controlled.

Comparison of the dynamic behaviors of both controlled and uncontrolled Haken-Lorenz systems is made by numerical simulation. The values of σ,μ and b remain unchanged in the numerical simulation. By adjusting the value of coefficient k, through, iteration of 2.0×10^4 times with a step of 0.01. we can compare the variation of the characteristic quantities which describe the system before and after the addition of the control, such as the maximum Lyapunov exponent, the phase maps, and the reconstruction of phase space.

The evolution of the maximum Lyapunov exponent of the Haken-Lorenz system, against the parameter μ is shown in Fig. 1 (a). When the system parameters $\sigma=10$, $\mu=30$, $b=8/3$, the maximum Lyapunov exponent of the Haken-Lorenz system is $\lambda_{max}=1.58$. When a nonlinear feedback control is added with feedback coefficient $k=0.03$,

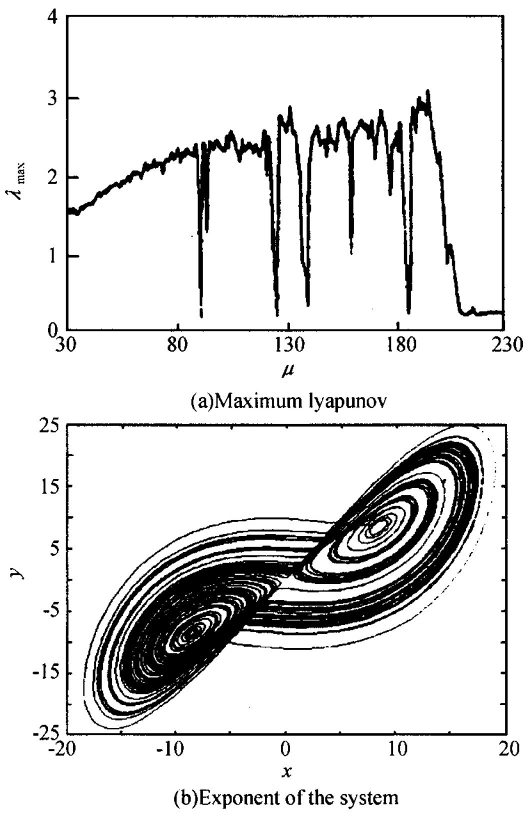


Fig. 1 Maximum lyapunov exponent and phase map of the controlled system

the maximum Lyapunov exponent $\lambda_{max} = -0.22$. That is, the maximum Lyapunov exponent of the system is changed after the addition of the control from positive to negative. Therefore the dynamic property of the system is also changed from a chaotic state into a periodic state.

The phase maps of the uncontrolled system are shown in Fig. 1(b). As shown, all the orbits are in finite regions, orbits have the ergodic property and special self-similar structures. The orbits form bands. If the phase space is magnified, more bands of orbits in the original bands can be found. All of these is the typical character of chaotic attractor.

Fig. 2 (a), (b), (c) and (d) are the phase maps of the controlled system. The orbits are obviously changed into n closed curves, which is the typical character of periodic orbits. It shows that the chaotic attractor of an infinite number of embedded self-similar structures, which is limited in a finite phase space, has been changed into periodic orbits. Numerical simulating shows that a chaotic system can be controlled into $2^n \times 3^m p$ multifarious stable periodic orbits (n, m are integers).

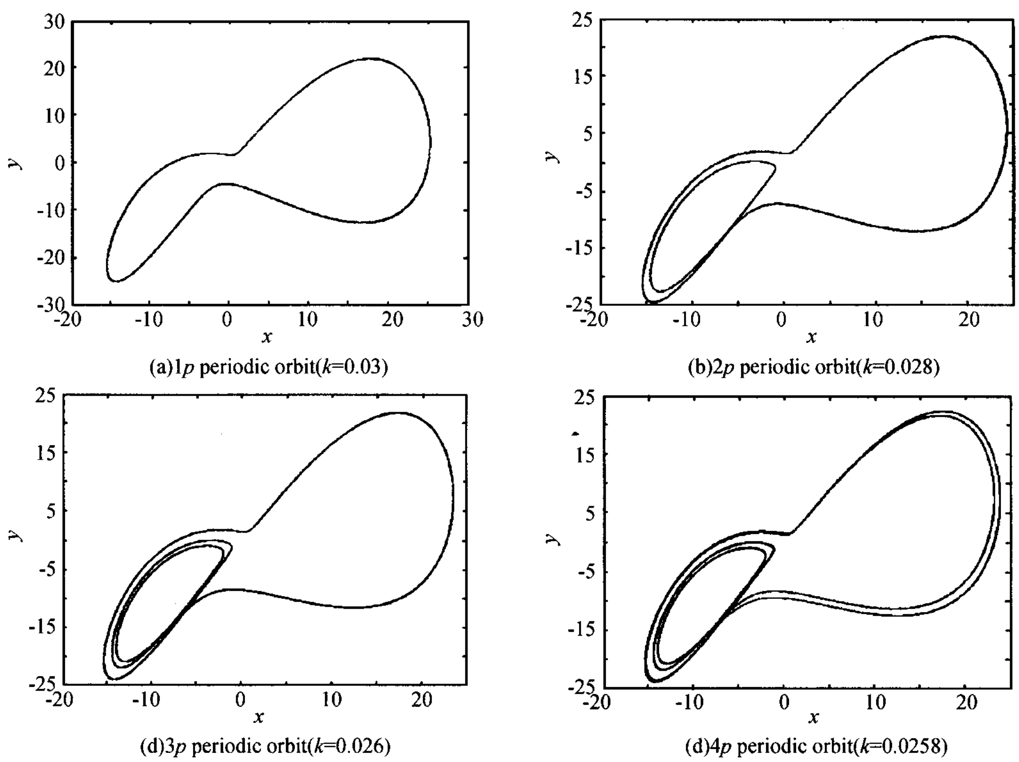
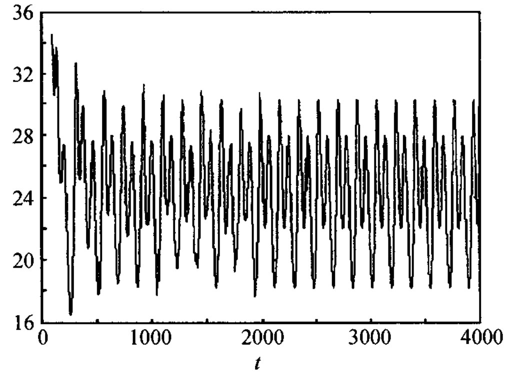
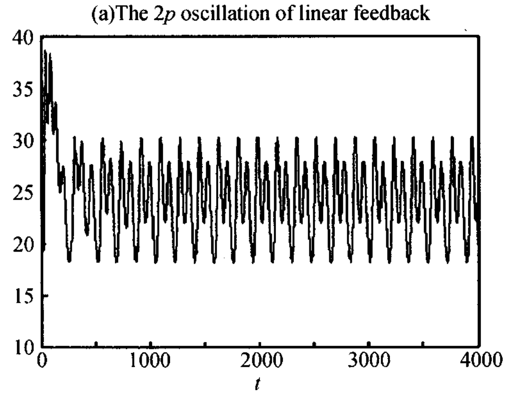


Fig. 2 The periodic orbits in the controlled system

Comparison of the numerical results for a linear feedback and a nonlinear feedback, is made as follows. When the linear feedback method is used, the feedback term is structured as $F_i = -kx_i$. The oscillation picture is shown in Fig. 3 (a), which describes that the Haken-Lorenz system is changed from a chaotic state into $2p$ periodic orbit. The oscillation picture, when a nonlinear feedback is used, is shown in Fig. 3 (b). Comparing the two figures, it's easy to find that a chaotic system can be stabilized quickly into $2p$ periodic orbit by a nonlinear feedback, while the linear feedback is much slower, the system is stabilized after the time sequence is more than 2000. The difference is significant, which shows that the nonlinear feedback method is quite efficient and fast.



(a)The $2p$ oscillation of linear feedback



(b)The $2p$ oscillation of nonlinear feedback

Fig. 3 Numerical vesults of the linear feedback and the nonlinear feedback

3 Conclusion

The Haken-Lorenz chaotic system is well controlled by using the method presented in this paper. Numerical simulation results show that the maximum Lyapunov exponent, the phase maps and the reconstruction of phase space all can be changed when control is added. It's found that when the system parameters $\sigma=10, \mu=30, b=8/3$, the maximum Lyapunov exponent of Haken-Lorenz system changes from 1.58 to -0.22, the orbits of the system in the phase maps change from the chaos attractor into $2^n \times 3^m p$ (n、m are integers) periodic orbits. What's more, the speed and the effect of the control are much better than those of linear control.

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非线性反馈控制单模激光 Haken-Lorenz 混沌系统

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摘 要 提出一种变量非线性反馈(VNF)方法控制混沌系统．介绍了该方法的控制原理以及反馈系数的选取原则，以单模激光 Haken-Lorenz 系统为例对非线性反馈控制方法进行了理论研究．仿真结果显示，通过恰当的选择反馈系数 k，使系统的最大李雅普诺夫(Lyapunov)指数由正值转变为负值，相图中系统的轨迹由混沌吸引子转变为周期数为 $2^n \times 3^m p$(n、m 为整数)的周期轨道．通过与线性反馈控制结果对比发现，非线性反馈控制方法简便有效，控制速度快．

关键词 非线性光学；混沌控制；非线性反馈；Haken-Lorenz 系统；周期轨道；Lyapunov 指数

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