Seidel Aberration Properties of Flat Slab Lens Systems Made from Left-handed Materials*

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Abstract A general study of the Seidel aberration properties of the flat slab lens systems made from left-handed materials was presented according to the primary aberration theory. Seidel aberration sums of such a system were derived based on the addition theorem. The way to find a flat slab lens system free from all five Seidel aberrations was described accordingly. Finally, two special examples of the singlet and doublet LHM slab lens systems without Seidel aberrations were given and discussed respectively.

Keywords Left-handed materials; Negative refractive index; Primary aberrations; Seidel aberration sums; Slab lens

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0 Introduction

More than three decades ago, the so-called Left-handed materials (LHMs) were first postulated by Veselago[1] who predicted that such materials could have negative indices of refraction. However, they are not naturally occurring as the conventional media, named right-handed materials (RHMs), with positive refractive indices. Therefore their properties were not extensively investigated at that time. Recently, the LHMs have been constructed in the microwave region by the combination of the conductive metallic elements deposited on a substrate[2] or by the transmission line methods[3]. Further, photonic-gap materials were also proposed that they could behave as effective negative index materials at the optical frequencies[4]. Furthermore, the capability of creating lens elements with LHMs has been demonstrated[5] showing that they still obey the standard laws and formulas with negative indices in the traditional geometrical optics.

A thick LHM plane parallel plate was first envisioned by Veselago that it could be served as an unusual converging lens[1]. Pendry also predicted that the LHM flat slab lens of $n=-1$ has the ability to achieve subwavelength focusing[6]. All of these make it alluring and crucial to promote many intensive researches on the peculiar properties and designs of LHM slab lenses in theorization and experimentation. In this paper, it will present a thorough study on the primary aberration properties of the LHM slab lens system and describe the way to find the systems that are free from all five primary (Seidel) aberrations in the benefit of negative index of refraction. It will show that a single LHM slab lens of $n=-1$ is sufficient to form a real image having unity magnification and free from all five Seidel aberrations. The example of a doublet LHM slab lens is also given partly for the original consideration that more flexible parameters of the system can be selected as compared to the case of 'perfect' lens.

1 Seidel aberration sums

The five basic types of primary aberrations, known as Seidel aberrations after *L*. Seidel who in 1856 gave explicit formulae for calculating them, are the third-order wavefront aberrations that arise due to geometrical deviations from Gaussian theory corresponds to the approximation that $\sin\theta\approx\theta-\theta^3/6$. The Seidel theory of aberrations is often used for designs of new types of optical systems. We first introduce the general formulas for the calculation of Seidel aberration sums of LHM flat slab lens system as shown in Fig. 1. The parameters involved in such a system composed of k slab lenses and j ($j=k+1$) planar refracting surfaces where the ith ($i=1,2,\cdots,j-1$) surface is departing from its next one with a distance d_i. The aperture stop (AS) is placed left to the first surface with a distance t_1 to take into account stop-shift effects on the off-axis aberrations. According to the geometrical optics, the position of entrance pupil (EnP) is exactly coincided in the plane of the aperture stop while the exit pupil (ExP) is in the

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image plane of the stop by the slab lens system. Since Seidel aberrations are applied for the edge of the entrance pupil and edge of the field, the paraxial ray-tracing paths of the marginal and principal meridional rays are also noted in red and green. In Fig. 1, h is the object height; a is the stop semi-aperture; n_i is the refractive indices between the ith and $i+1$th surfaces and $n_j=n_0$ are refractive indices of the surrounding medium the system located; s_i is distance from the ith surface to the object plane (if $i=1$), or to its Gaussian image formed by the first $(i-1)$ surface(s) (if $i>1$); s_i' is the distance from surface to the Gaussian image of the object plane formed by the first i surface(s); u_i is the convergence angle of the marginal ray from the axial object point for the ith surface and $u_1=a/(t_1-s_1)$; $\bar{u}_i$ is the convergence angle of the principal ray for the ith surface and $\bar{u}_1=h/(t_1-s_1)$; h_i is the height from the axis at which the marginal ray intersects the ith surface and $h_1=as_1/(t_1-s_1)$ in the paraxial approximation.

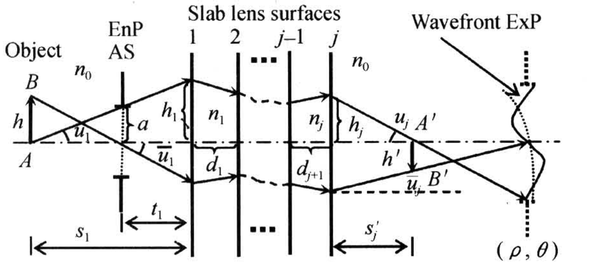

Fig. 1 Pictorial illustration of an LHM slab lens system located in the surrounding medium n_0 with a remote aperture stop

It has been shown that the primary aberration coefficients can be expressed by means of quantities, which only relate to the paraxial ray from the axial object point[8]. Thus, it may first perform a paraxial raytracing process of LHM slab lens system. Without loss of generality, only relevant parameters of the ith and $i+1$th surfaces are depicted in Fig. 2 in order to deduce the general relation formulas. According to Snell's law at the planar refracting surface between the adjacent LHM and RHM, we get from the Gaussian paraxial

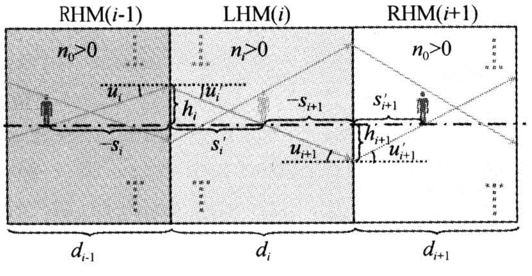

Fig. 2 General notations of the ith and $i+1$th surfaces for the paraxial raytracing of a paraxial marginal ray

optics that

$$\begin{cases} s_i'=s_i n_i/n_{i-1} \\ u_{i+1}=u_i'=u_i n_{i-1}/n_i \\ h_{i+1}=h_i s_{i+1}/s_i' \end{cases} \tag{1}$$

Since the Gaussian image formed by the first i surfaces of the system is the object for the $(i+1)$th surface, we have the transfer formula

$$s_{i+1}=s_i'-d_i \tag{2}$$

The five types of Seidel aberration sums (Spherical aberration S_{I}, Coma S_{II}, Astigmatism S_{III}, Curvature of field $S_{\mathrm{III}}+S_{\mathrm{IV}}$, Distortion S_{V}) presented here are taken from Born and Wolf[8], only being slightly modified for the fact that the surfaces are planar where $r_i=\infty$ (for the same reason, Petzval curvature of field S_{IV} always keeps zero)

$$\begin{cases} S_{\mathrm{I}}=\sum_{i=1}^{j} h_i^4 K_i^2 S_i \\ S_{\mathrm{II}}=-H\sum_{i=1}^{j} h_i^2 K_i(a+k_i h_i^2 K_i)S_i/a \\ S_{\mathrm{III}}=H^2\sum_{i=1}^{j}(a+h_i^2 k_i K_i)^2 S_i/a^2 \\ S_{\mathrm{IV}}=H^2\sum_{i=1}^{j}\frac{1}{r_i}\left(\frac{1}{n_i}-\frac{1}{n_{i-1}}\right)\equiv 0 \\ S_{\mathrm{V}}=-H^3\sum_{i=1}^{j}k_i(a+h_i^2 k_i K_i)(2a+h_i^2 k_i K_i)\cdot \\ \qquad S_i/a^3-\frac{a+h_i^2 k_i K_i}{a h_i^2}\left(\frac{1}{h_i^2}-\frac{1}{n_{i-1}^2}\right) \end{cases} \tag{3}$$

where H stands for the Helmholtz-Lagrange invariant of the system given by $-n_0\bar{u}_1 a$ and K_i, S_i, k_i are defined as

$$\begin{cases} K_i=-n_{i-1}/s_i \\ S_i=1/(n_i s_i')-1/(n_{i-1}s_i) \\ k_{i+1}=k_1+a\sum_{m=1}^{i} d_m/(n_m h_m h_{m+1}) \\ k_1=t_1(t_1-s_1)/(n_0 a s_1) \end{cases} \tag{4}$$

According to the addition theorem for the primary aberrations, firstly it would be better to derive the corresponding aberration coefficients associated with the individual surface and then it can get the total sums for primary aberrations of a specific lens system by direct addition of them. Substitution from equations (1) and (4) into the Seidel formulae leads to the following expressions of the Seidel aberrations introduced by the ith planar surface

$$\begin{cases} S_{\mathrm{I}}^{(i)}=u_i^4 n_{i-1}\left(\frac{n_{i-1}^2-n_i^2}{n_i^2}\right)s_i \\ S_{\mathrm{II}}^{(i)}=-\left(\frac{H}{a}\right)u_i^3\left(\frac{n_{i-1}^2-n_i^2}{n_i^2}\right)s_i \\ S_{\mathrm{IV}}\equiv 0 \\ S_{\mathrm{III}}^{(i)}=\left(\frac{H}{a}\right)^2 u_i^2\frac{1}{n_{i-1}}\left(\frac{n_{i-1}^2-n_i^2}{n_i^2}\right)s_i \\ S_{\mathrm{V}}^{(i)}=-\left(\frac{H}{a}\right)^3 u_i\frac{1}{n_{i-1}^2}\left(\frac{n_{i-1}^2-n_i^2}{n_i^2}\right)s_i \end{cases} \tag{5}$$

Then from the addition theorem and by substituting equations (1 ~ 2) into (5), get the required form of the Seidel formulae of a flat slab lens system composed of k LHM slabs that provides a basic platform for further discussions

$$\begin{cases} S_{\mathrm{I}} = \sum\limits_{i=1}^{k} S_{\mathrm{I}}^{(i)} = n_0^2 u_1^4 \sum\limits_{i=1}^{k} \dfrac{(n_0^2 - n_i^2)}{n_i^3} d_i \\ S_{\mathrm{II}} = \sum\limits_{i=1}^{k} S_{\mathrm{II}}^{(i)} = (\dfrac{H}{a}) u_1^3 n_0 \sum\limits_{i=1}^{k} \dfrac{(n_0^2 - n_i^2)}{n_i^3} d_i = \dfrac{\bar{u}_1}{u_1} S_{\mathrm{I}} \\ S_{\mathrm{IV}} \equiv 0 \\ S_{\mathrm{III}} = \sum\limits_{i=1}^{k} S_{\mathrm{III}}^{(i)} = (\dfrac{H}{a})^2 u_1^2 \sum\limits_{i=1}^{k} \dfrac{(n_0^2 - n_i^2)}{n_i^3} d_i = (\dfrac{\bar{u}_1}{u_1})^2 S_{\mathrm{I}} \\ S_{\mathrm{V}} = \sum\limits_{i=1}^{k} S_{\mathrm{III}}^{(i)} = (\dfrac{H}{a})^3 u_1 \dfrac{1}{n_0} \sum\limits_{i=1}^{k} \dfrac{(n_0^2 - n_i^2)}{n_i^3} d_i = (\dfrac{\bar{u}_1}{u_1})^3 S_{\mathrm{I}} \end{cases} \quad (6)$$

From equation(6), know that for a LHM slab lens system: S_{IV} is always zero; $S_{\mathrm{I}}^{(i)}$ is proportional to d_i and u_i^4 by the coefficient $n_0^2(n_0^2-n_i^2)/n_i^3$; S_{II}, S_{III} and S_{V} all are linearly dependent on S_{I} that considerably simplifies the expression for Seidel aberration sums. Thus the system is free from all five Seidel aberrations when the condition $S_{\mathrm{I}}=0$ is satisfied, that is

$$\sum_{i=1}^{k} \frac{(n_0^2 - n_i^2)}{n_i^3} d_i = 0 \quad (7)$$

An apparent solution of equations (7) is that $n_i = n_0$, making a singlet LHM slab lens without Seidel aberrations possible. In the case of a collimated beam from the object at infinity where $u_1=0$, such a system only shifts the beam without introducing any primary aberrations. In the following sections, it will detailedly discuss the primary aberration properties of the special systems such as singlet and doublet LHM slab lenses, which are much valuable in view of application.

2 Singlet LHM slab lens

For conventional plane-parallel plate with positive indices of refraction ($n>0$), it can't form a real image for an object located in front it and often are inserted into an imaging system to compensate for redundant spherical aberration or to produce required displacements. However, the emergence of LHM with negative refractive index allows an LHM flat slab to behave as an imaging lens for the optical system. As shown in Fig. 3, an LHM flat slab lens is composed of two planar surfaces ($r_1 = r_2 = \infty$) and has a negative-index body ($n<0$) made from LHM. Inherently, it can focus the light rays emanating from an object that is placed at a distance $-s_1$ from the slab lenses into an inner and an outer real images. By use of Snell's law for small angle of incidence and from equations(1~2),

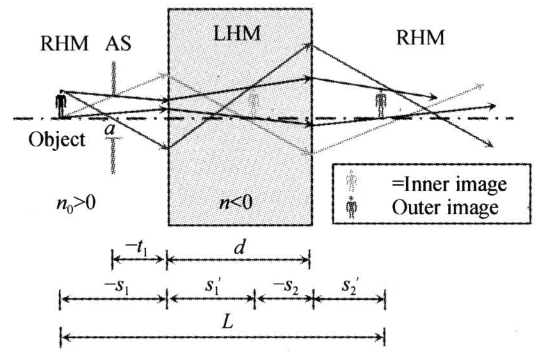

Fig. 3　Passage of paraxial rays through a singlet LHM flat slab lens with a negative-index body surrounded by a medium n_0

we obtain

$$\begin{cases} s_1' = (n/n_0) s_1 \\ s_2 = s_1' - d = (n/n_0) s_1 - d \\ s_2' = (n_0/n) s_2 = s_1 - (n_0/n) d \\ u_2 = (n_0/n) u_1 \end{cases} \quad (8)$$

Thus the axial displacement between the object and the outer image is found to be

$$L = -s_1 + d + s_2' = (1 - n_0/n) d \quad (9)$$

which is independent of the object distance $-s_1$ and can be lengthened by applying an LHM slab with a lower absolute index for L is reciprocal to n. In order to form real images, the image distance $s_2'>0$ should be satisfied, that is

$$0 > s_1 > (n_0/n) d \quad (10)$$

Let $k=1$ and substitute it into equation(6), we obtain the according expressions for the Seidel sums of a singlet LHM slab lens having two planar surfaces

$$\begin{cases} S_{\mathrm{I}} = S_{\mathrm{I}}^{(1)} + S_{\mathrm{I}}^{(2)} = n_0^2 (n_0^2 - n^2) u_1^4 d / n^3 \\ S_{\mathrm{II}} = S_{\mathrm{II}}^{(1)} + S_{\mathrm{II}}^{(2)} = (\dfrac{H}{a}) u_1^3 n_0 (\dfrac{n_0^2 - n^2}{n^3}) d = \dfrac{\bar{u}_1}{u_1} S_{\mathrm{I}} \\ S_{\mathrm{III}} = S_{\mathrm{III}}^{(1)} + S_{\mathrm{III}}^{(2)} = (\dfrac{H}{a})^2 u_1^2 (\dfrac{n_0^2 - n^2}{n^3}) d = (\dfrac{\bar{u}_1}{u_1})^2 S_{\mathrm{I}} \\ S_{\mathrm{IV}} \equiv 0 \\ S_{\mathrm{V}} = S_{\mathrm{V}}^{(1)} + S_{\mathrm{V}}^{(2)} = (\dfrac{H}{a})^3 u_1 \dfrac{1}{n_0} (\dfrac{n_0^2 - n^2}{n^3}) d = (\dfrac{\bar{u}_1}{u_1})^3 S_{\mathrm{I}} \end{cases} \quad (11)$$

From equation (11), it is found that the magnitudes of all the Seidel aberrations vary linearly with the LHM slab thickness and all the five sums are actually independent of incidence heights if u_1 and $\bar{u}_1$ are unchanged. Fig. 4 shows the effect of varying the refractive index of singlet LHM slab lens located in air ($n_0=1$) on the Seidel aberration sums when u_1 and $\bar{u}_1$ are kept at 0.5 rad and 0.25 rad. From Fig. 4, we know that $S_{\mathrm{I}} \sim S_{\mathrm{III}}$ and S_{V} are positive if $n<-1$, negative if $-1<n<0$ and zero if and only if $n=-1$ while S_{IV} is always being zero. Meanwhile, $S_{\mathrm{I}} \sim S_{\mathrm{III}}$ and S_{V} simultaneously

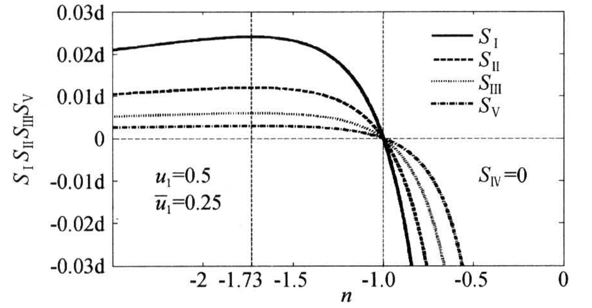

Fig. 4 Seidel aberration sums are shown as functions of the refractive index n for $u_1=0.5\text{rad}, \bar{u}_1=0.25$ rad and $n_0=1$

approach the maximum positive values when $n\rightarrow -\sqrt{3}$ and naturally incline to zero when $n\rightarrow -\infty$. It is worth noting that all the five primary aberrations are zero for a slab lens of a unity negative index $n=-1$. As a matter of fact, the same is true for the total monochromatic aberrations to any higher order as illustrated in Fig. 5 by way of the rigid raytracing. This intriguing feature confirms the Pendry's concept of a 'perfect' slab lens[3] in some extent in view of the aberration theories. Such a slab lens is ideal for the formation of a sharp image and relatively easy to manufacture for its flat shape. However, the 'perfect' slab lens is not all-powerful. It is inherently little nearsighted in the consideration that L is only twice as large as its thickness d. The system's magnification always holds positive unity and such a lens does not focus the plane waves yet many optical applications require the practical ability of lens to focus radiation from a remote objects. However, this problem can be solved by using the LHM lenses of curved surfaces[7] though with a little degenerated performance.

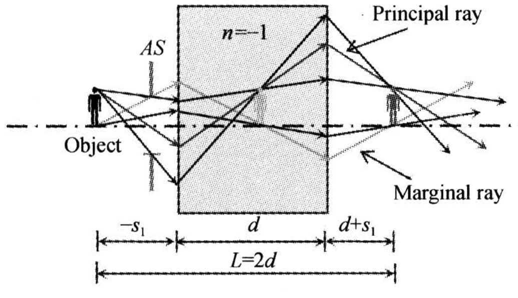

Fig. 5 Rigid raytracing of a 'perfect' slab lens with $n=-1$ made of LHM that is free of total monochromatic aerrations

3 Doublet LHM slab lens

Then is it possible to design a compound LHM doublet slab lens of $n\neq -1$ without Seidel aberrations? The answer is sure. As mentioned above, $S_I \sim S_{III}$ and S_V all are monotone decreasing functions having opposite signs for $n<-1$ and for $-1<n<0$. So it provides an effective means to counteract the Seidel aberrations inherently introduced by an LHM slab lens of $-1<n<0$ with another one of $n<-1$ when suitable thickness and refractive index are selected. Fig. 6 (a) shows the sketch of a doublet LHM slab lens composed of two parts with different negative indices of refraction free from all five Seidel aberrations .

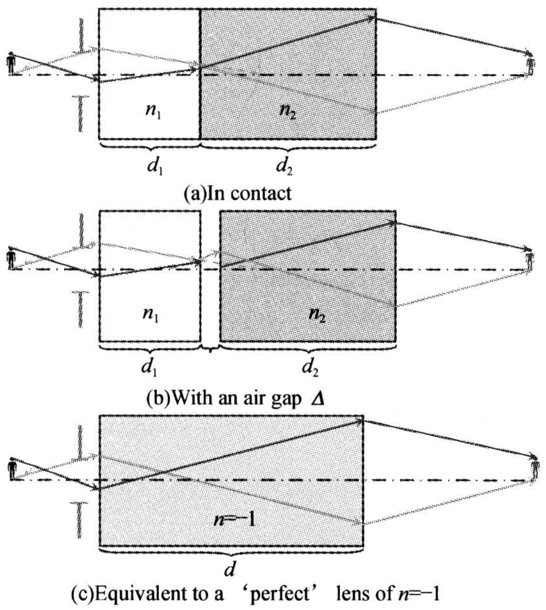

Fig. 6 Lens systems free from all five Seidel aberrations composed of two LHM slabs with $n_1=-1.732$ and $n_2=-0.927$

From equation (5), obtain the spherical aberration sum of the doublet LHM slab lens having three planar surface ($i=1,2,3$) located in air that

$$S_I=S_I^{(1)}+S_I^{(2)}+S_I^{(3)}=u_1^4\left[\left(\frac{1-n_1^2}{n_1^3}\right)d_1+\left(\frac{1-n_2^2}{n_2^3}\right)d_2\right] \tag{12}$$

For simplicity, to normalize the system, the unit of length is assumed to be such that the rear lens thickness $d_2=1$ and here we choose intentionally that $n_1=-\sqrt{3}$ to provide a minimum thickness since it reaches the maximum positive value for $(1-n_1^2)/n_1^3$. Then let $S_I=0$, we obtain from equation(10) that

$$d_1=-\frac{n_1^3}{1-n_1^2}\frac{1-n_2^2}{n_2^3}=-\frac{1-n_2^2}{0.3849n_2^3} \tag{13}$$

where the other four primary aberration sums are also zero as what can be seen from equation(9). Now the axial displacement between the object and the outer image becomes

$$L=L_1+L_2=(1-1/n_1)d_1+(1-1/n_2)d_2=(n_2^3+3.098n_2^2-4.098)/n_2^3 \tag{14}$$

As a practical example, design a normalized doublet LHM slab lens that is free from the Seidel aberrations with the preconditions: $-s=0.5$, $L=3$, $\Delta\approx 0$. Here the air gap Δ is preferable to be zero in order to offer a maximum free working space. Substituting them into equations (13~14), have $n_2=-0.912$ and $s'=0.927$. Thus from equation (10), soon get $d_1=0.573$. By separating this doublet LHM slab lens into two LHM slab lenses with an air gap Δ as shown in Fig. 6(b), get the spherical aberration sum of the separated LHM slab lens system from the equation(6) that has the same form as shown in equation(10) showing that the air gap doesn′t introduce any Seidel aberrations. Meanwhile, the doublet is equivalent to the singlet LHM slab lens with $n=-1$ and $d=L/2=1.5$ as shown in Fig. 6(c). It is found that the doublet LHM slab lens is slightly thicker than its equivalent 'perfect' lens for that $d_1+d_2=1.573>d=1.5$. What′s more, the 'perfect' lens with $n=-1$ is exactly impedance matched to the surrounding media (air) implying that none of the incident light energy is reflected at all, while it is not the case for the doublet LHM slab lens designed above.

4 Conclusion

Study of LHMs with negative indices is a subject with constant capacity for surprise, which is ready for rewriting the laws of optics and leading to unexpected and profound consequences. In recent years, there have been many studies on LHMs[8~10] due to the experimental verification of negative index artificial materials[11] and the introduction of the perfect lens with imaging resolution exceeding the diffraction limit by Pendy[3]. In this paper, it presents a general discussion about the Seidel aberration properties of the LHM slab lens systems in detail. Seidel aberration sums for such a system are first derived in favour of the addition theorem for the primary aberrations. Based on them, describe the way to find LHM slab lens system without all five Seidel aberrations. It is also shown that it is possible for a thick singlet LHM slab lens of $n=-1$, labeled as 'perfect' lens, to be free from monochromatic aberrations. A doublet LHM slab lens is also discussed as a practical example, which is free from Seidel aberrations and composed of two LHM parts with $n\neq-1$. Finally, we also point out that the 'perfect' lens has some advantages over its equivalent doublet or other types of LHM slabs. Although LHM slab lenses are inherently comfined to the near-distance optical applications, the emergence of them inevitably expands the theories of optical designs and opens the door for a new breed of optical devices.

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左手性介质平板透镜系统的赛德尔像差特性

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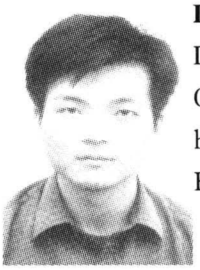
摘　要　根据初级像差理论广泛研究了由左手性介质构造的平板透镜系统的赛德尔像差特性．由初级像差加和定理推导了适合于这种系统的赛德尔和数公式，并指出了寻找消赛德尔像差的平板透镜系统的方法．作为两个特例，分别讨论了消赛德尔像差的左手性介质单透镜和双胶合透镜系统．

关键词　左手性介质；负折射率；初级像差；赛德尔像差和数；平板透镜



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