Optical Propagation Characteristics of All-metal-cladding Rib Waveguide

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Abstract The convenient analytical method for the all-metal-cladding rib waveguide is presented. The metal-cladding rib waveguide is converted into the two-dimension slab one by using the effective refractive method, then the equation of satisfying effective refractive index and the formula of absorption loss coefficient are derived by the derivation method for the all- metal-cladding rib waveguide. Taking this method as an example, optical propagation constant and the loss characteristics for SiO_2 have been calculated actually. Some available results are obtained.

Keywords Optical waveguide; Metal cladding; Absorption loss; Effective refractive index

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0 Introduction

Ridged optical waveguide plays an essential role in constructing guided-wave optical components such as switching/modulating devices[1~4]. When metal electrodes (that is metal cladding) is added on these devices, it has an effect on mode-propagation characteristics of ridge waveguide and lead to absorption loss. The loss of metal cladding waveguide is caused by the complex of the metal permittivity. As the actual metal electrical conductivity is definite, there is a closed relationship between the conductivity and Joule heat for conductor, so that the electromagnet power loss is caused. The waveguide loss is shown by the propagation constant, which is caused by metal permittivity. Because of complicating boundary and absorption affection, it is difficult to analyze optical characteristics accurately. The characteristic equation for optical guided waves is a complex transcendental equation, and the numerical analysis is usually very complicated. The convenient analytical method for the all-metal-cladding rib waveguide is presented in this paper. The metal-cladding rib waveguide is converted into the two-dimension slab one by using the effective refractive method, then the equation of satisfying the effective refractive index and the formula of absorption loss coefficient are derived by the derivation method in section Ⅱ. Using this method as a example, we have calculated optical propagation characteristics of all-metal-cladding rib waveguide and the loss one for SiO_2 in the section Ⅲ. The conclusion is given in the section Ⅳ.

1 Theoretical analysis

1.1 The dielectric constants for all-electrode rib waveguide

Fig. 1 shows the cross section of metal-all ridge waveguide. The rib width is a, and the inner rib height is b, and the outer regions of the rib have a thickness of b-h. The refractive indexes of the waveguide core and the substrate are n_1 and n_2, respectively. The electrode which is metal cladding is absorbing medium. Its complex refractive index, $\hat{n}_m$, is given by[5]

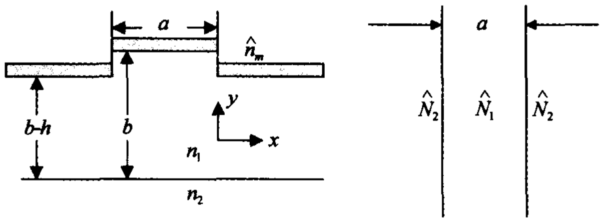

Fig. 1 Metal cladding rib waveguide and its effective slab one

$$\hat{n}_m = n_m - jk_m \tag{1}$$

where n_m and k_m are metal real refractive index and extinction coefficient, respectively. The relative permittivity of each medium layer in the waveguide are expressed as

$$\varepsilon_i = n_i^2 \quad (i=1,2) \tag{2}$$

$$\hat{\varepsilon}_m = \hat{n}_m^2 = (n_m - jk_m)^2 = n_m^2 - k_m^2 - j2n_mk_m \equiv -\varepsilon_m - jK_m \tag{3}$$

where ε_m and K_m are the absolute value of the real and of $\hat{\varepsilon}_m$, respectively. For metal, $n_m < k_m$, therefore, the real and the imaginary of the metal relative dielectric constants, $\hat{\varepsilon}_m$, are negative, So we have

$$\varepsilon_m = k_m^2 - n_m^2 \quad K_m = 2n_mk_m \tag{4}$$

1.2 The absorption loss of all- metal-cladding rib waveguide

We only consider TE-modes here. According to the method of effective refractive index, the rib waveguide in Fig. 1 may be translated into the two-dimensional slab one in which X-direction thickness is a, and the refractive indexes are $\hat{N}_2$, $\hat{N}_1$ and $\hat{N}_2$, respectively. Here, $\hat{N}_1$ is the complex effective refractive index of the core in the slab waveguide in which Y-direction thickness is b and

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the refractive indexes of the three layers are n_2, n_1 and $\hat{n}_m$, respectively; $\hat{N}_2$ is the complex effective refractive index of the cladding and the substrate in the slab waveguide, in which Y-direction thickness is b-h and the refractive indexes of the three layers are also n_2, n_1 and $\hat{n}_m$, respectively.

In order to simplify the expression, we assume that $\hat{E}_1 = \hat{N}_1^2$ and $\hat{E}_2 = \hat{N}_2^2$. So $\hat{E}_1$ and $\hat{E}_2$ satisfy the TE-mode equations of the slab guide as follow

$$k_0(\varepsilon_1 - \hat{E}_1)^{1/2} b = n\pi + \arctan\frac{(\hat{E}_1 - \varepsilon_2)^{1/2}}{(\varepsilon_1 - \hat{E}_1)^{1/2}} + \arctan\frac{(\hat{E}_1 - \hat{\varepsilon}_m)^{1/2}}{(\varepsilon_1 - \hat{E}_1)^{1/2}} \tag{5}$$

$$k_0(\varepsilon_1 - \hat{E}_2)^{1/2}(b-h) = n\pi + \arctan\frac{(\hat{E}_2 - \varepsilon_2)^{1/2}}{(\varepsilon_1 - \hat{E}_2)^{1/2}} + \arctan\frac{(\hat{E}_2 - \varepsilon_3)^{1/2}}{(\varepsilon_1 - \hat{E}_2)^{1/2}} \tag{6}$$

where $n = 0, 1, 2, \cdots$ is the mode-order number in the Y direction, and $k_0 = 2\pi/\lambda_0$ is the wave number in the vacuum, and λ_0 is the laser wavelength in the vacuum. It is convenient to introduce the following parameters: $V_1 = k_0(\varepsilon_1 - \varepsilon_2)^{1/2} b$, $\hat{P}_1 = (\varepsilon_1 - \hat{E}_1)^{1/2}/(\varepsilon_1 - \varepsilon_2)^{1/2}$, $\hat{Q}_1 = \hat{Q}_2 = (\varepsilon_1 - \varepsilon_2)^{1/2}/(\varepsilon_1 - \hat{\varepsilon}_m)^{1/2}$, $V_2 = k_0(\varepsilon_1 - \varepsilon_2)^{1/2}(b-h)$, $\hat{P}_2 = (\varepsilon_1 - \hat{E}_2)^{1/2}/(\varepsilon_1 - \varepsilon_2)^{1/2}$. From these parameters, the equation (5) and (6) are simplified as

$$V_1\hat{P}_1 = (n+1)\pi - \sin^{-1}\hat{P}_1 - \sin^{-1}\hat{Q}_1\hat{P}_1 \tag{7}$$

$$V_2\hat{P}_2 = (n+1)\pi - \sin^{-1}\hat{P}_2 - \sin^{-1}\hat{Q}_2\hat{P}_2 \tag{8}$$

Generally, $|Q_1|, |Q_2| \ll 1$. When b and b-h are large enough, $V_1, V_2 \gg 1$, and $|\hat{P}_1|, |\hat{P}_2| \ll 1$. On the conditions above, equations (7) and (8) can be approximately expressed as

$$V_1\hat{P}_1 = (n+1)\pi - \hat{P}_1 - \hat{Q}_1\hat{P}_1 \tag{9}$$

$$V_2\hat{P}_2 = (n+1)\pi - \hat{P}_2 - \hat{Q}_2\hat{P}_2 \tag{10}$$

The solutions of the above equations can be written as

$$\hat{P}_1 = (n+1)\pi/(1 + V_1 + \hat{Q}_1) \tag{11}$$

$$\hat{P}_2 = (n+1)\pi/(1 + V_2 + \hat{Q}_2) \tag{12}$$

From (11), (12) and the formulas of $V_1, V_2, \hat{P}_1, \hat{P}_2, \hat{Q}_1, \hat{Q}_2$ parameters, we have respectively

$$\hat{E}_1 = E_1 - jL_1 \tag{13}$$

$$\begin{cases} E_1 = \varepsilon_1\left\{1 - \dfrac{(n+1)^2\pi^2}{\varepsilon_1[(\varepsilon_1-\varepsilon_2)^{-1/2} + k_0 b]^2}\left[1 - \dfrac{2\cos\left[\dfrac{1}{2}\arctan\dfrac{K_m}{\varepsilon_1+\varepsilon_m}\right]}{[(\varepsilon_1+\varepsilon_m)^2 + K_m^2]^{1/4}[(\varepsilon_1-\varepsilon_2)^{-1/2} + k_0 b]}\right]\right\} \\ L_1 = \dfrac{2(n+1)^2\pi^2\sin\left[\dfrac{1}{2}\arctan\dfrac{K_m}{\varepsilon_1+\varepsilon_m}\right]}{[(\varepsilon_1+\varepsilon_m)^2 + K_m^2]^{1/4}[(\varepsilon_1-\varepsilon_2)^{-1/2} + k_0 b]^3} \end{cases} \tag{14}$$

$$\hat{E}_2 = E_2 - jL_2 \tag{15}$$

$$\begin{cases} E_2 = \varepsilon_1\left\{1 - \dfrac{(n+1)^2\pi^2}{\varepsilon_1[(\varepsilon_1-\varepsilon_2)^{-1/2} + k_0(b-h)]^2}\left[1 - \dfrac{2\cos\left[\dfrac{1}{2}\arctan\dfrac{K_m}{\varepsilon_1+\varepsilon_m}\right]}{[(\varepsilon_1+\varepsilon_m)^2 + K_m^2]^{1/4}[(\varepsilon_1-\varepsilon_2)^{-1/2} + k_0(b-h)]}\right]\right\} \\ L_2 = \dfrac{2(n+1)^2\pi^2\sin\left[\dfrac{1}{2}\arctan\dfrac{K_m}{\varepsilon_1+\varepsilon_m}\right]}{[(\varepsilon_1+\varepsilon_m)^2 + K_m^2]^{1/4}[(\varepsilon_1-\varepsilon_2)^{-1/2} + k_0(b-h)]^3} \end{cases} \tag{16}$$

The complex effective refractive index of two-dimensional slab waveguide in X-direction, satisfying the characteristic equation of TM mode in the way of the concealed function, is[6]

$$F(\hat{N}, \hat{E}_1, \hat{E}_2) = m\pi + 2\arctan\frac{\hat{E}_1(\hat{N}^2 - E_2)^{1/2}}{E_2(\hat{E}_1 - \hat{N}^2)^{1/2}} - k_0(\hat{E}_1 - \hat{N}^2)^{1/2} a = 0 \tag{17}$$

where $m = 0, 1, 2 \cdots$ is the order number of the modes. In practical calculation, L_1 and L_2 in the equation (14) and (16) are small in quantities, therefore, $\hat{E}_1$ and $\hat{E}_2$ defined by equation (13) and (15) can be considered as the imaginary adding quantities on the basis of E_1 and E_2, which are $\Delta\hat{E}_1 = -jL_1$, $\Delta\hat{E}_2 = -jL_2$. Assuming that N is a real solution of the equation (17) when $L_1 = 0$ and $L_2 = 0$, because the L_1 and L_2 are small in quantities, therefore, when E_1 is changed into $\hat{E}_1$ by $\Delta\hat{E}_1$, E_2 is changed into $\hat{E}_2$ by $\Delta\hat{E}_2$. So we can think approximately that the imaginary adding quantity for N is $\Delta\hat{N} = -j\Delta N$, then $\hat{N} = N - j\Delta N$. From (17), we have

$$\left[\frac{\partial F}{\partial \hat{N}}\right]_0 \Delta\hat{N} + \left[\frac{\partial F}{\partial \hat{E}_1}\right]_0 \Delta\hat{E}_1 + \left[\frac{\partial F}{\partial \hat{E}_2}\right]_0 \Delta\hat{E}_2 = 0 \tag{18}$$

where $()_0$ is quantity at $L_1 = 0$ and $L_2 = 0$. From (18), we have

$$\Delta N=-\left[L_1\left(\frac{\partial F}{\partial \hat{E}_1}\right)_0+L_2\left(\frac{\partial F}{\partial \hat{E}_2}\right)_0\right]\Big/\left[\left(\frac{\partial F}{\partial \hat{N}}\right)_0\right] \quad (19)$$

The derivatives of the equation (17) with respect to $\hat{N}$, $\hat{E}_1$ and $\hat{E}_2$ are, respectively

$$\left(\frac{\partial F}{\partial \hat{N}}\right)_0=\frac{N}{(E_1-N^2)^{1/2}}\left\{k_0a+\frac{2E_1E_2(E_1-E_2)}{(N^2-E_2)^{1/2}[E_2^2(E_1-N^2)+E_1^2(N^2-E_2)]}\right\} \quad (20)$$

$$\left(\frac{\partial F}{\partial \hat{E}_1}\right)_0=-\frac{1}{2(E_1-N^2)^{1/2}}\left\{k_0a+\frac{2E_2(N^2-E_2)^{1/2}[E_1-2(E_1-N^2)]}{E_2^2(E_1-N^2)+E_1^2(N^2-E_2)}\right\} \quad (21)$$

$$\left(\frac{\partial F}{\partial \hat{E}_2}\right)_0=-\frac{E_1(E_1-N^2)^{1/2}}{(N^2-E_2)^{1/2}}\frac{E_2+2(N^2-E_2)}{E^2(N^2-E_2)+E_2^2(E_1-N^2)} \quad (22)$$

By substituting the equation (20), (21) and (22) into (19), we have

$$\Delta N=\frac{L_1}{2N}\left\{1-\frac{2E_2(E_1-N^2)[E_1+2(N^2-E_2)]}{2E_1E_2(E_1-E_2)+k_0a(N^2-E_2)^{1/2}[E_2^2(E_1-N^2)+E_1^2(N^2-E_2)]}\right\}+\frac{L_2}{N}\frac{E_1(E_1-N^2)[E_2+2(N^2-E_2)]}{E_1E_2(E_1-E_2)+k_0a(N^2-E_2)^{1/2}[E_2^2(E_1-N^2)+E_1^2(N^2-E_2)]} \quad (23)$$

then the absorbing loss coefficient, α, is

$$\alpha=-2k_0I_m(\hat{N})=2k_0\Delta N=\frac{k_0L_1}{N}\left\{1-\frac{2E_2(E_1-N^2)[E_1+2(N^2-E_2)]}{2E_1E_2(E_1-E_2)+k_0a(N^2-E_2)^{1/2}[E_2^2(E_1-N^2)+E_1^2(N^2-E_2)]}\right\}+\frac{2k_0L_2}{N}\frac{E_1(E_1-N^2)[E_2+2(N^2-E_2)]}{E_1E_2(E_1-E_2)+k_0a(N^2-E_2)^{1/2}[E_2^2(E_1-N^2)+E_1^2(N^2-E_2)]} \quad (24)$$

From above analysis, the effective refractive index (N) of the TE mode for all-metal-cladding rib waveguide is given by the following equation

$$k_0(E_1-N^2)^{1/2}a=m\pi+2\arctan\frac{E_1(N^2-E_2)^{1/2}}{E_2(E_1-N^2)^{1/2}} \quad (25)$$

It is the equation (17) at $L_1=0$, $L_2=0$. The optimum sizes for all-metal cladding rib waveguide can be given by numerical calculation of the equations (24) and (25).

For all-metal cladding rib waveguide, the equations (24) is the formula of the absorbing loss coefficient, the equations (25) is the eigen equation of the guide mode.

2 The calculation examples

The TE-mode effective refractive index and absorption loss coefficient for different doped SiO_2 all-metal cladding rib waveguide are practically calculated as the examples. Here, $\lambda_0=1.3$ μm, $n_1=1.60$ (doped), $n_2=1.445$. In the core, $n_m=0.19$ (for Au) and the extinction coefficient is $k_m=6.1$. The numerical results are shown in Fig. 2 and Fig. 3.

From the Fig. 2 (a), we know the effective refractive index N will increas with the raise of the waveguide core thickness (inner rib) b, and the rib width a, and will decrease with the raise of the rib height h; the larger waveguide core (inner rib) thickness (for example, $b>6$ μm), corresponds to the smaller the effection of the rib height and width on the effective refractive index. When the waveguide core (inner rib) thickness is a constant, the larger the rib width corresponds to (for example, $a>2b$) the smaller the effection of the inner rib height on the effective refractive index.

In Fig. 2(b), it is clear that the absorption loss of metal will decrease with the increase of the

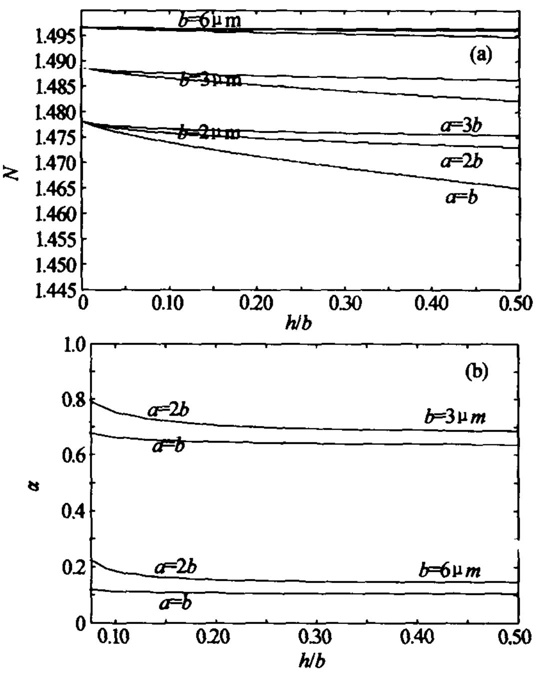

Fig. 2 The effective refractive index, N, and the absorption loss coefficient, α, vary with the normalization rib height, h/b, for E_{00}^x mode

waveguide core (inner rib) thickness. In other words, the larger the waveguide core (inner rib) thickness corresponds to the smaller the absorption loss coefficient. In the design of the real waveguide devices, the waveguide core (inner rib) thickness is designed as large as possible in order to decrease the metal absorption loss. The effection of the inner rib width and normalization height on the metal absorption loss is designed much smaller.

Fig. 3 shows that the effective refractive index and the absorption loss coefficient vary with waveguide core thickness b, when $h=b/2$ and $a=2b$. As shown in Fig. 3, the single mode can be propagated in the rib waveguide when only $b \approx 1.25 \sim 3.1$ μm, when $b \approx 3.1$ μm, E_{01}^{x} mode have been closed to cutting off. Assuming $b \approx 3.1$ μm, $a=2b=6.2$ μm and $h=1.55$ μm, the domain mode E_{00}^{x} is only propagated in the rib waveguide, and

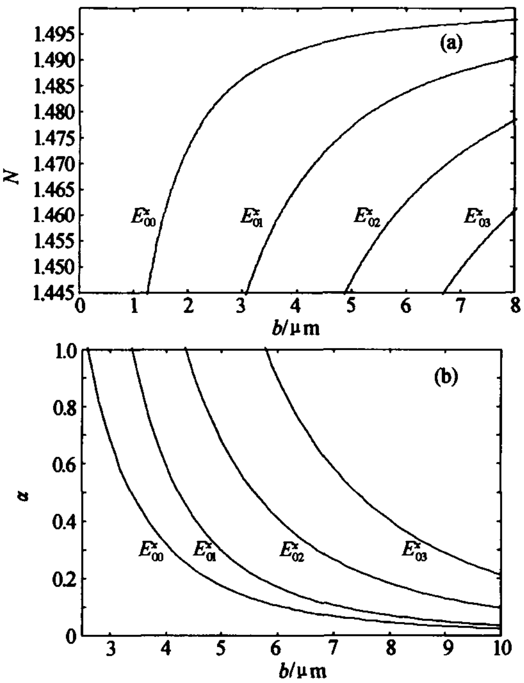

Fig. 3 The effective refractive index, N, and the absorption loss coefficient, α, vary with the rib height for the E_{0n}^{x} modes

absorption loss coefficient of the mode is small, that is $\alpha \approx 0.65$ cm^{-1}, in other words, E_{00}^{x} mode is propagated with low loss in the all-electrode rib waveguide.

3 Conclusion

Using the method proposed in this paper, the optical propagation characteristics and the absorption loss of the all-metal cladding rib waveguide are calculated conveniently and effectively. The equation (29) is the formula of the absorbing loss coefficient, and the equation (30) is the eigen equation of the guide mode. The absorption loss coefficient of the single mode is $\alpha \approx 0.65$ cm^{-1} for SiO_2 all-electrode rib waveguide.

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全电极脊形波导光传输特性

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摘　要　给出一种较为简便的全电极脊形波导的分析方法,即采用有效折射率方法把此金属包层脊形波导化成一个二维等效平板波导,然后利用微分法得到模的有效折射率满足的方程和吸收损耗系数的解析表达式．作为一个特例,用这种方法对 SiO_2 全电极脊形波导光传输特性如光传播常数和损耗进行了计算,并得到一些有益的结论．

关键词　光波导;金属包层;吸收损耗;有效折射率

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