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Power Exchange between Two Nonparallel Waveguides

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Abstract A concise analytical method (extending coefficient) of coupling between two nonparallel waveguides is reported in this paper. By this method, a novel coupled equations are derived for two non-parallel waveguides. From this, the perfect analytic solutions between two nonparallel waveguides are obtained. In according with these analytic solutions, the propagation distance and spaces of input/output-guide ports and angle may be optimized for the nonparallel waveguide structures.

Keywords Integrated optics; Non-parallel waveguides; Coupled equation
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0 Introduction;

There is many applications for nonparallel waveguides in integrated optics and relative optoelectronics devices such as tapered coupler[1] and input/output waveguides of X-type and some MEMS optical switch[2~4]. Coupled-mode theory based on local normal modes is often adopted for the analysis of the tapered and nonparallel waveguide structures[5~7]. Two types of local modes are employed in the analysis: the local array modes defined as the normal modes of the parallel waveguides at a position along the z axis and the local waveguide modes defined as the normal modes of individual uniform waveguides at z. The conventional orthogonal coupled theory[4~7] can be applied to the description of the mode coupling between the local array modes. If the local waveguide modes are chosen as the base functions in the trial solutions, the nonorthogonal coupled - mode formalism[8~11] can be used. The coupled-mode formulation based on the exact local array modes contains less approximation than the one based on waveguide modes, but usually more complicated and difficult to apply to practical waveguide structures such as circular fiber and waveguides. So far, two self-consistant nonorthogonal coupled-mode formulations have been proposed: a scalar theory by Syms and Peall[12,13] and a vector theory by Haus and Huang[14,15]. In these analytical methods, the change of nonparallel waveguide's geometric shape is translated into the one of the refractive index. Their analytical formulations and calculation is usually very complicated. In general, the coupled mode equations are merely solved by numerical techniques[16].

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In this paper the method of the extending coefficient is reported: the change of nonparallel waveguide geometry is directly translated into the one of the distance between two parallel waveguides in the coupling coefficient. In other words, the coupling coefficient is extended technologically, and novel coupling-mode equations are derived, in which the change of the distance between two parallel waveguides is included in coupling equations by exponent. The perfect analytical solutions of coupled system for nonparallel waveguides are readily obtained by using a technological transformation. Some salient features of the power exchange between two nonparallel waveguides are revealed and discussed.

1 Basic theory of optical waveguide coupling

1.1 Basic coupling equations

The analysis hereafter is carried out for two-dimensional configurations , but the results are applicable for three-dimensional structures as well by utilizing the effective index method.

Coupling equations between two parallel waveguides are expressed as[17]

$$\frac{dA_1}{dz} = -jK_{12}A_2\exp(-j2\delta z) \tag{1}$$

$$\frac{dA_2}{dz} = -jK_{21}A_1\exp(j2\delta z) \tag{2}$$

Where A_1 and A_2 is the field amplitudes, $2\delta=\beta_2-\beta_1$, β_1 and β_2 are the propagation constants of guide 1 and 2, respectively. K_{12} and K_{21} are coupling coefficient, and

$$K_{21} = \omega\varepsilon_0 \frac{\iint (n^2-n_2^2)e_1^* e_2 dxdy}{\iint (e_1^* \times h_1 + e_1 \times h_1^*) \hat{z} dxdy} \tag{3}$$

$$K_{12} = \omega\varepsilon_0 \frac{\iint (n^2-n_1^2)e_2^* e_1 dxdy}{\iint (e_2^* \times h_2 + e_2 \times h_2^*) \hat{z} dxdy} \tag{4}$$

where

$$n^2 = (n_1^2-n_0^2)+(n_2^2-n_0^2)+n_0^2 \tag{5}$$

is refractive index.

The equations (1)～(4) are general formalism of coupling between optical waveguides, where integrated range in the equations (3) and (4) is the area of whole cross sections of coupling optical waveguide.

Note that the self-coupling coefficient K_{11} and K_{22} in the coupling equations (1) and (2) have been neglected, because they are very small generally, that is $|K_{11}|, |K_{22}| \ll |K_{12}|, |K_{21}|$.

1.2 Coupling coefficient with waveguide structure parameters

The coupling coefficient between the parallel optical waveguides whose material and size are the same, are obtained from the equation (3) and (4) as

$$K_{12}=K_{21}=K=\frac{2ph^2}{\beta(w+2/p)(h^2+p)}\cdot \exp(-ps) \quad (6)$$

where

$$p^2=\beta^2-n_2^2k^2 \quad (7)$$

$$h^2=n_1^2k^2-\beta^2 \quad (8)$$

where $k=2\pi/\lambda$, and β is the propagation constants of guide 1 and 2.

2 A novel coupling analytical methods between nonparallel waveguides

2.1 The coupling equations between two nonparallel waveguides

A schematic drawing of two-nonparallel-coupling-waveguides system is shown in Fig. 1. This structure consists of two single-mode coupled waveguides. In order to simplify analysis, the assumption are made as follow:

1) The sizes and material for two waveguides are the same.

2) The phases of two coupled modes is matched in propagation direction, which is

$$\beta_1=\beta_2 \text{ or } 2\delta=0 \quad (9)$$

3) The taper varies slowly so that the coupling to the radiation modes and /or the effect of wavefront tilt may be neglected.

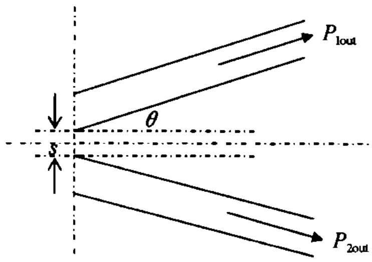

Fig. 1 The coupling system of nonparallel double waveguides

The equation (6) shows that the coupled coefficient changes by the exponent of the interval between two parallel waveguides. We assume that nonparallel waveguides in Fig. 1. can be considered as a superposition of two parallel waveguides. Therefore, the coupled system of the nonparallel guides may be expressed as the one of the parallel guides by the continuing variation of the interval of the parallel guides defined at z-direction. Therefore the interval s of parallel guides in the equation (6) is translated into

$$s \to s+2z\tan\theta \quad (10)$$

By substituting (10) into (6), the coupling coefficients of two nonparallel waveguides can be expressed as

$$K_{\text{no-parallel}}=\frac{2ph^2}{\beta w(h^2+p^2)}\exp(-ps-2pz\tan\theta)= K\exp(-2pz\tan\theta) \quad (11)$$

By substituting (11) and $K_{12}=K_{21}=K_{\text{no-parallel}}$ into equation (1) and (2), we obtain

$$\frac{dA_1}{dz}=-jKA_2\exp(-2pz\tan\theta) \quad (12)$$

$$\frac{dA_2}{dz}=-jKA_1\exp(-2pz\tan\theta) \quad (13)$$

which include all information such as the angle of two nonparallel guides, the guide interval of the input ports, and propagation distance and the power exchanging with these parameters. From coupling equations derived here between two nonparallel waveguides, we can present perfect analytical solutions.

2.2 Analytical solutions

A set of effective analytical solutions of the equation (12) and (13) can be derived using a mathematical transformation.

By substituting the equation (13) into the derivative of the equation (12) with respect to z, we have

$$\frac{d^2A_1}{dz^2}+2p\tan\theta\frac{dA_1}{dz}+K^2A_1\cdot \exp(-4pz\tan\theta)=0 \quad (14)$$

In a similar way, we have also

$$\frac{d^2A_2}{dz^2}+2p\tan\theta\frac{dA_2}{dz}+K^2A_2\cdot \exp(-4pz\tan\theta)=0 \quad (15)$$

To solve the equation (14) and (15)

$$t=f(z) \quad (16)$$

is defined, so the equations (14) is

$$\frac{d^2A_1}{dt^2}[f'(z)]^2+\frac{dA_1}{dt}[f''(z)+2p\tan\theta f'(z)]+ K^2A_1\exp(-4pz\tan\theta)=0 \quad (17)$$

Assuming

$$f''(z)+2p\tan\theta f'(z)=0 \quad (18)$$

then

$$f'(z)=\exp(-2pz\tan\theta) \quad (19)$$

From(16)and(18), we have

$$t=-\frac{1}{2p\tan\theta}\exp(-2pz\tan\theta) \qquad (20)$$

By substituting (18)and (19) into (12), we have

$$\frac{d^2A_1}{dt^2}+K^2A_1=0 \qquad (21)$$

The general solution of equation (21) may be expressed as

$$A_1=C_1\cos(Kt)+C_2\sin(Kt) \qquad (22)$$

By using the equation (12) and (20), the initial conditions of the equation (21) are written as

$$t|_{z=0}=-\frac{1}{2p\tan\theta} \qquad (23)$$

$A_1(z)|_{z=0}=0$(the input condition assumed),

$$\frac{dA_1}{dz}|_{z=0}=-jK \qquad (24)$$

$A_2(z)|_{z=0}=0$(the input condition assumed),

$$\frac{dA_2}{dz}|_{z=0}=0 \qquad (25)$$

By substituting (23)and(24)into(22), we have

$$C_1\cos\left[\frac{K}{2p\tan\theta}\right]-C_2\sin\left[\frac{K}{2p\tan\theta}\right]=0 \qquad (26)$$

$$C_1\sin\left[\frac{K}{2p\tan\theta}\right]+C_2\cos\left[\frac{K}{2p\tan\theta}\right]=-j \qquad (27)$$

Then

$$C_1=-j\sin\left[\frac{K}{2p\tan\theta}\right], C_2=-j\cos\left[\frac{K}{2p\tan\theta}\right] \qquad (28)$$

By substituting (28) into (22), we obtain

$$A_1=-j\sin\left[\frac{K}{2p\tan\theta}\right]\cos(Kt)-j\cos\left[\frac{K}{2p\tan\theta}\right]\cdot\sin(Kt)=-j\sin K\left[\frac{K}{2p\tan\theta}+t\right] \qquad (29)$$

By substituting (20) into (29), we have

$$A_1=-j\sin\left\{\frac{K}{2p\tan\theta}[1-\exp(-2pz\tan\theta)]\right\} \qquad (30)$$

In similar way, we obtain

$$A_2=\cos\left\{\frac{K}{2p\tan\theta}[1-\exp(-2pz\tan\theta)]\right\} \qquad (31)$$

The output power from the waveguide 1 and 2 are, respectively

$$P_1=|A_1|^2=\sin^2\{\frac{K}{2p\tan\theta}[1-\exp(-2pz\tan\theta)]\} \qquad (32)$$

$$P_2=|A_2|^2=\cos^2\{\frac{K}{2p\tan\theta}[1-\exp(-2pz\tan\theta)]\} \qquad (33)$$

which are perfect solutions of the coupling between two nonparallel waveguides. These theoretical results provide a simple analytical method for the design of other similar nonparallel waveguide devices, and avoid complicated numerical analysis for the similar devices.

2.3 Discussion of the solutions

1) The power is conservative throughout the whole propagation process of the nonparallel waveguide, that is

$$|A_1|^2+|A_2|^2=1, \text{and } \frac{d}{dz}(|A_1|^2+|A_2|^2)=0 \qquad (34)$$

which shows that the results derived in this paper is self-consistent.

2) When the nonparallel-waveguide propagation distances are enough large, that is $z\to\infty$, the output powers are

$$P_1|_{z=\infty}\to\sin^2\left[\frac{K}{2p\tan\theta}\right],$$

$$P_2|_{z=\infty}\to\cos^2\left[\frac{K}{2p\tan\theta}\right] \qquad (35)$$

which are the functions of $\tan\theta$.

3) At the output ports, the output power exchange may be all or part due to the coupling between two nonparallel guides. When θ is constant, the max-power positions and min-ones of the power exchange are given as

when P_2 is at the max-power positions

$$z=-\frac{1}{sp\tan\theta}\ln\left[1-\frac{2m\pi p\tan\theta}{K}\right],$$

$$m=0,1,2\cdots,m<\left[\frac{K}{2\pi p\tan\theta}\right] \qquad (36)$$

when P_2 is at the min-power positions

$$z=-\frac{1}{2p\tan\theta}\ln\left[1-\frac{2\left[m+\frac{1}{2}\right]\pi p\tan\theta}{K}\right],$$

$$m=0,1,2\cdots m<\left[\frac{K}{2\pi p\tan\theta}-\frac{1}{2}\right] \qquad (37)$$

4) the coupling length between two nonparallel waveguides is given as

$$L_{\text{no-parallel}}=-\frac{1}{2p\tan\theta}\ln\left[1-\frac{2\pi\tan\theta}{K}\right] \qquad (38)$$

which is the results with $m=0$ and $m=1$. With respect to the enough large m, there is no coupling length between two nonparallel guides. The result is different from parallel waveguides coupling[18].

The calculation example is shown in Fig. 2. The input conditions are assumed to be $A_1(0)=0$ and $A_2(0)=1$. The width of each guide is $w=6$ μm. The refractive indexes are $n_1=1.4888$ and $n_2=1.4660$, The effective refractive index is $N=1.4863$. The output power as function of the propagation distance z are shown in Fig. 2. (a)～(c) . When the title angle θ and the interval s between two input guides ports are smaller, the power at the output ports may exchange totally as shown in Fig 2(a). When the title angle θ and the interval s between two input ports are larger, the power at the output ports may exchange partly as shown in Fig. 2. (c).

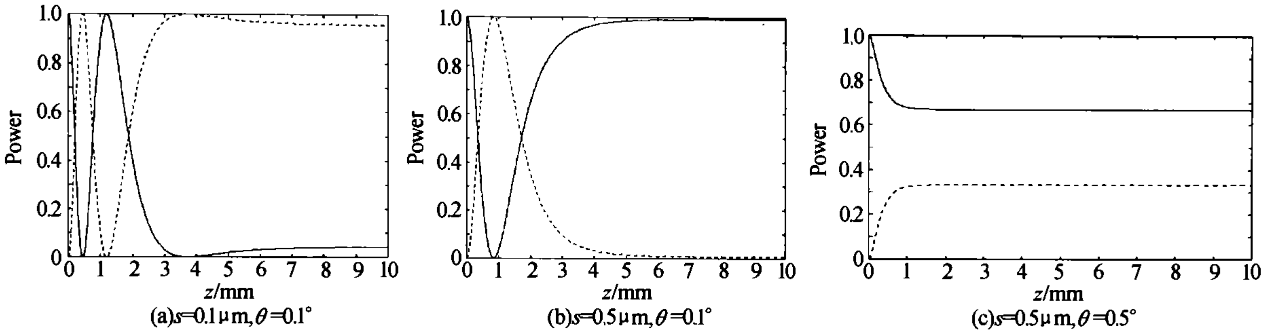

Fig. 2 Tthe curves of the output power as a function of the propagation distance z for different angle θ and interval s of the input ports. Here $A_1(0)=0$ and $A_2(0)=1$

3 Conclusion

There are many reports for the coupling of the nonparallel waveguides, in which change of nonparallel waveguide′ s geometric shape is translated into the one of the refractive index. Their analysis and calculation is very complicated and difficult. In general, the coupled mode equations are merely solved by numerical techniques.

The novel method of the extending coefficient is presented in this paper: the change of nonparallel waveguide geometric shape is directly translated into the one of the interval between two parallel waveguides in the coupling coefficient. In other words, the coupling coefficient is extended technologically, and the novel coupling-mode equations are derived, in which the change of the interval between two parallel waveguides is included in coupling equations by exponent. The perfect analytical solutions of the coupled system for the nonparallel waveguides are readily obtained by using a technological transformation. Some salient features of the power exchange between two nonparallel waveguides are revealed and discussed in the section 2. In short, a concise analytical method (extending coefficient) is presented in this paper for the nonparallel-waveguide devices in integrated optics and relative optoelectronics devices.

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两个非平行波导间的能量转换

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摘 要 提出一种分析非平行波导间耦合的简明方法-耦合系数推广法．利用这种方法，导出一种新的非平行双波导的耦合方程，由此得到一组非平行波导耦合的完美的分析解，依据这些分析解可以优化非平行波导的传输距离、输入/输出端口波导间距和夹角．
关键词 集成光学；非平行波导；耦合方程

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