Dispersion-Managed Soliton Interaction in an Optical Time-Division Multiplexed System with Random Dispersion Map *

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Abstract Dispersion-managed soliton interaction is investigated in an optical time-division multiplexed system with random dispersion map by the variational approach and the numerical simulation. It is found that the perturbation caused by random dispersion map enhances the interaction between neighboring solitons, and results in fluctuations of amplitude and separation which submerge solitons. The interaction distance relates to the standard deviation, the initial relative phases, the pulse width and the relative amplitude.

Keywords Dispersion-managed soliton; Optical time-division multiplexed system; Random dispersion map

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0 Introduction

Recently both bright and dark soliton propagation in dispersion-managed system are considered most attractive for long-haul optical communication, where dispersion-managed solitons are new types of optical solitons. For these dispersion-managed solitons, such effects as Gorden-Haus jitter, emission of radiation, and phase-match four-wave mixing are strongly reduced. But perturbations in the dispersion-managed system do harm to soliton propagation for long-haul optical communication. For example, stochastic perturbation caused by random dispersion map leads to disintegration of soliton, and degrades the stability of soliton propagation[1~7].

Optical time division multiplexing (OTDM) is a very powerful technique for the ultrahigh-speed communication. Optical time-division multiplexed systems have several advantages for system operation such as high bit-rate-to-bandwidth ratio, natural accommodation of higher bit rate payloads, and ease of supervising the multiplexed line if error checking of aggregated streams is feasible. However the mutual interaction between neighboring pulses becomes a serious obstacle in the optical time-division multiplexed system due to their periodical changes of pulse-width[8].

In this work, the dispersion-managed soliton interaction is considered in an OTDM system with random dispersion map, and some novel conclusions are obtained.

1 Theoretical analysis

The propagation of dispersion-managed soliton is governed by the nonlinear Schrödinger equation for a dimensionless envelop of the electric field[5,6]

$$\mathrm{j}\frac{\partial u}{\partial Z}+\frac{d(Z)}{2}\frac{\partial^2 u}{\partial \tau^2}+|u|^2u=0 \tag{1}$$

where $Z=z/z_d$, $z_d=\tau_0^2/|\bar{d}|$, $\tau=t/\tau_0$ · τ_0, z_d, $\bar{d}$ and $d(Z)$ are the pulse-width, dispersion length, the path-average dispersion and the dispersion map, respectively.

When $d(Z)$ is designed as an optimal dispersion map, thus all ideal soliton properties are recovered in a dispersion-managed soliton system. In a practical dispersion-managed soliton system, the dispersion map $d(Z)$ consists of a periodical constant $d_0(Z)$ and a small quantity $\delta d(Z)$ because of randomicity of periodic dispersion map[7,8]. The small quantity relates to deviation of both dispersion magnitude and the span lengths of the periodic dispersion map respectively, and becomes the function of propagation distance. For example, $d(Z)$ may be the periodical stepwise function of propagation distance in many dispersion-managed schemes.

When the random small quantity $\delta d(Z)$ is considered as the Gaussian white-noise model, we can obtain

$$\begin{aligned}&\langle \delta d(Z)\rangle=0\\&\langle \delta d(Z)\delta d(Z')\rangle=2D^2\delta(Z-Z')\end{aligned} \tag{2}$$

where D is standard deviation of dispersion caused by the random dispersion map $d(Z)$, namely, $D\sim\delta d/d_0$ or $D\sim\delta Z/Z_{+,-}$. Z_+ and Z_- are the normalized segment lengths of positive and negative dispersion fibers in a span of the dispersion map.

Replace u with u_1+u_2 in Eq. (1), the equations

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for u_1 and u_2 are given by

$$\mathrm{j}\frac{\partial u_i}{\partial Z}+\frac{d(Z)}{2}\frac{\partial^2 u_i}{\partial \tau^2}+(|u_i|^2+2|u_{3-i}|^2)u_i=0$$

$$(i=1,2) \tag{3}$$

where the fast varying terms $u_{3-i}^2 u_i^*$ ($i=1,2$) are neglected.

Lagrangian density of Eq. (3) can be defined as usual variational approach

$$L(Z,\tau)=\frac{\mathrm{j}}{2}u_i*\frac{\partial u_i}{\partial Z}+\frac{\mathrm{j}}{2}u_i*\frac{\partial u_i}{\partial \tau}+\frac{1}{4}|u_i|^4-\frac{\mathrm{d}(Z)}{4}\left|\frac{\partial u_i}{\partial \tau}\right|^2+2|u_i|^2|u_{3-i}|^2 \tag{4}$$

Then we adopt as trial function

$$u_i(Z,\tau)=\sqrt{A_i}\exp\left[-(1+jb_i)\frac{(\tau-\tau_{0i})^2}{2W_i^2}-\mathrm{j}\omega_i(\tau-\tau_{0i})+\mathrm{j}\phi_i\right] \tag{5}$$

where A_i, W_i, b_i, τ_{oi}, ω_i and ϕ_i ($i=1,2$) are the intensity, width, chirp coefficient, central position, frequency and phase of every pulse, respectively, and they all are the functions of propagation distance.

Averaged Lagrangian is defined as

$$\bar{L}(Z)=\int_{-\infty}^{\infty}L(Z,\tau)\mathrm{d}\tau \tag{6}$$

Substitution of Eqs. (4) and (5) into Eq. (6) gives

$$\bar{L}=\sum_{i=1}^{2}\left[-E_i\left(-\frac{1}{4}\frac{\mathrm{d}b_i}{\mathrm{d}Z}+\frac{b_i}{2W_i}\frac{\mathrm{d}W_i}{\mathrm{d}Z}+\omega_i\tau_{oi}+\frac{\mathrm{d}\phi_i}{\mathrm{d}Z}\right)-\frac{d(Z)}{2}\left(\frac{1+b_i^2}{2W_i^2}+\omega_i^2\right)+\frac{E_i^2}{2\sqrt{2\pi}W_i}\right]+2\frac{E_1E_2}{\sqrt{\pi}W}\exp\left(-\frac{\Delta^2}{W^2}\right) \tag{7}$$

where $E_i=\int_{-\infty}^{\infty}|u_i|^2\mathrm{d}\tau=\sqrt{\pi}A_iW_i$ is constant.

Varying Eq. (7) leads to the equations

$$\frac{\mathrm{d}\Delta}{\mathrm{d}Z}=-d(Z)\Omega$$
$$\frac{\mathrm{d}\Omega}{\mathrm{d}Z}=\frac{4E\Delta}{\sqrt{\pi}W^3}\exp\left[-\frac{\Delta^2}{W^2}\right] \tag{8}$$

where $\Delta=\tau_{01}-\tau_{02}$, $E=E_1+E_2$, and $\Omega=\omega_1-\omega_2$ are the relative separation, the total energy and the frequency difference, respectively, and $W^2=W_1^2+W_2^2$.

A rough estimation can be made below. When the relative separation Δ is far more larger than the pulse-width in propagation line, namely $\Delta \gg W$. So we assume that Ω is approximately constant by Eq. (8), then

$$\Delta-\Delta_0=-\Omega\int_0^z \mathrm{d}(x)\mathrm{d}x \tag{9}$$

where Δ_0 is the initial relative separation.

When there are m spans in the dispersion-managed map, and m is far more than 1 (long distance), namely $Z=m(Z_+ + Z_-)$, and $m \gg 1$, we can get

$$\Delta(Z)=-\Omega\left[m\left(\int_0^{z_+}d_1(1+D)\mathrm{d}x+\int_{z_+}^{z_++z_-}d_2(1+D)\mathrm{d}x\right)\right]+\Delta_0 \tag{10}$$

where d_1 and d_2 are the normalized dispersion amplitudes of positive and negative dispersion fibers corresponding to the path-average dispersion $\bar{d}$ in a span of dispersion map.

The fluctuation of pulse separation is given by

$$[\delta\Delta(Z)]^2=m^2\Omega^2D^2(d_1^2Z_+^2+d_2^2Z_-^2)=\frac{1}{4}\Omega^2D^2Z^2(d_1^2+d_2^2) \tag{11}$$

where we have resumed $Z_+=Z_-$.

As a rough estimation, the timing shifts of neighboring soliton are given by

$$\delta\Delta\approx\sqrt{[\delta\Delta(Z)]^2}=\frac{1}{2}|\Omega DZ|\sqrt{(d_1^2+d_2^2)} \tag{12}$$

As the reference [8], in an optical time-division multiplexed system the interaction distance of two neighboring solitons is defined as the distance where the timing shifts of neighboring pulses exceed a half of their pulse-width. So we can get the interaction distance

$$L_{\text{interaction distance}}=1.76/|\Omega D|\sqrt{(d_1^2+d_2^2)} \tag{13}$$

where the full width at half maximum (FWHM) of soliton is 1.76.

We can see the interaction distance is inversely proportional to the frequency difference, the standard deviation and the dispersion amplitude when pulse separation is far more larger than the pulse-width.

2 The effect of random dispersion map on soliton interaction

Interaction between adjacent solitons may induce the soliton collision and the time jitter, thus increase the bit-error-rate, and becomes a serious obstacle in the optical time-division multiplexed system. Moreover in the optical time-division multiplexed dispersion-managed soliton system with random dispersion map, the situation is very distinguished from that of normal soliton system.

Although Eq. (1) has a Hamiltonian structure, it is not integrable because of the inhomogeneity (Z dependent coefficient). Such a behave of u_i ($i=1,2$) may be obtained as its response averaged over $d(Z)$, which results in path-averaged soliton. However, taking a simple average of $d(Z)$ fails to provide the proper response because of the correlations with variations of u_i ($i=1,2$) and $d(Z)$. In this work, we hope that the numerical simulations may provide the proper and clear response combining the variational approach.

The numerical simulations of pulse propagation are performed in the optical fiber link with a random dispersion map, the fiber segments z_+ and z_- are 50 km. The path-average dispersion is $\bar{d}=-1.27$ ps^2/km, their fiber dispersions are $d_1=22$ (corresponding to

$-28.00\ ps^2/km$) and $d_2 = -20$ (corresponding to $25.50\ ps^2/km$). If we take the pulse-width 10 ps, the dispersion length is about 122 km corresponding the path-average dispersion. The randomly modulation with standard deviation D is used in dispersion amplitude of every fiber segment. As usual, an average of a few different random sequences of standard deviation has been used in the below numerical simulations.

Fig. 1 shows the interaction distance versus the standard deviation for different situations, the solided line is the result from the Eq. (13), and the dotted line is the result of the numerical simulation. Initially input pulse is $1.2 \times \{\exp[-(\tau-5)^2] + \exp[-(\tau+5)^2 - j \times 10^{-3}(\tau+5)]\}$. In the dispersion managed soliton system with random dispersion map, we can see, the interaction distance from numerical simulation is around about the results calculated by Eq. (13). The result from the variational approach is in agreement with that of the numerical simulation when the relative separation is far more larger than the pulse-width in propagation line. So we can make a rough estimation about the interaction distance by the variational approach.

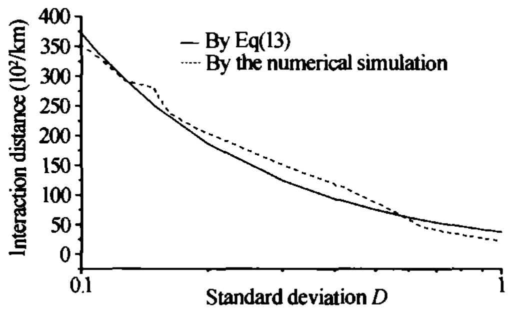

Fig. 1 The interaction distance versus the standard deviation for the results by the Eq. (13) and the numerical simulation (10 ps soliton)

Large fluctuation in soliton separation will lead to large fluctuation of amplitude, and result in the submergence of soliton. Fig. 2 shows the interaction distance versus the relative amplitude for different initial relative phases, Fig. 3 and Fig. 4 are the normalized intensity of two solitons versus the propagation distance for the different initial relative phases and the relative amplitude. The initially input soliton pulse is $1.2 \times [\text{sech}(\tau - \Delta/2) + B \times \text{sech}(\tau + \Delta/2) \times e^{j\varphi}]$, where the separation of solitons is $\Delta = 7$ (corresponding to about 15 Gbit/s for 10 ps pulse), and $\phi = 0$ (in phase) or $\phi = \pi$ (out of phase). The standard deviation is $D = 0.5$ and the relative amplitude is chosen. In the dispersion managed soliton system with random dispersion map, we can see that the perturbation enhances the interaction between solitons, and results in collision which submerges solitons. The reason is that the perturbation amplifies the line wave with small amplitude which affects neighbouring solitons, further correlates the two solitons and then enhances their interaction. The effect relates to the standard deviation, the initial relative phases, the pulse width and the relative amplitude.

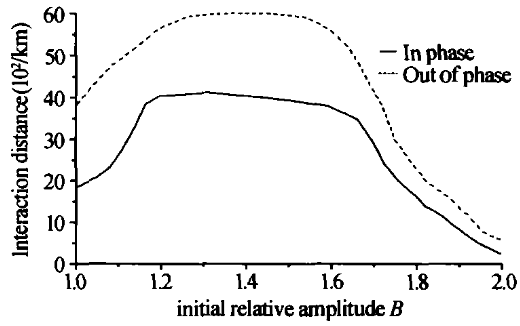

Fig. 2 The interaction distance versus the initial relative amplitude for the different initial phases (10 ps soliton)

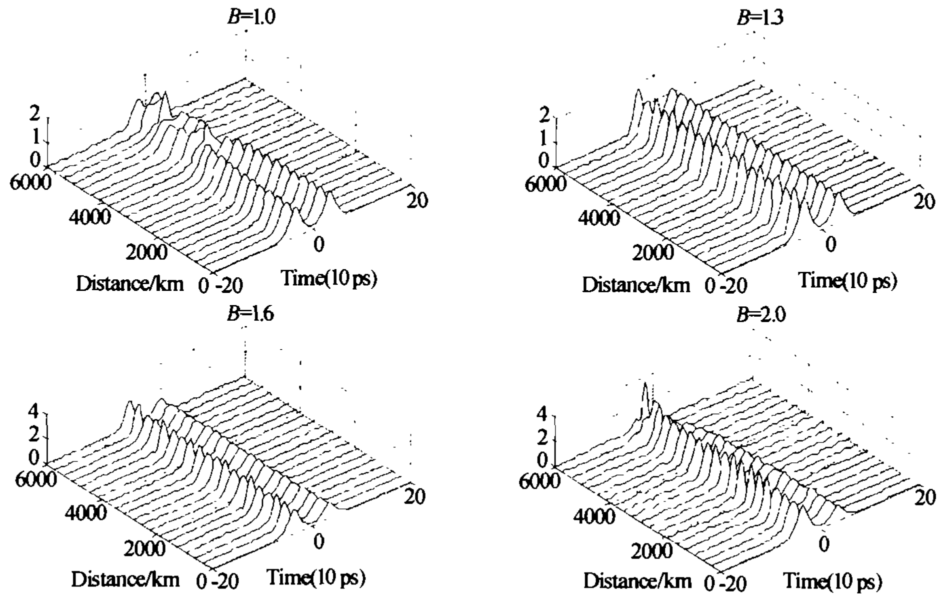

Fig. 3 The normalized intensity of two solitons (in phase) versus the propagation distance for the different relative amplitudes (10 ps soliton)

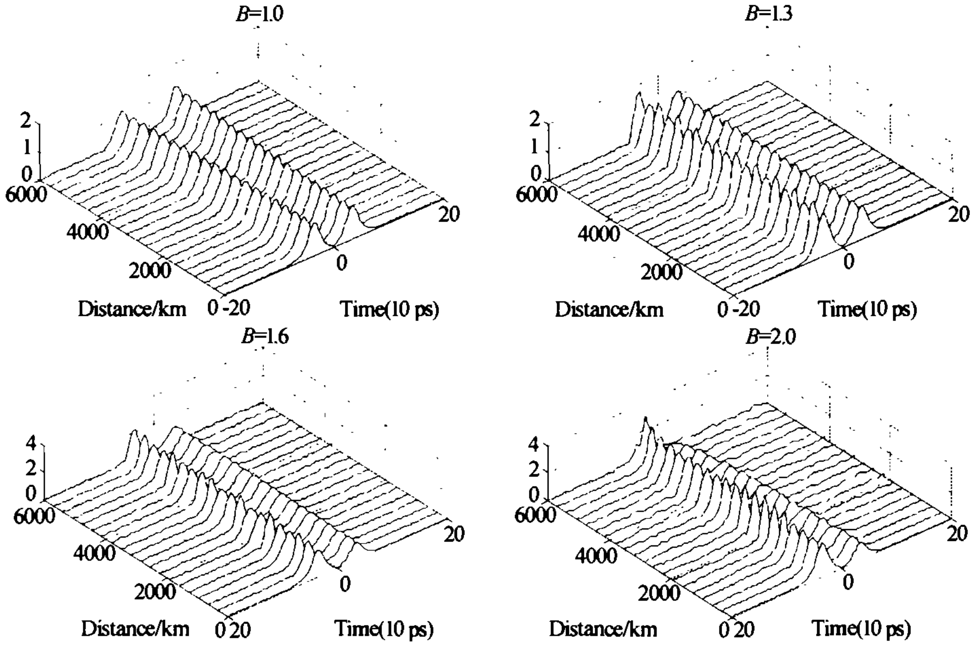

Fig. 4 The normalized intensity of two solitons (out of phase) versus the propagation distance for the different initial relative amplitudes (10 ps soliton)

3 Conclusion

Dispersion-managed soliton interaction is considered in an optical time-division multiplexed system with random dispersion map, and the effect of random dispersion map on soliton interaction is investigated by the variational approach and the numerical simulation. The result from the variational approach is in agreement with that of the numerical simulation when the relative separation is far more larger than the pulse-width in propagation line. It is found that the perturbation caused by random dispersion map enhances the interaction between neighboring solitons, and results in fluctuations of amplitude and separation which submerge solitons. The perturbation amplifies the line wave with small amplitude which affects neighbouring solitons, further correlates the two solitons and then enhances their interaction. The effect relates to the standard deviation, the initial relative phases, the pulse width and the relative amplitude.

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具有随机色散的光时分复用系统中色散管理孤子相互作用

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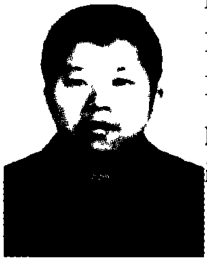
摘 要 本文采用变分法和数值方法研究了具有随机色散的光时分复用系统中色散管理孤子相互作用. 发现随机色散增强了孤子间相互作用,产生了导致孤子淹没的振幅和间隔涨落. 相互作用距离与初始间隔、初始相对相位、脉宽和初始相对振幅有关.
关键词 色散管理孤子;光时分复用;随机色散



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