Study on Unique Saturation Properties in Fiber Optical Parametric Amplifiers

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Abstract Two very pithy and powerful analytical expressions for saturated signal gain and signal output power of fiber optical parametric amplifiers are educed with numerical analysis. Those are realized in three steps, first, a series of numerical values are worked out by numerically solving the couple NLS equations that governed the fiber parameter process. Then the function form for saturated signal gain is figured out by control variates method. At last, adopting least square method, three coefficients are finely obtained with the maximum relative error (in terms of calculated saturated signal gain with numerical integration) no more than 0.46‰. At the same time, the analytical expression for saturated signal output power is worked out as well. Results of this work agree well with existing experiment results.

Keywords Optical fiber communication technology; Optical fibers; Optical parametric amplifiers (OPAs); Saturated signal gain

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0 Introduction

With the increasing availability of high output power EDFA as well as the development of highly nonlinear fiber (HNLF), more and more fiber optical parametric amplifiers (OPAs) are operated near or in the saturated-gain region rather than small-signal gain region[1,2]. Several studies on small-signal gain have been reported for parametric amplifiers or wavelength converters based on fiber nonlinearity[3,4]. However, there has been no study on the calculation of signal gain in the gain-saturated condition. Kyo Inoue and Takaaki Mukai have investigated noise characteristics and the wavelength dependence of saturated - gain for fiber OPAs[5,6], but not the input pump and signal powers dependence of gain saturation. In this letter, taking into account the pump depletion, we investigate the nonlinearity coefficient, the input pump and signal powers dependence of gain saturation, respectively. By adopting numerical integration, control variates method and least square method, two very pithy and powerful analytical expressions for saturated signal gain and saturated signal output power are educed. These expressions indicate some unique saturation properties in fiber OPAs.

1 Calculating saturated signal gain

Focusing on general application of fiber parametric amplifier, we consider an intense pump, a weak signal, and a generated idler, with respective carrier w_p, w_s and w_i (satisfying $2w_p = w_i + w_s$), and electric field $A_p(z)$, $A_s(z)$ and $A_i(z)$. All electric fields are in the same state of linear polarization. Neglecting attenuation in HNLF, the couple NLS equations governing the fiber parameter process may be written as[4,7]

$$\frac{\mathrm{d}P_p}{\mathrm{d}z} = -4\gamma(P_p^2 P_s P_i)^{1/2}\sin\theta \tag{1}$$

$$\frac{\mathrm{d}P_s}{\mathrm{d}z} = 2\gamma(P_p^2 P_s P_i)^{1/2}\sin\theta \tag{2}$$

$$\frac{\mathrm{d}P_i}{\mathrm{d}z} = 2\gamma(P_p^2 p_s p_i)^{1/2}\sin\theta \tag{3}$$

$$\frac{\mathrm{d}q}{\mathrm{d}z} = \Delta\beta + \gamma(2P_p - P_s - P_i) + \gamma(P_p^2 P_s/P_i)^{1/2} + (P_p^2 P_i/P_s)^{1/2} - 4(P_s P_i)^{1/2}\cos\theta \tag{4}$$

Where z is the distance from the beginning of HNLF, γ is the nonlinearity coefficient, P_p, P_s, P_i are the pump, signal and idler powers, respectively, $\theta = \Delta\beta z + 2\varphi_p(z) - \phi_s(z) - \phi_i(z)$, $\phi_p(z)$, $\phi_s(z)$ and $\phi_i(z)$ are the phase of the pump, signal, and idler, respectively, and $\Delta\beta$ is the linear phase mismatch given by[8]

$$\Delta\beta \approx -\frac{2\pi}{\lambda_0^2}\frac{\mathrm{d}D}{\mathrm{d}\lambda}(\lambda_p - \lambda_0)(\lambda_p - \lambda_s)^2 \tag{5}$$

By positioning the pump wavelength in the anomalous dispersion region ($\Delta\beta < 0$), it is possible to compensate for the nonlinear phase mismatch ($\gamma(2P_p - P_s - P_i)$) by the linear phase mismatch $\Delta\beta$ at $z = 0$. Thus, the phase matching condition can be written in the form

$$\Delta\beta = -\gamma(2P_{p0} - P_{s0} - P_{i0}) \tag{6}$$

Here, the phase mismatch parameter K should be introduced[9].

$$k = \Delta\beta + \gamma(2P_p - P_s - P_i) \tag{7}$$

The idler is assumed to be zero at $z = 0$. For this special

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case, $\theta = \frac{\pi}{2}$ at $z = 0$[5]. Following (3) and (4), this has the consequence that the signal and the idler will start to grow immediately in the fiber, consequently the nonlinear phase mismatch and K will decrease. Sin θ will oscillate between + 1 and − 1, the power will oscillate between the pump and the signal as well.

In order to calculate the saturated signal gain, equations (1) ~ (4) have been numerically solved by taking into account the pump depletion along the fiber, which is usually neglected. Fig. 1 shows the calculated pump power (dashed line) and signal power (solid line) as a function of the fiber length. As the fiber length increased, the signal output increased until reached peak value and saturated, and then decreased. Here, saturated signal gain and saturated signal output power refer to the peek values of signal gain and signal output power respectively. Then, those equations have been repeatedly solved with different values of γ, input pump power P_{p0} and input signal power P_{s0}. We obtained a series of numerical results about saturated signal gain and output power. In Fig. 2, we reported calculation results. The saturated signal gain fluctuated from 24.4 dB to 50.8 dB with input pump power ranging from 0.2 W to 4.2 W and input signal power ranging from 0.01 mW to 0.21 mW.

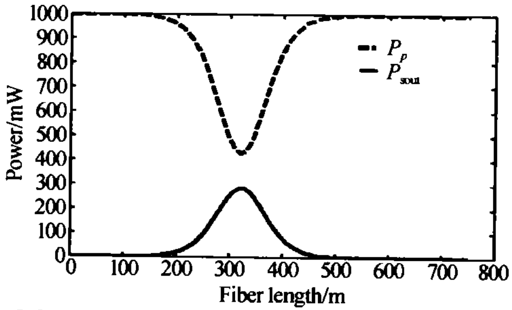

Fig. 1 Calculated pump and signal powers as a function of fiber length. $\gamma = 17\ \mathrm{W}^{-1}\mathrm{km}^{-1}$, $P_{s0} = 0.1$ mW

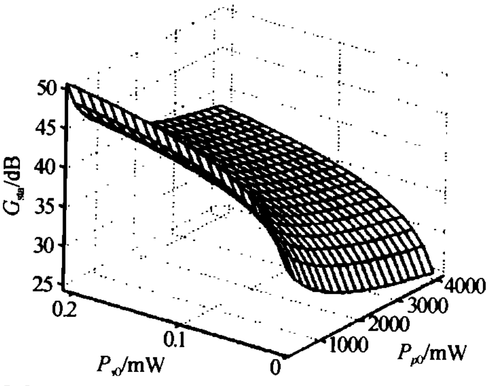

Fig. 2 Calculated saturated signal gain as a function of input pump and signal powers, $\gamma = 17\ \mathrm{W}^{-1}\mathrm{km}^{-1}$

2 Analyzing saturated signal gain properties

For the sake of clarity, we adopt control variates method to further investigate the saturated signal gain properties. At first, we study the relationship between saturated signal gain and γ, with P_{p0} and P_{s0} fixed. The results are reported in Fig. 3 for typical values of pump and signal powers. It demonstrated that γ have nothing to do with the value of saturated signal gain. It comes to a conclusion that saturated signal gain could not be improved by increasing nonlinearity coefficient of HNLF. Detailed studying found that γ affects the fiber length needed to fulfill the gain saturated condition. Their relationship is inverse ratio, which could be deduced from equations (1) ~ (4) with equation (6) satisfied.

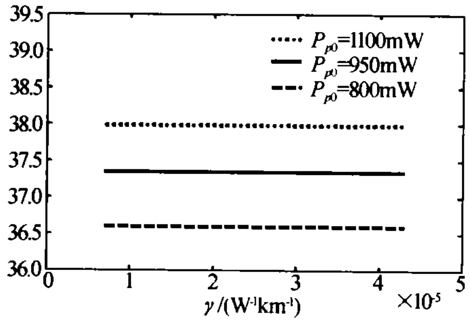

Fig. 3 Calculated saturated signal gain versus nonlinear coefficient g using three different pump powers, $P_{s0} = 0.05$ mW

Then, with γ and P_{s0} fixed, we investigate the saturated signal gain as a function of P_{p0}. Fig. 4 (a) shows that the saturated signal gain increases as pump power increases, more important, the curve is very similar to that of logarithm function. Fig. 4 (b) shows the curves for the saturated signal gain as a function of $\log_{10}(P_{p0})$. In fact, they are beelines rather than curves. Noticing that slopes are identical while intercepts are different for different P_{s0}, we can note

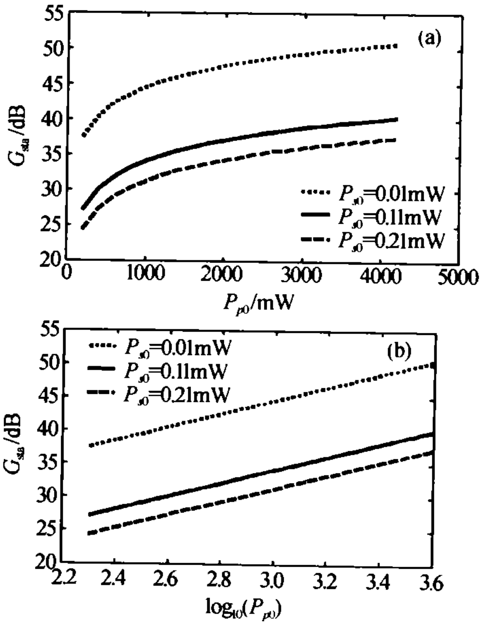

Fig. 4 Saturated signal gain evolution as a function of (a) pump power, (b) the logarithm of pump power, for a fiber OPA with three different signal input power, $g = 17\mathrm{W}^{-1}\mathrm{km}^{-1}$

the saturated signal gain G_{sat} as follows

$$G_{sat}=A\log_{10}(P_{p0})+f(P_{s0}) \tag{8}$$

Where A is a constant, while f is a function of P_{s0}.

Last, but not the least, we study the saturated signal gain as a function of P_{s0} with γ and P_{p0} fixed. Fig. 5 (a) shows that the saturated signal gain decreases as P_{s0} increases. We estimate the saturated signal gain is linear with P_{s0}^{-1}. Fig. 5 (b) shows the saturated signal gain as a function of P_{s0}^{-1}, which is more similar to logarithm curve than beeline. Fig. 5 (c) demonstrates that the saturated signal gain is linear with $\log_{10}(P_{s0}^{-1})$ or $\log_{10}(P_{s0})$. Noticing that slopes are identical while intercepts are different for different P_{p0}, G_{sat} may be written as

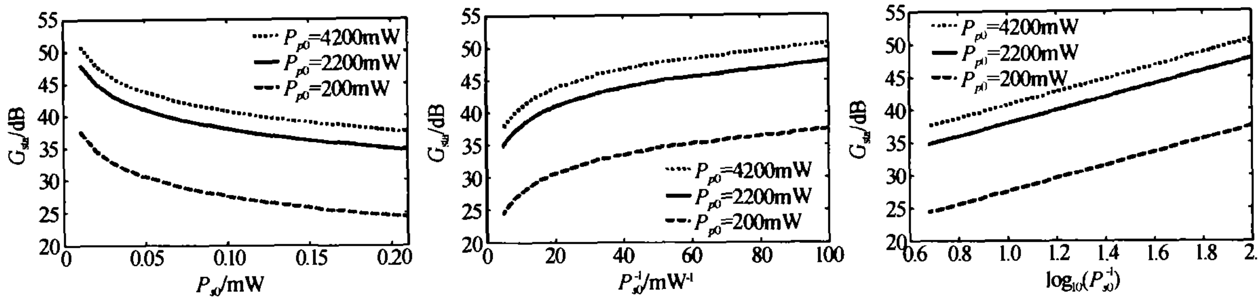

Fig. 5 Saturated signal gain evolution as a function of (a) signal input power, (b) the reciprocal of signal input power, (c) the logarithm of the reciprocal of signal input power for a fiber OPA with three different pump powers, $\gamma=17\ \mathrm{W^{-1}km^{-1}}$

$$G_{sat}=B\log_{10}(P_{s0})+g(P_{p0}) \tag{9}$$

where B is a constant, while g is a function of P_{p0}. As a matter of fact, equations (8) and (9) should be satisfied simultaneously. Generally, G_{sat} can be written as

$$G_{sat}=A\log_{10}(P_{p0})+B\log_{10}(P_{s0})+C \tag{10}$$

where C, as well as A and B, is a constant waiting for giving specific values.

Here, P_{s0} and P_{p0} are two independent variables. Adopting multivariable least square method, three coefficients are finely worked out with the maximum relative error (in terms of calculated saturated signal gain with numerical integration) no more than 0.46‰. Correspondingly, (10) is written as

$$G_{sat}=-5.4178+9.9943\times\log_{10}(P_{p0})+9.9974\times\log_{10}(P_{s0}) \tag{11}$$

Approximatively, (11) could be rewritten as

$$G_{sat}=10\log_{10}\left(\frac{P_{p0}}{3.4816\times P_{s0}}\right) \tag{12}$$

In this form, the maximum relative error (in terms of calculated saturated signal gain with numerical integration) is no more than 0.614‰.

In order to test and verify the correctness of this expression, we calculate the saturated signal gain using (12) with $P_{p0}=1148.2$ mW (after considering 0.4 dB insertion loss for the pump power 31.0 dB), $P_{s0}=0.0041$ mW (or −23.9 dBm as reported in [10]). The calculated result equals to 49.05 dB which do well with the experiment result 49 dB reported by Jonas Hansryd[10].

As is well known, signal gain in decibel units is defined as

$$G_s=10\log_{10}\left(\frac{P_{sout}}{P_{s0}}\right) \tag{13}$$

where P_{sout} is the signal output power of fiber OPAs. Hence the saturated output power P_{s_sat} may be written as

$$P_{s\_sat}=\frac{P_{p0}}{3.4816} \tag{14}$$

It shows that 28.72% of pump power could be transferred to signal power.

It is necessary and very interesting to compare saturated signal gain G_{sat} with small-signal gain G_{small}, which is given by[2]

$$G_{small}=10\log_{10}(e^2)P_{p0}L_g-6 \tag{15}$$

From (12) and (15), it comes to three conclusions: first, both G_{sat} and G_{small} can be improved by increasing P_{p0}, but the efficiencies are quite different. If P_{p0} were doubled, G_{small} would be more than doubled, while G_{sat} would only increase 3 dB. Second, G_{small} can be linearly increased with γ increased, while G_{sat} would not be affected. Last, the larger P_{s0}, the smaller G_{sat}, whereas G_{small} have nothing to do with P_{s0}. All in all, saturated signal gain is quite different from small signal-gain.

3 Summary

We have studied the fiber optical parametric amplifier in gain-saturated region, where the gain properties is quite different from that of small-signal gain. Analytical expressions for saturated signal gain and output power were presented with a condition that the phase matching (6) is satisfied by appropriately positioning the pump and signal wavelengths. The saturated signal gain determined by P_{p0} and P_{s0} while saturated signal output power determined only by P_{p0}. Their common property is that both have nothing to do with γ and fiber length. In other words, we couldn't improve saturated signal gain or output power by increasing γ or fiber length, while this is usually used to improve small - signal gain. Fortunately, not only small - signal gain but also the saturated signal gain could be improved by improving pump power P_{p0}, even more than 28% of pump power could be transferred to signal power in the gain-saturated condition. These are very useful for guiding the design and analyses of fiber OPAs.

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光纤参量放大饱和增益特性研究

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摘　要　通过数值分析的方法推导出光纤参量放大饱和信号增益和信号输出功率的数学表达式．计算分三步，首先数值求解描述参量放大过程的非线性耦合方程得出一系列数值，然后用控制变量法找到饱和信号增益的函数形式，最后用最小二乘法拟合出系数（与数字积分结果比较，最大相对误差不超过0.46‰）．同样也得出了饱和信号输出功率的表达式．计算结果与已有实验结果相吻合．

关键词　光纤通信技术；光纤；光纤参量放大；饱和信号增益

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