Emission Probability of a Non-degenerate Two-photon Mazer*

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Abstract The quantum theory of the two-photon mazer, previously limited to the degenerate case, is extended to the non-degenerate situation. The analytical expression for the atomic emission probability is obtained and the corresponding behavior is investigated. The results show that the emission probability behaviors in the non-degenerate two-photon mazer differ from those in the degenerate two-photon mazer, since two cavity modes are allowed in the mazer so that there are two freedoms changing the potentials that the atoms experience as they transit the cavity.

Keywords Emission probability; Non-degenerate two photon process; Mazer

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0 Introduction

A quantum-mechanical treatment of the center-of-mass (c. m.) motion is required in the limitation of ultracold atoms since the kinetic energy of c. m. motion may be comparable to the atom-field interaction energy. This treatment is shown to result in a completely new kind of induced emission in a cavity. In this way, Scully et al[1] have introduced a new concept called mazer (microwave amplification via z-motion-induced emission of radiation) and have done a series of researches[2]. The dressed state analysis shows that the cavity field can act like a quantum-mechanical potential for an ultracold incident atom. The nature of the potentials depends on the mode profile of the cavity field as well as on the atom-field coupling strength. Thus, the atom, besides changing its electronic states, can be either reflected or transmitted through the cavity. Works in Refs[1,2] are limited to the one-photon interaction between one-mode cavity field and two-level atoms. It is well known that the two-photon process is also an important process in the interaction between atoms and quantum electromagnetic fields. Many nonclassical effects, for example, the squeezed states of light, involve the two-photon processes. The two-photon micromaser has been studied experimentally and theoretically[3]. Zhang et al[4] extended the concept of mazer to the two-photon process in an effective two-level-atom system and introduced the concept of two-photon mazer. However, it is limited to the degeneration case, i. e., $\omega = 2\omega_1$, here ω is the atomic transition frequency and ω_1 is the frequency of the cavity field. In paper, the restriction and the quantum theory for the non-degenerate two-photon mazer was established.

1 Analysis of non-degenerate two-photon process

A two-state atom is considered (the upper state $|a\rangle$ and the lower state $|b\rangle$) to move along the z direction on the way to a cavity of length l. The atom is coupled resonantly to two modes of the quantized cavity field, satisfying the relation $\omega=\omega_1+\omega_2$, here ω_1 and ω_2 denote the frequencies of the two cavity-modes, respectively. The atomic c. m. motion is described quantum- mechanically and the atom-field Hamiltonian operator reads in the dipole and rotating-wave approximation

$$H=\frac{p_z^2}{2M}+H_0+\hbar gu(z)(\sigma a_1^\dagger a_2^\dagger+\sigma^\dagger a_1 a_2) \quad (1)$$

where M is the atomic mass, p_z the atomic momentum operator for the c. m. motion, $u(z)$ the mode function of the cavity field, and

$$H_0=\sum_{i=1}^{2}\hbar\omega_i a_i^\dagger a_i+\hbar\omega\sigma^\dagger\sigma \quad (2)$$

with $\sigma^\dagger=|a\rangle\langle b|$ ($\sigma=|b\rangle\langle a|$) the atomic raising (lowering) operator, g the atom-field coupling constant and $a_i^\dagger$ (a_i) the photon creation (annihilation) operators.

In an interaction picture, the Hamiltonian operator is

$$H_I=\frac{p_z^2}{2M}+V(z) \quad (3)$$

where $V(z)=\hbar gu(z)(\sigma a_1^\dagger a_2^\dagger+\sigma^\dagger a_1 a_2)$. It is found

$$V(z)|\phi_{n_1,n_2}^{\pm}\rangle=V_{n_1,n_2}^{\pm}(z)|\phi_{n_1,n_2}^{\pm}\rangle=\pm\hbar gu(z)[(n_1+1)(n_2+1)]^{1/4}|\phi_{n_1,n_2}^{\pm}\rangle \quad (4)$$

where the eigenvalues are $V_{n_1,n_2}^{\pm}(z)=\pm u(z)(\hbar k_{n_1,n_2})^2/2M$ with $k_{n_1,n_2}=\kappa\mu_{n_1,n_2}$, $\kappa=(2Mg/\hbar)^{1/2}$ and $\mu_{n_1,n_2}=[(n_1+1)(n_2+1)]^{1/4}$. The eigenstates are

$$|\phi_{n_1,n_2}^{\pm}\rangle=\frac{1}{\sqrt{2}}(|a,n_1,n_2\rangle\pm|b,n_1+1,n_2+1\rangle) \quad (5)$$

Naturally, this kind of problem can be treated as a

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scattering process. It is considered where an atom is in the upper state $|a\rangle$ and with a c. m. momentum $\hbar k$ is incident upon a cavity field in the number state $|n_1, n_2\rangle$, i. e. , n_1 photons in mode 1 and n_2 photons in mode 2. The atom-field system before the scattering process is characterized by the state

$$|\Psi(z,0)\rangle=\theta(-z)e^{ikz}|^{a,n_1,n_2}\rangle=\frac{1}{\sqrt{2}}\theta(-z)e^{ikz}(|\phi^+_{n_1,n_2}\rangle+|\phi^-_{n_1,n_2}\rangle) \quad (6)$$

where the Heaviside's unit step function θ merely indicates on which side of the cavity the atom can be found. The state of the system after the interaction evolves in

$$|\Psi(z,t)\rangle=\frac{1}{\sqrt{2}}e^{-i\frac{\hbar k^2}{2M}t}\{[R_{a,n_1,n_2}(k)e^{-ikz}\theta(-z)+T_{a,n_1,n_2}(k)e^{ik(z-l)}\theta(z-l)]|a,n_1,n_2\rangle+[R_{b,n_1+1,n_2+1}(k)e^{-ikz}\theta(-z)+T_{b,n_1+1,n_2+1}(k)\cdot e^{ik(z-l)}\theta(z-l)]|b,n_1+1,n_2+1\rangle\} \quad (7)$$

where T and R are determined by the suitable combination of the reflection amplitudes $\rho^{\pm}_{n_1,n_2}$ and the transmission amplitudes $\tau^{\pm}_{n_1,n_2}$ as follows

$$R_{a,n_1,n_2}(k)=\frac{1}{2}[\rho^+_{n_1,n_2}+\rho^-_{n_1,n_2}] \quad (8)$$

$$T_{a,n_1,n_2}(k)=\frac{1}{2}[\tau^+_{n_1,n_2}+\tau^-_{n_1,n_2}] \quad (9)$$

which are the probability amplitudes that the atom is reflected or transmitted with the atom-field state remaining in the initial state $|a, n_1, n_2\rangle$ and

$$R_{b,n_1+1,n_2+1}(k)=\frac{1}{2}[\rho^+_{n_1,n_2}-\rho^-_{n_1,n_2}] \quad (10)$$

$$T_{b,n_1+1,n_2+1}(k)=\frac{1}{2}[\tau^+_{n_1,n_2}-\tau^-_{n_1,n_2}] \quad (11)$$

which are the probability amplitudes that the atom is reflected or transmitted when the atom-field state makes a transition from the initial state $|a, n_1, n_2\rangle$ to the state $|b, n_1+1, n_2+1\rangle$. Note that all the physical characteristics regarding the interaction of ultracold atoms with a high quality cavity can be calculated in terms of the above formalism that is suitable for arbitrary mode function $u(z)$.

For simplicity, it is restricted to the mesa mode function and study the emission probability properties of the system. When an initially excited two-state atom is incidently upon the cavity containing (n_1, n_2) photons in the two modes(1, 2), respectively, the probability that the atom goes to the state $|b\rangle$ and emits a photon in mode 1 together with a photon in mode 2 simultaneously is

$$P_{\text{emission}}(n_1,n_2)=|R_{b,n_1+1,n_2+1}(k)|^2+|T_{b,n_1+1,n_2+1}(k)|^2 \quad (12)$$

In the meantime, for the mesa-function form, $u(z)=1$ for $0<z<l$ and $u(z)=0$ elsewhere, the reflection amplitudes $\rho^{\pm}_{n_1,n_2}$ and the transmission amplitudes $\tau^{\pm}_{n_1,n_2}$ can be calculated analytically

$$\rho^{\pm}_{n_1,n_2}=\frac{i}{2}\left(\frac{k^{\pm}_{n_1,n_2}}{k}-\frac{k}{k^{\pm}_{n_1,n_2}}\right)\sin(k^{\pm}_{n_1,n_2}l)\tau^{\pm}_{n_1,n_2} \quad (13)$$

$$\tau^{\pm}_{n_1,n_2}=\left[\cos(k^{\pm}_{n_1,n_2}l)-\frac{i}{2}\left(\frac{k^{\pm}_{n_1,n_2}}{k}+\frac{k}{k^{\pm}_{n_1,n_2}}\right)\cdot\sin(k^{\pm}_{n_1,n_2}l)\right]^{-1} \quad (14)$$

with $k^{\pm}_{n_1,n_2}=(k^2\mp k^{\pm}_{n_1,n_2})^{1/2}$. It is seen that $P_{\text{emission}}(n_1,n_2)$ depends on the photon number n_1 and n_2, the atomic center-of-mass momentum k and the cavity length l. When the system is in the thermal-atom regime, i. e. , the atomic kinetic energy is larger than the atom-field coupling energy $k/k_{n_1,n_2}\gg 1$, it can be seen that Eq. (11) has the form of the Rabi oscillation because in this case

$$\rho^{\pm}_{n_1,n_2}\approx 0, \tau^{\pm}_{n_1,n_2}\approx\exp\left[ik\left(1\mp\frac{k^2_{n_1,n_2}}{2k^2}l\right)\right] \quad (15)$$

then Eq. (12) reduces to $P_{\text{emission}}(n_1, n_2)=\sin^2[(k^2_{n_1,n_2}/2k^2)l]$. It is a natural consequence as the incident thermal atom can transmit through the potential region nearly freely. When the system is in the ultracold-atom regime, i. e. , $k/k_{n_1,n_2}\ll 1$, some reasonable approximations can be made on Eqs. (13) and (14) so that it is gotten that

$$P_{\text{emission}}(n_1,n_2)=\frac{1/2+1/4\sin(2k_{n_1,n_2}l)}{[1+(k_{n_1,n_2}/2k)^2\sin^2(k_{n_1,n_2}l)]} \quad (16)$$

The above discussions are in the case that the cavity field is initially in a Fock state $|n_1, n_2\rangle$, the case can also be considered where the cavity modes are initially in coherent state $|\alpha\rangle$ and $|\beta\rangle$. When an excited atom is incident upon the two cavity modes in which the first is initially in a coherent state $|\alpha\rangle$ and the other is in a Fock state $|n_2\rangle$, the emission probability will be

$$P_{\text{emission}}=\sum_{n_1=0}^{\infty}p(n_1)P_{\text{emission}}(n_1,n_2) \quad (17)$$

where $p(n_1)$ is a Poisson distribution of photon numbers for a coherent state $|\alpha\rangle$ and determined by $p(n_1)=\exp(-\bar{n}_1)(\bar{n}_1)^{n_1}/n_1!$ in which $\bar{n}_1$ is the mean number of photons. If the two cavity modes are initially in the coherent state $|\alpha\rangle$ and $|\beta\rangle$, respectively, the emission probability will be

$$P_{\text{emission}}=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}p(n_1)p(n_2)P_{\text{emission}}(n_1,n_2) \quad (18)$$

Next some pictures are drawn according to Eqs. (12) ~ (18) in order to understand the emission probability behaviors in the different atom regimes. Fig. 1(a) is for the thermal-atom regime with $k/\kappa=0$, the first cavity mode is initially in a coherent state with $n_1=8$ and the second cavity mode is initially in vacuum state. Clearly the features of the collapse-revivals in the non-degenerate two-photon process can be seen, which is different from the degenerate case[4]. Fig. 1 (b) is for the ultracold-atom regime with $k/\kappa=0.01$ (solid) and $k/\kappa=0.1$ (dotted), both the two cavity modes are initially in vacuum state, and some resonant peaks can be seen. The smaller the kinetic energy of the incident atoms, the narrower the resonant peaks. When the cavity length is adjusted to $\kappa l[(n_1+1(n_2+1)]^{1/4}=m\pi$ with m an integer, the resonant peaks appear. Fig. 1(c) and (d) are for the ultracold-atom regime with $k/\kappa=0.1$, in (c) the first cavity mode is initially in a coherent state $\bar{n}_1=8$ and the second is initially in vacuum state, while in (d) the cavity modes are in the coherent states $\bar{n}_1=8$ and $\bar{n}_2=4$, respectively. It is seen that the collapses-revivals don' t take place in the ultracold-atom regime.

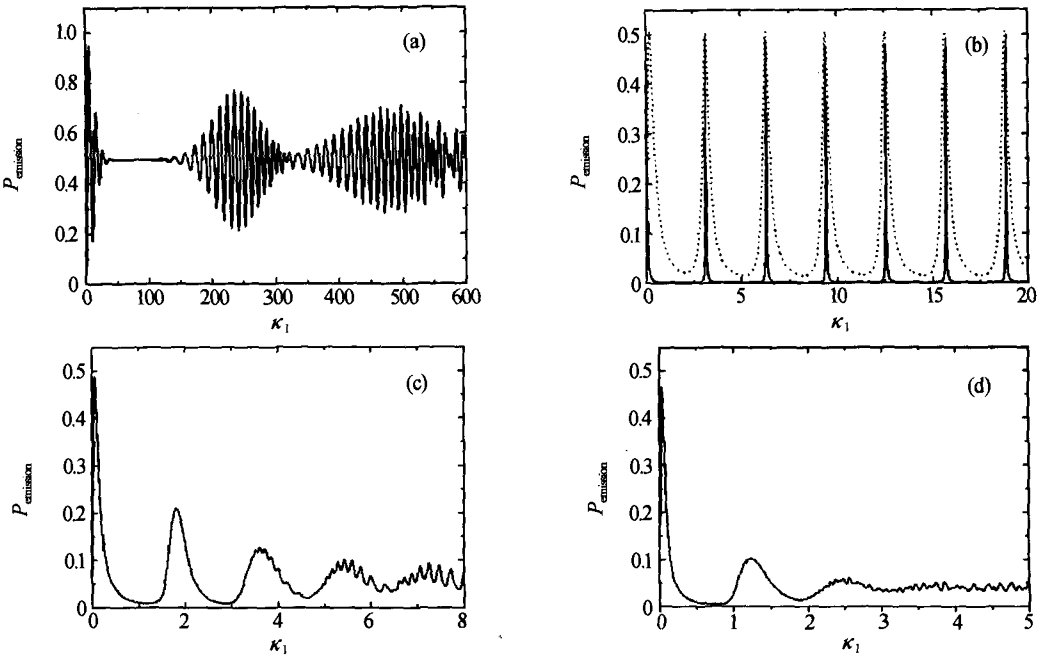

Fig. 1 Emission probability $P_{emission}$ (a) the first mode is initially in the coherent state $\bar{n}_1=8$ and the second in vacuum state with $k/\kappa=10$; (b) the two modes are initially in vacuum state with $k/\kappa=0.1$(dotted) and $k/\kappa=0.01$(solid); (c) the first mode is initially in the coherent state $\bar{n}_1=8$ and the second in vacuum state with $k/\kappa=0.1$; (d) the two modes are initially in the coherent state $\bar{n}_1=8$ and $\bar{n}_2=4$, respectively, with $k/\kappa=0.1$

2 Conclusion

In conclusion, the quantum theory was established for the non-degenerate two-photon mazer and the properties of the emission probability was studied. It should be emphasized that the emission probability behaviors in the non-degenerate two-photon mazer differ from those in the degenerate two-photon mazer, after all, the two cavity modes were allowed in the mazer so that there were two freedoms changing the potentials that the atoms experienced as they transitted the cavity.

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非简并双光子微脉塞的发射几率

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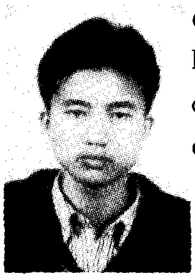
摘　要　将简并双光子微脉塞的量子理论推广到非简并情形，给出了原子发射几率的解析表达式并且分析了相应的性质. 发现由于在微脉塞中存在两种腔场模式，导致有两个自由度来改变原子在经过腔场时所受到的势场，因此非简并双光子微脉塞的发射几率的行为不同于简并双光子情况下的发射几率.

关键词　发射几率；非简并双光子过程；微脉塞



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