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# An Adaptive Wavelet Transform via Lifting for Image Compression

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**Abstract** A method for the construction of adaptive wavelet transforms using median filtering based on lifting scheme is introduced. Adaptive predictors followed by adaptive update algorithm are designed to better approximate signals and create more compact signal representations. In image compression, the experiments show that the proposed method doesn't match PSNR performance of D9/7 transform, but the visual quality of the method is better.

**Keywords** Adaptive wavelet transform; Median filtering; Image compression; Lifting scheme
**CLCN** TP919.81 **Document Code** A

## 0 Introduction

The merit of the wavelet transform stems from the fact that DWT tends to compress signals into a few coefficients of large magnitude. Compression follows from the "vanishing moments" property of wavelets, which guarantees that the wavelet coefficients of low-order polynomial signals are zero[1]. Thus, if a signal is exactly polynomial, it can be described using scaling coefficients alone. However, most signals in realworld are not polynomial, such as image signals. Because wavelet functions have localized support, they can be well-approximated by a piecewise polynomial function.

Image compression relies on efficient representations of image, the wavelet transform provides such a representation. There are two major methods to compute wavelet transform of an image: classical method[2] and lifting scheme[3,4]. The classical method is the oldest and involves application of separate lowpass and highpass filters often with floating point arithmetic. The lifting scheme is very general and highly flexible tool for building new wavelet decompositions. It might improve a given wavelet transform or build a new wavelet on irregular case. Furthermore, it offers the possibility of replacing linear filters by nonlinear ones[5]. The nonlinear method opens up the possibility of adapting the wavelet transform to image content and involves the computation of wavelet transform coefficients.

Motivated by Claypoole[6,7], we incorporate adaptivity into the wavelet transform because it can be adapted to the space frequency characteristics of image. Our goal is designing adaptive predictor to better approximate signal and create more compact signal. It still retains linear properties and show great potential to retain sharp edges while reducing artifacts. The mechanism utilizes the lifting scheme based on median filtering. The method is simple but effective.

## 1 Lifting scheme

Lifting scheme exploits a spatial domain prediction-error interpretation of the wavelet transform and provides a powerful framework for designing customized transforms[3,4]. We present here an overview of the lifting concept. The algorithm consists of three basic operations (see Fig. 1).

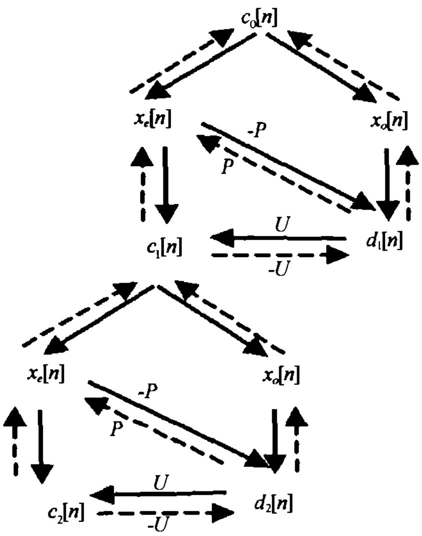


Fig. 1 Lifting scheme of wavelet transform

(1) Split: Divide the original data into disjoint subsets. In the standard lifting scheme we split the original data set $x[n]$ into $x_e[n]=x[2n]$, the even points, and $x_o[n]=x[2n+1]$, the odd points.

(2) Predict: Generate the wavelet coefficients $d[n]$ as the error in predicting $x_o[n]$ from $x_e[n]$ using prediction operator $P$

$$d[n]=x_o[n]-P(x_e[n]) \quad (1)$$

(3) Update: The scaling coefficients $c[n]$ are accomplished by applying an update operator $U$ to $d[n]$ and adding to $x_e[n]$

$$c[n]=x_e[n]+U(d[n]) \quad (2)$$

These three steps form a lifting stage. Iteration of the lifting stage on the output $c[n]$ creates the

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complete set of DWT scaling and wavelet coefficients $c_j[n]$ and $d_j[n]$ $(j=1,2,\cdots)$. The lifting steps are easily inverted. Rearranging (1) and (2), we have

$$x_e[n]=c[n]-U(d[n]),x_o[n]=d[n]+P(x_e[n]) \quad (3)$$

In the simplest case, the predict and update operator are linear. The standard high-pass and low-pass decomposition filters $g[n]$ and $h[n]$, respectively, are replaced with the predict and update filters $P(Z)$ and $U(Z)$ (see Fig. 2), the parameters $u_i, p_i$ $(i=1,2,\cdots)$ can be accomplished using the method in[7].

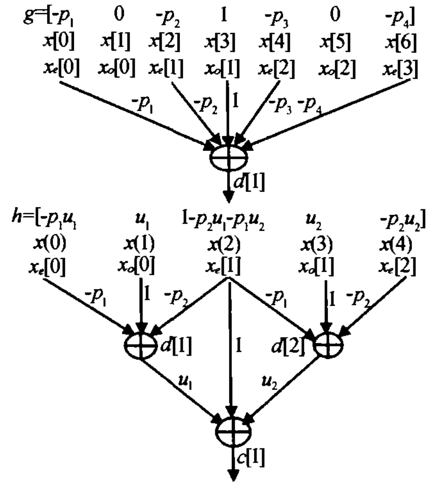


Fig. 2 Illustration of predict (up, $N=4$ point linear predict filter $P(Z)$ yields $g$) and update (down, $N=2$ point linear predict followed by $N'=2$ point linear update yields $h$), $g$ and $h$ denote corresponding high-pass and low-pass filters of the wavelet transform

## 2 Adaptive lifting wavelet transform based on median filtering

The lifting approach of wavelet design gives us a great deal of flexibility. We can use any linear, nonlinear or space-varying predict or update, and the lifting construction ensures that the transform is invertible. The lifting approach for adaptive DWT that optimizes data-based prediction measures might match the characteristics of a given signal. This leads to more efficient signal representations. It is key to successful signal compression.

Median filters are well known nonlinear filters, and have been successfully incorporated into multiresolution signal decompositions by Goutsias and Heijmans[8], and Hampson and Resquet[9]. They show great potential for image compression, and their structures are similar to lifting scheme. Therefore, we incorporate the median filter into adaptive scheme. Consider an $N$ point median filter predictor, for each odd coefficient $x[2n+1]$, the predictor $P$ chooses the median value of the $N$ nearest even coefficients $x[2(n-k)]$ ($N$ point data window) as the prediction. The wavelet coefficient is $d[n]=x[2n+1]-P(x[2(n-k)])$, where $P(x[2(n-k)])$ is the median value for each $N$ point data window.

We utilize median filter in the predict step then followed this operation by an adaptive update step to design the scaling coefficients $c[n]$. Here, we employ the predict/update framework of figure 1. Firstly, we compute $d[n]$ using the median filter. Then a set of update coefficients $u_k$ must be found for each $n$ to ensure each scaling coefficient $c[n]$ has a valid polynomial interpretation, i. e., a certain scalar quantity is preserved. Therefore, despite the application of the nonlinear median filter in the predict step, the update coefficients uk adapt to ensure $c[n]$ satisfy linear polynomial constraints and are a low-pass representation of the original data set. The tree structure constructed to trace each $c[n]$ up to the original data $x[n]$ is shown in Fig. 3.

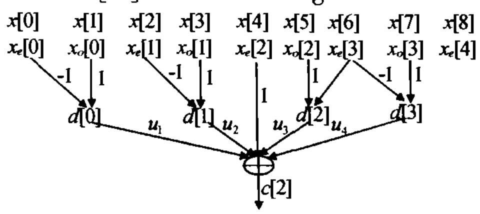


Fig. 3 Median prediction followed by linear update (An $N=5$ point median filter predict followed by $N'=4$ point update)

If we desire an update filter of length $N'$, we must generate $N'$ update constraints. If $N'=1$, then the 1 point update filter coefficient is $u=1/2$ for any choice of median filter and any data. Thus, it is better suited for image compression. Especially, it can be easily modified to accomplish integer to integer arithmetic using the method in[10].

If $N'>1$, we should use all the free variables to satisfy polynomial constraints. However, it is possible that a single data point will be the output of multiple median predict filter as shown for $x_e[3]$ in Fig. 3. Thus, the branches of the update filter tree overlap, and the resulting linear update constraints may be incompatible. In this case, we incorporate the first few polynomial constraints, and use the remaining free variables to minimize the energy of the update filter. This keeps the update filter coefficients from becoming high unbalanced.

We can also incorporate a median filter into the update step. Regardless of the median filter choices, we form each scaling coefficient by adding the even data to the output of the update median filter multiplied by 1/2. This is guaranteed to satisfy the zeroth polynomial constraint, so it can be viewed as optimal polynomial approximation.

## 3 Application in image compression

In this section, we demonstrate the application of the adaptive algorithm in image compression. Here we utilize the 5 point median filter predict followed by 3 point median filter update. Note that the output of the update median filter is multiplied by 1/2. We use an EZW encoder[11], and compress the zero tree symbol stream with an adaptive arithmetic encoder[12]. We choose "Lena" as the original image for testing. The performance of the popular D9/7 transform is the reference for comparability. Fig. 4 shows the PSNR performance of the experiment. Fig. 5 illustrates the difference of the visual quality of the experiment. We use the MPQM quality metric[13] which measures the quality of the image according to CCIR-500 recommendation.

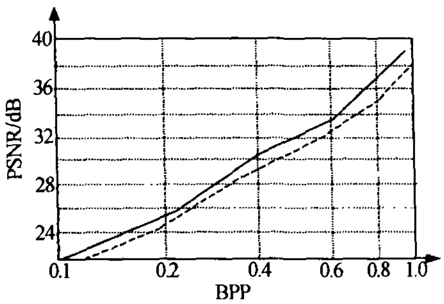


Fig. 4 PSNR curves of D9/7 transform (solid line) and our algorithm (dash) for "Lena" image

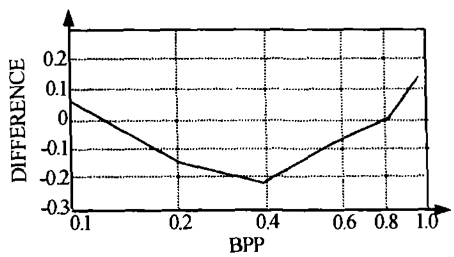


Fig. 5 Difference of MPQM quality between D9/7 transform and our algorthm for "Lena" image

It is well known that PSNR is not always well correlated to the visual quality. From the simulation results, we can see that the PSNR of our algorithm is a little poor than that of the D9/7 transform, but the visual quality of our algorithm is better than that of the D9/7 transform for reduction in artifacts. Furthermore, another advantage of our algorithm is that boundary extension is very simple, it will reduce the power dissipation and memory requirements when implemented in hardware. For example, we only add two zeroes to both the left and the right of the dataset when we employ a 5 point median filter prediction.

## 4 Conclusion

Generally in image compression, the optimal predictor is nonlinear for lifting scheme. Therefore, it is interesting to construct nonlinear predictors. However, we should consider other new adaptive lifting transforms for further improving its PSNR performance and visual quality. We intend to modify our transform for further improving its performance. Future work involves to apply update-first lifting and combing adaptive predictor and adaptive update algorithm to image compression.

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# 基于自适应提升小波变换的图像压缩

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**摘 要** 介绍一种用基于提升算法的中值滤波构造自适应小波变换方法.其思想是设计自适应的"预测算子"及自适应的"更新"运算以更好的逼近信号,从而达到更"紧凑"的信号表示.实验表明,在图像压缩应用中,在峰值信噪比(PSNR)上,虽然本文算法比不上D9/7小波,但在主观评价上更好.
**关键词** 自适应小波变换;中值滤波;图像压缩;提升格式

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