Preparation of Entangled Squeezed Vacuum States via Atom-cavity-field Raman Interaction*

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Abstract A scheme for preparing bipartite and multipartite entangled squeezed vacuum states (SVSs) via the Raman interaction of an Λ-type three-level atom with squeezed vacuum states cavity fields is presented. In this scheme, an atom initially prepared in its ground state is in proper order injected into several cavities initially prepared each in the SVS with same interaction time. After the atom interacting with these cavities, the entanglement of atom-cavity-field is generated. Afterwards, by measuring the atom the cavity-field entangled SVSs are obtained. Also, the entanglement properties of bipartite entangled SVSs as well as multipartite entangled SVSs are discussed.

Keywords Quantum entanglement; Squeezed vacuum state; Raman interaction

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0 Introduction

Quantum entanglement, first noted by Einstein-Podolsky-Rosen (EPR)[1] and Schrödinger[2], is one of the essential features of quantum mechanics. For decades, entanglement has been the focus of much work in the foundations of quantum mechanics, being associated particularly with quantum nonseparability and the violation of Bell's inequalities. In recent years, however, entanglement has generated more interests in the quantum information processing such as quantum teleportation[3], superdense coding[4], quantum key distribution[5] and telecoloning[6].

There are different types of entanglement for physics systems. The original studies on quantum entanglement focused on the systems with a finite-dimensional (discrete variable) state space, such as the polarizations of a photon or the discrete levels of an atom. For instance, in the teleportation experiments from the Innsbruck group[7], it is the polarization directions of single photons that are entangled. Since the experimental demonstration of quantum teleportation of coherent states[8], many interests have arisen in continuous variable quantum information processing[9,10], Thereupon, the entangled states with continuous variable receive much attention in the study of quantum entanglement[11,12]. In the Caltech teleportation experiment[8], two EM field modes are entangled with respect to photon numbers and the state used for teleportation is a two-mode squeezed state. Recently, S. J. van Enk et al[13]. investigated a third type of entangled states of two modes of the light field, namely entangled coherent states. More recently, we have investigated a new type of entangled state of two modes of electromagnetic field, namely an entangled squeezed vacuum state (SVS). A scheme for distilling maximally entangled SVS from partially entangled pure states of single mode SVS was suggested in Ref. [14]. A proposal for teleporting a superposition state of SVSs by using entangled SVS as the quantum channel was presented in Ref. [15]. In this paper, we present a new method for preparing entangled SVS, it is via the Raman interaction of an Λ-type three-level atom with SVS cavity-field.

1 Raman interaction

First, let's briefly review the Raman interaction of atom-cavity-field. For a system consisting of a degenerate Λ-type three-level atom and a single mode field, the Hamiltonian is given by ($\hbar=1$)

$$\hat{H}=\omega\hat{a}^{+}\hat{a}+\omega_f|f\rangle\langle f|+\omega_0(|e\rangle\langle e|+|g\rangle\langle g|)+ g_1(\hat{a}^{+}|g\rangle\langle f|e^{-i\Delta t}+\hat{a}|f\rangle\langle g|e^{i\Delta t})+ g_2(\hat{a}^{+}|e\rangle\langle f|e^{-i\Delta t}+\hat{a}|f\rangle\langle e|e^{i\Delta t}) \quad (1)$$

where $|e\rangle$ and $|g\rangle$ are two degenerate lower levels of the atom, $|f\rangle$ is the upper level of the atom, $\hat{a}^{+}$ and $\hat{a}$ are the bosonic creation and annihilation operators of the cavity field, g_1 (g_2) is the coupling constant between the atomic transition $|f\rangle\rightarrow|g\rangle$ ($|e\rangle$) and cavity mode, ω_0 and ω_f are the energies of the lower-level and upper-level of the atom, Δ is the detuning, respectively. If the atomic transition frequency is highly detuned from the cavity field frequency, the upper level $|f\rangle$ can be adiabatically eliminated. Under this condotion the effective Hamiltonian of this system

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is given by

$$\hat{H}_{\rm eff} = -\lambda \hat{a}^+\hat{a}(|e\rangle\langle g| + |g\rangle\langle e|) - \hat{a}^+\hat{a}(\beta_1|g\rangle\langle g| + \beta_2|e\rangle\langle e|) \quad (2)$$

where $\lambda = g_1g_2/\Delta, \beta_1 = g_1^2/\Delta, \beta_2 = g_2^2/\Delta, \Delta = (\omega_f - \omega_0) - \omega$. The parameters β_1 and β_2 denote the intensity dependent Stark shifts of states $|g\rangle$ and $|e\rangle$, respectively. For convenience, let $g_1 = g_2 = g$, thus $\lambda = \beta_1 = \beta_2 = g^2/\Delta$.

Suppose that the atom is initially prepared in the ground state $|g\rangle$, and the cavity field is initially prepared in the SVS $|\xi\rangle$, the initial atom-cavity state is

$$|\Psi(0)\rangle_{a\text{-}f} = |g\rangle \otimes |\xi\rangle \quad (3)$$

where $\xi = re^{i\varphi}$ is any complex number with modulus r and argument φ. An expansion in terms of Fock states is

$$|\xi\rangle = \sqrt{\operatorname{sech} r}\sum_{n=0}^{\infty}\frac{\sqrt{(2n)!}}{n!}\left(-\frac{1}{2}e^{i\varphi}\tanh r\right)^n|2n\rangle \quad (4)$$

In interaction picture, the time evolution of state vector of system is given by Schrödinger equation

$$i\frac{d|\Psi(t)\rangle}{dt} = \hat{H}_{\rm eff}|\Psi(t)\rangle \quad (5)$$

The state vector $|\Psi(t)\rangle$ is given by

$$|\Psi(t=\tau)\rangle = \exp(-i\hat{H}_{\rm eff}t)|\Psi(0)\rangle_{a\text{-}f} = 2^{-1}[(|\xi\rangle + |\xi e^{4i(g^2/\Delta)\tau}\rangle)|g\rangle - (|\xi\rangle - |\xi e^{4i(g^2/\Delta)\tau}\rangle)|e\rangle] \quad (6)$$

where τ is the interaction time between the atom and cavity field, it can be controlled by adjusting the velocity of the atom. With choosing $(g^2/\Delta)\tau = \pi/4$, we obtain

$$|\Psi(\tau)\rangle_{a\text{-}f} = 2^{-1}[(|\xi\rangle + |-\xi\rangle)|g\rangle - (|\xi\rangle - |-\xi\rangle)|e\rangle] \quad (7)$$

Similarly, if the atom is initially prepared in its excited state $|e\rangle$, we can obtain

$$|\Psi(\tau)\rangle_{a\text{-}f} = 2^{-1}[(|\xi\rangle + |-\xi\rangle)|e\rangle - (|\xi\rangle - |-\xi\rangle)|g\rangle] \quad (8)$$

2 Preparation of entangled SVSs

According to the description above, it is easy to realize the generation of the entangled SVSs. We assume that there are two cavities (referred to as 1 and 2), which are both initially prepared in the SVS $|\xi\rangle$, and that the atom is initially prepared in the ground state $|g\rangle$. Then the initial state of the whole system is given by

$$|\Psi(0)\rangle_{a\text{-}f} = |g\rangle \otimes |\xi\rangle_1 \otimes |\xi\rangle_2 \quad (9)$$

Let atom enter cavity 1 and 2 in proper order with same interaction time τ. After atom passes through these cavities, the atom-field state is

$$|\Psi'\rangle_{a\text{-}f} = 2^{-1}(|\xi\rangle_1|\xi\rangle_2 + |-\xi\rangle_1|-\Psi\rangle_2)|g\rangle - 2^{-1}(|\xi\rangle_1|\xi\rangle_2 - |-\xi\rangle_1|-\xi\rangle_2)|e\rangle \quad (10)$$

Thereupon, we perform an atom measurement. When the atom is detected in the ground state or excited state, the cavity field collapses correspondingly into

$$|\Psi''\rangle_f \rightarrow |\xi\rangle_1|\xi\rangle_2 + |-\xi\rangle_1|-\xi\rangle_2 \quad (11)$$

or

$$|\Psi''\rangle_f \rightarrow |\xi\rangle_1|\xi\rangle_2 - |-\xi\rangle_1|-\xi\rangle_2 \quad (12)$$

After normalization we get two entangled SVSs

$$|\phi^{\pm}\rangle_{1,2} = \frac{1}{\sqrt{N_{\pm}}}(|\xi\rangle_1|\xi\rangle_2 \pm |-\xi\rangle_1|-\xi\rangle_2) \quad (13)$$

where $N_{\pm} = 2(1 \pm k^2)$ is normalization constant and $k = \langle\xi|-\xi\rangle = \sqrt{\sec h(2r)}$ is the overlap.

If the two cavity-fields are initially prepared in the SVSs with equal-amplitude and opposite-phase, the initial state of the whole system is given by

$$|\Psi(0)\rangle_{a\text{-}f} = |g\rangle \otimes |\xi\rangle_1 \otimes |-\xi\rangle_2 \quad (14)$$

After atom interacts with two cavity-fields, the system evolves to

$$|\Psi'\rangle_{a\text{-}f} = \frac{1}{2}(|\xi\rangle_1|-\xi\rangle_2 + |-\xi\rangle_1|\xi\rangle_2)|g\rangle - \frac{1}{2}(|\xi\rangle_1|-\xi\rangle_2 - |-\xi\rangle_1|\xi\rangle_2)|e\rangle \quad (15)$$

By performing a similar measurement as above, we get two entangled SVSs

$$|\Psi^{\pm}\rangle_{1,2} = \frac{1}{\sqrt{N_{\pm}}}(|\xi\rangle_1|-\xi\rangle_2 \pm |-\xi\rangle_1|\xi\rangle_2 \quad (16)$$

Synthesizing Eq. (13) and Eq. (16), we obtain a set of quasi-Bell states based on SVSs in 2 × 2 Hilbert space.

It is possible to generate the entangled SVS with N subsystems. We assume that there are N cavities which are initially prepared each in the SVS. By adopting the similar method as stated above, we can obtain the entangled SVSs with N subsystems.

$$|\phi\rangle_{12\cdots N}^{\pm} = \frac{1}{\sqrt{N_{\pm}}}(|\xi\rangle_1|\xi\rangle_2\cdots|\xi\rangle_N \pm |-\xi\rangle_1|-\xi\rangle_2\cdots|-\xi\rangle_N) \quad (17)$$

where $N'_{\pm} = 2(1 \pm k^N)$ is normalization constant.

3 The feature of entanglement

The methods for preparing the entangled SVSs have been discussed; now we analyze the features for these states. For bipartite entangled SVSs described by Eq. (13) and Eq. (16), they form a set of quasi-Bell basic states in 2 ×2 Hilbert space

$$\begin{cases} |\phi\rangle_{1,2}^{+} = \frac{1}{\sqrt{N_+}}(|\xi\rangle_1|\xi\rangle_2 + |-\xi\rangle_1|-\xi\rangle_2) \\ |\phi\rangle_{1,2}^{-} = \frac{1}{\sqrt{N_-}}(|\xi\rangle_1|\xi\rangle_2 - |-\xi\rangle_1|-\xi\rangle_2) \\ |\phi\rangle_{1,2}^{+} = \frac{1}{\sqrt{N_+}}(|\xi\rangle_1|-\xi\rangle_2 + |-\xi\rangle_1|\xi\rangle_2) \\ |\phi\rangle_{1,2}^{-} = \frac{1}{\sqrt{N_-}}(|\xi\rangle_1|-\xi\rangle_2 - |-\xi\rangle_1|\xi\rangle_2) \end{cases} \quad (18)$$

They are not orthogonal each other. In the limiting case of $r \rightarrow \infty$, the 4 entangled SVSs are orthogonal each other. Then these states reduce to standard Bell-states.

Now we discuss the entropy of entanglement for the above states. Let $\rho_1^{(i)}$ $(i=1,2,3,4)$ denote the deduced density operators of the quasi - Bell states $(|\phi\rangle^+, |\phi\rangle^-, |\Psi\rangle^+, |\Psi\rangle^-)$ respectively. Their concrete forms are given by

$$\rho_1^{(1)}=\rho_1^{(3)}=\frac{1}{N_+}(|\xi\rangle_1\langle\xi|+k|\xi\rangle_1\langle-\xi|+k|-\xi\rangle_1\langle\xi|+|-\xi\rangle_1\langle-\xi|) \quad (19)$$

$$\rho_1^{(2)}=\rho_1^{(4)}=\frac{1}{N_-}(|\xi\rangle_1\langle\xi|-k|\xi\rangle_1\langle-\xi|-k|-\xi\rangle_1\langle\xi|+|-\xi\rangle_1\langle-\xi|) \quad (20)$$

The two eigenvalues of every deduced density operators are given by

$$\lambda_\pm^{(1)}=\lambda_\pm^{(3)}=\frac{(1\pm k)^2}{2(1+k^2)},\lambda_\pm^{(2)}=\lambda_\pm^{(4)}=2^{-1} \quad (21)$$

The entropies of entanglement are then

$$E(\phi^+)=E(\Psi^+)=-\frac{(1+k)^2}{2(1+k^2)}\log_2\frac{(1+k)^2}{2(1+k^2)}-\frac{(1-k)^2}{2(1+k)^2}\log_2\frac{(1-k)^2}{2(1+k^2)} \quad (22)$$

$$E(\phi^-)=E(\Psi^-)=1 \quad (23)$$

which indicates that entangled SVSs $|\phi\rangle^-$ and $|\Psi\rangle^-$ are maximally entangled state, their amounts of entanglement are exactly one ebit and the entanglement is independent of the parameters involved. However, the states $|\phi\rangle^+$ and $|\Psi\rangle^+$ are not maximally entangled state except in the case of $r\to\infty$.

For multipartite entangled SVS given by Eq. (17), it is more complex to discuss their entanglement. Some of the entanglement properties of multipartite entangled SVS were discussed in Ref. [14]. Here we only consider two limits that arise from $r\to\infty$ and $r\to0$. In the asymptotic limit $r\to\infty$, the overlap $k=\sqrt{\operatorname{sech}(2r)}\to0$, then two states $|\xi\rangle$ and $|-\xi\rangle$ approach orthogonal. The states $|\phi\rangle_{12\cdots N}^{\pm}$ approach the multipartite maximally entangled states, namely multipartite GHZ state[16]

$$|\mathrm{GHZ}\rangle_N^{\pm}=\frac{1}{\sqrt{2}}(|\bar{0}\rangle_1|\bar{0}\rangle_2\cdots|\bar{0}\rangle_N\pm|\bar{1}\rangle_1|\bar{1}\rangle_2\cdots|\bar{1}\rangle_N) \quad (24)$$

where an orthogonal basis can be constructed such that $|\bar{0}\rangle\equiv|\xi\rangle_{r\to\infty}$ and $|\bar{1}\rangle\equiv|-\xi\rangle_{r\to\infty}$. For $r\to0$ the state $|\phi\rangle_{12\cdots N}^{-}$ becomes the W state[16], which can be proved easily. Expanding $|\phi\rangle_{12\cdots N}^{-}$ in Fock state, we obtain

$$|\phi\rangle^-_{12\cdots N}=\frac{(\operatorname{sech} r)^{N/2}}{\sqrt{2[1-(\operatorname{sech}2r)^{N/2}]}}\cdot\sum_{n_1n_2\cdots n_N}\frac{\sqrt{(2n_1)!\,(2n_2)!\cdots(2n_N)!}}{n_1!\,n_2!\cdots n_N!}\cdot\left(-\frac{1}{2}e^{i\varphi}\tanh r\right)^{n_1+n_2+\cdots+n_N}\cdot[1-(-1)^{n_1+n_2\cdots+n_N}]|2n_1,2n_2,\cdots,2n_N\rangle \quad (25)$$

When the r approaches zero, we can prove

$$\lim_{r\to0}\frac{\tanh r}{\sqrt{1-(\operatorname{sech}2r)^{N/2}}}=\frac{1}{\sqrt{N}} \quad (26)$$

Therefore we see that only the terms with $n_1+n_2+\cdots+n_N=1$ survive, and the resultant state is

$$|W\rangle_N=\frac{1}{\sqrt{N}}(|20\cdots0\rangle+|02\cdots0\rangle+\cdots+|00\cdots2\rangle) \quad (27)$$

But not like the general W state, there are two photons in every mold.

4 Summary

In summary, we have presented a scheme to prepare bipartite and multipartite entangled SVSs via the Raman interaction of a degenerate Λ-type three-level atom with cavity-fields in SVS. If initial cavity-field states are in two different SVSs $|\xi\rangle_1|\xi\rangle_2$ and $|\xi\rangle_1|-\xi\rangle_2$, we can obtain a set of quasi - Bell states $|\phi\rangle_{12}^{\pm}$, $|\Psi\rangle_{12}^{\pm}$ based on SVSs in 2×2 Hilbert space. When the squeezed parameter $r\to\infty$, the two SVSs $|\xi\rangle$ and $|-\xi\rangle$ become orthogonal, and then these quasi-Bell states reduce to standard Bell-states. For multipartite entangled SVS, we have proven that in the limits of $r\to\infty$ and $r\to0$, this state reduces correspondingly to GHZ state and W state.

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利用原子-腔场喇曼相互作用制备纠缠压缩真空态

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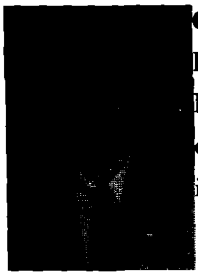
摘　要　提出了利用量子态腔场与原子的喇曼相互作用制备纠缠压缩真空态的方案．在该方案中，一个初始制备在基态的原子被依次送入几个初始制备在压缩真空态的微腔中．通过控制原子的运行速度，使原子与每一个腔具有相同的相互作用时间．当原子与腔场发生相互作用，原子与腔场产生纠缠之后，进行原子的测量．当原子被测量处于基态或激发态时，按照量子力学波包塌缩原理，腔场态将塌缩到相应的纠缠压缩真空态．对纠缠压缩真空态的纠缠性质也进行了简略的讨论．

关键词　量子纠缠；压缩真空态；喇曼相互作用



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