

非正交轴系激光经纬仪反向运动学线性模型

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摘要 为满足大尺寸测量仪器性能与制造水平提升的需求,基于非正交轴系架构理念对激光经纬仪进行设计。针对非正交轴系经纬仪在无参考末端情况下计算反向运动旋转角度的难题,提出反向运动线性模型,以实现旋转角度的快速、高精度计算。首先,基于李群李代数基本理论构建非正交轴系经纬仪正向运动学模型。其次,构建空间目标点与视准轴位姿参数间的约束关系,并结合旋转角度误差传递模型,确立用于求解旋转角度误差修正值的线性方程组。最终通过旋转角度初始估计值与误差修正值线性相加,获取高精度的反向旋转角度值。仿真结果表明,该方法所计算的旋转角度误差趋近于0,真实实验的旋转角度误差均小于0.02 mrad,验证了所提反向运动学线性模型的可行性与实用性。

关键词 光学仪器; 非正交轴系; 反向运动; 线性模型; 误差传递

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1 引言

制造产业是国民经济发展的基础,现阶段智能制造已成为传统产业改造升级和新型制造业发展的主流趋势^[1]。三维测量技术^[2]是实现智能制造的重要技术支撑,现代大尺寸测量技术对多学科进行了交叉融合,相关测量仪器也向着非接触、高精度、自动化、智能化的方向发展^[3]。目前常见的大尺寸三维测量仪器主要有经纬仪^[4]、全站仪^[5]、激光跟踪仪^[6]、激光追踪仪^[7]、激光雷达^[8-9]、室内全局定位测量系统^[10]和摄影测量^[11]等。经纬仪、全站仪以及跟踪仪等空间坐标测量仪器不仅对自身内部结构的正交性要求严格,对设计、材料选择、加工、装配、检校及维护等环节也需严格要求。将消除传统仪器轴系的正交性约束作为切入点,天津大学吴斌教授团队^[12-13]设计组装了非正交轴系全站仪与经纬仪测量系统并验证了相关设计理论的可行性。近年来,众多研究人员针对非正交轴系仪器测量理论展开研究,补充并丰富了相关基础原理^[14-18],本文在非正交轴系经纬仪的理论基础上展开相关研究,望推动国内高端仪器制造与理论的发展。

经纬仪是一类可用于水平和垂直角度高精度测量的光学非接触式仪器,目前基于非正交轴系理论已开发出相应的非正交轴系激光经纬仪^[19]。为满足后期测量精度理论研究的需要,亟须建立非正交轴系经纬仪的反向运动学模型。非正交轴系经纬仪的反向运动学

模型描述的是已知测量目标的空间坐标,求其对应水平与垂直方向的旋转角,是正向运动的逆过程。任意测点坐标都可映射为非正交轴系经纬仪精准的旋转角度,这有助于误差空间分布等相关精度理论研究,也可有效协助仪器自动瞄准目标点。杨峰亭^[20]首次提出用于协助仪器自动瞄准目标点的反向运动近似计算模型。该反向运动模型是以已知测量激光束所在空间直线位置为前提建立的,而在实际应用中激光束上无固定参考末端,无法根据被测点坐标推测激光视准轴当前准确的空间位姿参数,相应的旋转角度难以准确计算。此外,该模型是反向运动学的近似计算模型,不满足误差分布模拟仿真相关研究的需要。本文涉及的非正交轴系测量仪器是一类新概念仪器,其理论与技术支撑仍然存在大量空白。反向运动模型是非正交轴系测量仪器理论中缺失的重要部分,国内外研究人员除杨峰亭^[20]外没有人对此开展过相应的研究,因此本文立足于填补相关反向模型理论空白展开研究。

综上所述,亟须建立仅在被测目标点坐标已知情况下无参考末端的非正交轴系经纬仪的反向运动学模型。本文基于李群李代数基本理论建立了非正交轴系经纬仪正向运动学模型,推导了旋转角度误差与激光视准轴运动学误差间的传递模型。在正交近似条件下快速估计旋转角度初始值,并确立了以旋转角度误差修正值与初始值相加为主要形式的反向运动线性模型,最终实现了反向运动旋转角度的快速、准确计算。

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2 非正交轴系激光经纬仪基本原理

2.1 非正交轴系激光经纬仪结构及参数

如图 1 所示,非正交轴系经纬仪主要由一个高精度二维转台与一个准直激光器组成,激光器出射的激光束作为测量视准轴。与传统经纬仪有所区别,非正交轴系经纬仪的转台旋转轴与激光视准轴之间不满足正交条件。但为标识便利,仍以“竖直旋转轴”“水平旋转轴”以及“激光视准轴”对内部轴系进行命名。激光

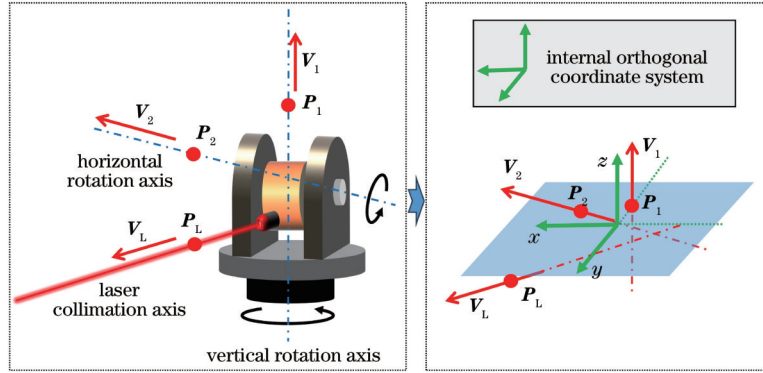


图 1 非正交轴系激光经纬仪结构

Fig. 1 Structure of the non-orthogonal shafting laser theodolite

2.2 基于李群李代数的正向运动建模

作为现代几何数学的重要工具,李群李代数借助了指数积公式在机器人运动学与动力学分析方面具有的表达简洁、物理意义清晰且计算效率高的特点^[21]。为了实现非正交轴系经纬仪正向运动过程中激光视准轴实时空间位姿参数的精确描述,利用李代数全局指数积公式对非正交轴系经纬仪进行正向运动学建模。

描述空间刚体运动的变换矩阵 g 可由 3×3 的旋转矩阵 R 与 3×1 的平移矩阵 b 表示,其描述形式为

$$g = \begin{bmatrix} R & b \\ 0 & 1 \end{bmatrix}, \quad (1)$$

根据李代数特性(螺旋运动特性),可将旋转轴看作是在参考坐标系下的一个运动旋量 ξ , 相对应的 6 维坐标形式为

$$\xi = \begin{bmatrix} \omega \\ v \end{bmatrix} = \begin{bmatrix} \omega \\ p \times \omega \end{bmatrix}, \quad (2)$$

式中: $\omega = [\omega_1 \ \omega_2 \ \omega_3]^T$ 代表旋转轴的方向向量; p 代表旋转轴上的固定点三维坐标; v 代表旋转轴上固定点与方向向量的叉乘结果。此外,运动旋量需满足

$$\begin{cases} \|\omega\| = 1 \\ \omega \cdot v = 0 \end{cases}. \quad (3)$$

将运动旋量从 6 维向量形式映射为矩阵形式,表示为

$$\hat{\xi} = \begin{bmatrix} \hat{\omega} & v \\ 0 & 0 \end{bmatrix}, \quad (4)$$

式中, $\hat{\omega}$ 代表 ω 的反对称矩阵,表示为

视准轴受旋转轴驱动,对空间中待测目标点进行瞄准。上述轴系空间位姿参数是非正交轴系经纬仪的关键结构参数,轴系空间位姿状态可通过轴上固定点坐标与方向向量进行描述。 $V_1(u_1, v_1, w_1)$ 代表竖直旋转轴的方向向量, $P_1(x_1, y_1, z_1)$ 代表竖直旋转轴上的固定点坐标; $V_2(u_2, v_2, w_2)$ 代表水平旋转轴的方向向量, $P_2(x_2, y_2, z_2)$ 代表水平旋转轴上的固定点坐标; $V_L(u_L, v_L, w_L)$ 代表激光视准轴的方向向量, $P_L(x_L, y_L, z_L)$ 代表激光视准轴上的固定点坐标。

$$\hat{\omega} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}. \quad (5)$$

如图 2 所示,非正交轴系经纬仪工作过程中,竖直旋转轴与水平旋转轴对应的旋量坐标分别为 ξ_1 与 ξ_2 。激光视准轴绕竖直旋转轴与水平旋转轴的旋转角度分别为水平角 θ_1 与俯仰角 θ_2 。它们各自对应的运动变换矩阵 $g_i (i=1, 2)$ 表示为

$$g_i = \exp(\hat{\xi}_i \theta_i) = \begin{bmatrix} R_i & b_i \\ 0 & 1 \end{bmatrix}, \quad (6)$$

$$R_i = I_3 + \frac{\sin(\|\omega_i\| \theta_i)}{\|\omega_i\|} \hat{\omega}_i + \frac{1 - \cos(\|\omega_i\| \theta_i)}{\|\omega_i\|^2} \hat{\omega}_i^2, \quad (7)$$

$$b_i = \begin{bmatrix} \theta_i I_3 + \frac{1 - \cos(\|\omega_i\| \theta_i)}{\|\omega_i\|^2} \hat{\omega}_i + \\ \frac{\|\omega_i\| \theta_i - \sin(\|\omega_i\| \theta_i)}{\|\omega_i\|^3} \hat{\omega}_i^2 \end{bmatrix} v_i, \quad (8)$$

式中, I_3 为三阶单位矩阵。

根据李群李代数理论,非正交轴系经纬仪正向运动变换矩阵的指数积形式为

$$g = \exp(\hat{\xi}_1 \theta_1) \cdot \exp(\hat{\xi}_2 \theta_2), \quad (9)$$

非正交轴系经纬仪激光视准轴在运动过程中的实时空间位姿表达式为

$$\begin{bmatrix} V_{LFin} & P_{LFin} \\ 0 & 1 \end{bmatrix} = g \begin{bmatrix} V_L & P_L \\ 0 & 1 \end{bmatrix}, \quad (10)$$

式中, V_{LFin} 、 P_{LFin} 分别表示发生运动后激光视准轴的单位方向向量与固定点坐标。至此, 非正交轴系经纬仪正向运动学模型建立完毕。

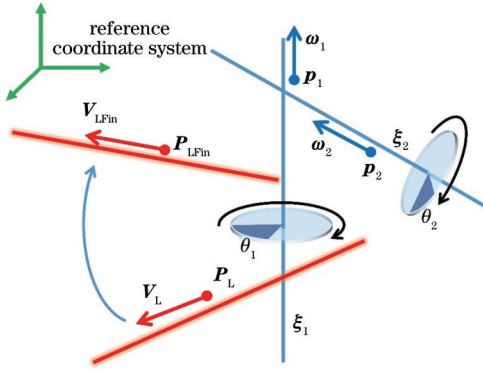


图 2 非正交轴系激光经纬仪工作原理

Fig. 2 Principle of the non-orthogonal shafting laser theodolite

3 反向运动线性模型

3.1 激光视准轴空间约束条件

非正交轴系经纬仪的反向模型指在已知被测点空间坐标的前提下获取相对应的旋转角度。设空间中任意一目标点 $P_{Tar}(x_{Tar}, y_{Tar}, z_{Tar})$, 根据非正交轴系经纬仪正向运动学原理, 解算得到瞄准该目标点时的视准轴上固定点 $P(x, y, z)$ 与方向向量 $V(u, v, w)$ 。当视准轴瞄准目标点时, 以上参数满足约束等式:

$$P_{Tar} - P = kV, \quad (11)$$

式中, k 代表目标点与视准轴上固定点的距离。由于经纬仪不具备空间距离测量功能, 即距离值 k 是未知的, 因此, 需要将式(11)进行改写, 得到

$$\begin{cases} (x_{Tar} - x)v - (y_{Tar} - y)u = 0 \\ (x_{Tar} - x)w - (z_{Tar} - z)u = 0, \\ (y_{Tar} - y)w - (z_{Tar} - z)v = 0 \end{cases} \quad (12)$$

式(12)即为求解反向运动旋转角度的基础约束等式。

3.2 初始旋转角度估计

若将正向运动学模型代入约束等式[式(12)]中对旋转角进行解算, 则计算过程较为复杂且极易出现错误。因此, 提出一种线性的反向计算模型, 可对非正交轴系经纬仪的旋转角度进行快速简便且精确的解算。首先, 设待解的旋转角度为 (θ_1, θ_2) , 并将其分解为

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \theta_{0,1} \\ \theta_{0,2} \end{bmatrix} + \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \end{bmatrix}, \quad (13)$$

式中: $\theta_{0,1}$ 、 $\theta_{0,2}$ 为旋转角度的初始估计值; $\delta\theta_1$ 、 $\delta\theta_2$ 为旋转角度的误差修正值。

误差修正值的准确计算是实现非正交轴系经纬仪反向运动计算的关键, 相关计算模型将在后文中详细描述。如图 3 所示, 将轴系近似看作正交关系即可快速解算旋转角度的初始估计值, 计算方式为

$$\begin{cases} \theta_{0,1} = -\arctan(x_{Tar}/y_{Tar}) \\ \theta_{0,2} = \arctan(z_{Tar}/y_{Tar}) \end{cases} \quad (14)$$

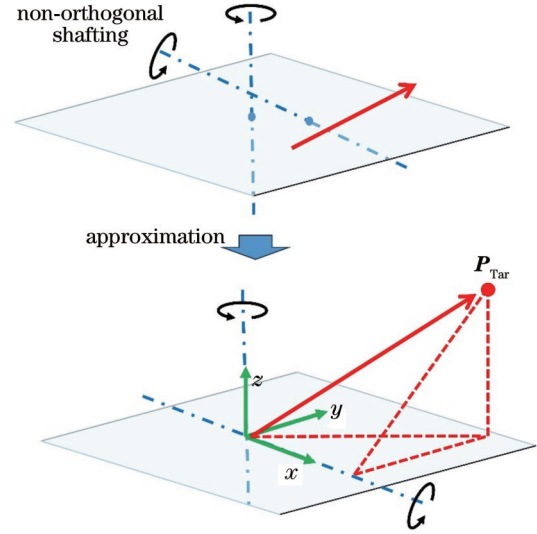


图 3 旋转角度的初始估计值

Fig. 3 Initial estimate of the rotation angle

3.3 反向运动角度线性计算

将上述解算得到的旋转角度初始值代入式(9)、(10)中计算得到激光视准轴空间位姿参数的初始值, 分别由点坐标 (x_0, y_0, z_0) 与方向向量 (u_0, v_0, w_0) 表示。初步解算得到的激光视准轴空间位姿必然存在误差, 因此视准轴的真实空间位姿参数可表示为

$$\begin{bmatrix} u \\ v \\ w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \\ w_0 \\ x_0 \\ y_0 \\ z_0 \end{bmatrix} + \begin{bmatrix} \delta u \\ \delta v \\ \delta w \\ \delta x \\ \delta y \\ \delta z \end{bmatrix}, \quad (15)$$

式中, 点坐标 $(\delta x, \delta y, \delta z)$ 与方向向量 $(\delta u, \delta v, \delta w)$ 分别表示误差值。

将式(15)代入式(12), 舍去极小值部分, 可得到

$$\begin{bmatrix} y_{Tar} - y_0 & -x_{Tar} + x_0 & 0 & v_0 & -u_0 & 0 \\ z_{Tar} - z_0 & 0 & -x_{Tar} + x_0 & w_0 & 0 & -u_0 \\ 0 & z_{Tar} - z_0 & -y_{Tar} + y_0 & 0 & w_0 & -v_0 \end{bmatrix} \begin{bmatrix} \delta u \\ \delta v \\ \delta w \\ \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} (x_{Tar} - x_0)v_0 - (y_{Tar} - y_0)u_0 \\ (x_{Tar} - x_0)w_0 - (z_{Tar} - z_0)u_0 \\ (y_{Tar} - y_0)w_0 - (z_{Tar} - z_0)v_0 \end{bmatrix} \quad (16)$$

基于 Selig^[21]提出的指数映射微分的显式表达式,可推导旋转角度误差传递模型。由旋转角度误差引起的误差表达式表示为

$$\begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{v} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\xi}_1 & \text{Ad}[\exp(\hat{\boldsymbol{\xi}}_1 \theta_1)] \boldsymbol{\xi}_2 \end{bmatrix} \begin{bmatrix} \delta \theta_1 \\ \delta \theta_2 \end{bmatrix}, \quad (17)$$

式中, $\text{Ad}(g)$ 表示 g 的伴随变换, 表示为

$$\text{Ad}(g) = \begin{bmatrix} \boldsymbol{R} & \mathbf{0} \\ \hat{\boldsymbol{b}}\boldsymbol{R} & \boldsymbol{R} \end{bmatrix}. \quad (18)$$

旋转角度误差最终会影响激光视准轴空间位姿参数的精度。激光视准轴方向向量误差值可表示为

$$\begin{bmatrix} \delta u \\ \delta v \\ \delta w \end{bmatrix} = \hat{\boldsymbol{\omega}} \boldsymbol{V}_0 = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} u_0 \\ v_0 \\ w_0 \end{bmatrix} = - \begin{bmatrix} 0 & -\omega_0 & v_0 \\ \omega_0 & 0 & -u_0 \\ -v_0 & u_0 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} = -\hat{\boldsymbol{V}}_0 \boldsymbol{\omega}, \quad (19)$$

同理, 激光视准轴上固定点坐标误差值可表示为

$$\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} = \hat{\boldsymbol{\omega}} \boldsymbol{P}_0 + \boldsymbol{v} = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix} \begin{bmatrix} x_0 \\ y_0 \\ z_0 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = - \begin{bmatrix} 0 & -z_0 & y_0 \\ z_0 & 0 & -x_0 \\ -y_0 & x_0 & 0 \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{bmatrix} + \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} = -\hat{\boldsymbol{P}}_0 \boldsymbol{\omega} + \boldsymbol{v}, \quad (20)$$

由式(19)与式(20)可得

$$\begin{bmatrix} \delta u \\ \delta v \\ \delta w \\ \delta x \\ \delta y \\ \delta z \end{bmatrix} = \begin{bmatrix} -\hat{\boldsymbol{V}}_0 & \mathbf{0} \\ -\hat{\boldsymbol{P}}_0 & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{\omega} \\ \boldsymbol{v} \end{bmatrix}. \quad (21)$$

设

$$\boldsymbol{J} = \begin{bmatrix} y_{\text{Tar}} - y_0 & -x_{\text{Tar}} + x_0 & 0 & v_0 & -u_0 & 0 \\ z_{\text{Tar}} - z_0 & 0 & -x_{\text{Tar}} + x_0 & \omega_0 & 0 & -u_0 \\ 0 & z_{\text{Tar}} - z_0 & -y_{\text{Tar}} + y_0 & 0 & \omega_0 & -v_0 \end{bmatrix} \begin{bmatrix} -\hat{\boldsymbol{V}}_0 & \mathbf{0} \\ -\hat{\boldsymbol{P}}_0 & \mathbf{1} \end{bmatrix} \begin{bmatrix} \boldsymbol{\xi}_1 & \text{Ad}[\exp(\hat{\boldsymbol{\xi}}_1 \theta_1)] \boldsymbol{\xi}_2 \end{bmatrix}, \quad (22)$$

可得

$$\begin{bmatrix} (x_{\text{Tar}} - x_0)v_0 - (y_{\text{Tar}} - y_0)u_0 \\ (x_{\text{Tar}} - x_0)\omega_0 - (z_{\text{Tar}} - z_0)u_0 \\ (y_{\text{Tar}} - y_0)\omega_0 - (z_{\text{Tar}} - z_0)v_0 \end{bmatrix} = \boldsymbol{J} \begin{bmatrix} \delta \theta_1 \\ \delta \theta_2 \end{bmatrix}. \quad (23)$$

将以上方程代入式(13), 可得:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} = \begin{bmatrix} \theta_{0,1} \\ \theta_{0,2} \end{bmatrix} + (\boldsymbol{J}^T \boldsymbol{J})^{-1} \boldsymbol{J}^T \begin{bmatrix} (x_{\text{Tar}} - x_0)v_0 - (y_{\text{Tar}} - y_0)u_0 \\ (x_{\text{Tar}} - x_0)\omega_0 - (z_{\text{Tar}} - z_0)u_0 \\ (y_{\text{Tar}} - y_0)\omega_0 - (z_{\text{Tar}} - z_0)v_0 \end{bmatrix}. \quad (24)$$

结合所得到的旋转角度初值, 最终反向旋转角度可由旋转角度初值与误差修正值线性相加得到, 式(24)即为计算最终旋转角度的线性方程。

4 仿真验证

对上述反向运动线性模型的可行性进行仿真验证。非正交轴系经纬仪的轴系参数使用激光跟踪仪等高精度空间坐标测量仪器进行标定, 具体标定细节可参考相关内部参数标定文献^[22-23], 标定结果如表1所示。在仿真模拟与实验验证中, 均采用该组内部轴系参数进行反向运动旋转角度的解算。如图4所示, 在

Solidworks 软件中制作非正交轴系激光经纬仪模型, 其轴系参数与表1保持一致。在仿真软件环境下模拟以固定间隔角度绕轴旋转后的激光视准轴, 旋转运动完成后, 在激光视准轴上采集被测目标点。采集的目标点坐标可在 Solidworks 软件下直接获取并作为参考值。将旋转的间隔角度设为 10° , 分别在激光视准轴上距离固定点 2、4、6、8、10 m 处采集点坐标。模拟的采集得到的空间被测目标点坐标如图5所示。

利用 Matlab 软件将轴系仿真参数及目标点坐标代入反向运动线性模型进行计算。当非正交轴系经纬仪旋转角度初始估计值与旋转角度实际值差距过大

表 1 轴系参数
Table 1 Shafting parameters

Shafting	Vector	Point coordinate /mm
Vertical rotation axis	(0, 1, 0)	(0, 0.081, 0)
Horizontal rotation axis	(0.999999, 0, -0.000328)	(0, 0, 0)
Laser axis	(0.004097, 0.999991, 0.000549)	(-0.511, 0, -19.473)

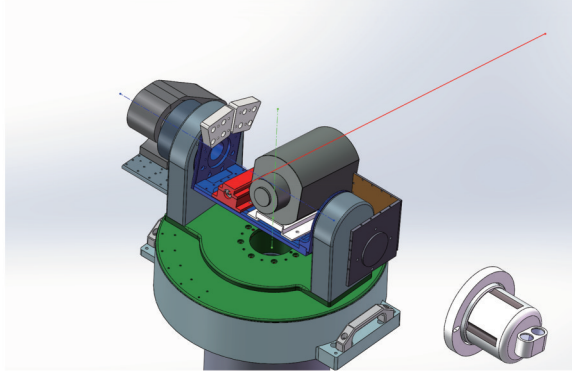


图 4 仿真环境

Fig. 4 Simulation environment

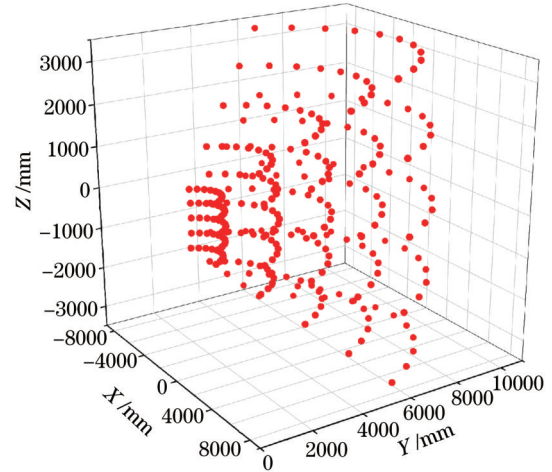


图 5 仿真被测目标点

Fig. 5 Simulation of the measured target points

时, 单次误差修正值的计算精度受限。为了获取高精度的误差修正值, 可以进行迭代优化, 即将上一次的旋转角度计算值作为初始估计值再次代入所提反向运动线性模型进行计算。非正交轴系经纬仪的反向运动学

旋转角度整体计算流程如图 6 所示。

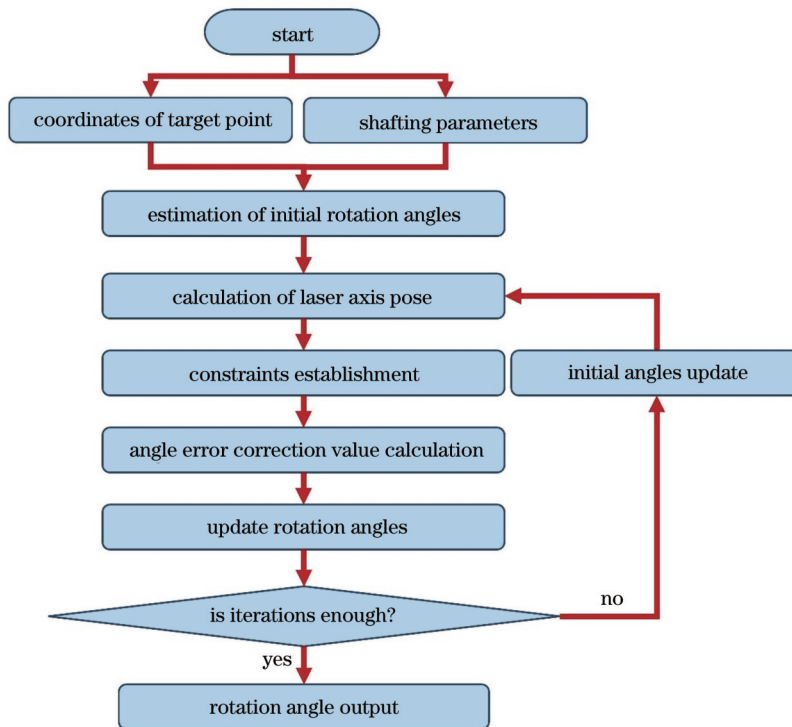


图 6 计算流程

Fig. 6 Calculation process

最终计算得到的旋转角度结果如表 2 与图 7 所示, 计算得到的旋转角度与真实角度的差距趋近于 0, 证

明所提反向运动线性模型是可行的且具备较高的精度。

表 2 旋转角度仿真计算结果

Table 2 Simulation results of the rotation angles

Simulation result	θ_1	θ_2
Mean error /mrad	1.68×10^{-11}	8.18×10^{-15}
RMSE (root mean square error) /mrad	5.95×10^{-11}	3.32×10^{-11}

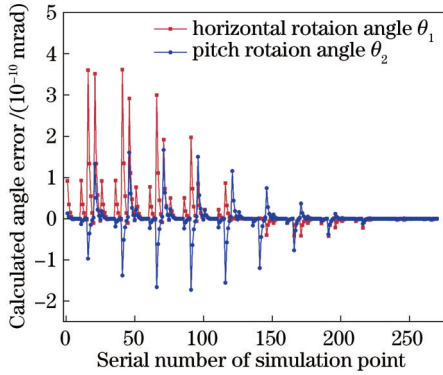


图 7 旋转角度误差仿真结果

Fig. 7 Simulation results of the rotation angle error

5 实验验证

非正交轴系激光经纬仪样机由高精度二维转台与准直激光器组成,实物如图 8 所示。二维转台选用美国 ALIO 公司的 AZ-EL GIMBAL 两轴转台,该转台应用了高精度精密交叉滚子轴承、圆光栅码盘,采用了全闭环控制及角度误差补偿等关键技术,可实现 $\pm 1''$ 角度定位重复性精度。激光器发散角为 0.6 mrad ,具备良好的准直性。非正交轴系激光经纬仪轴系参数由激光跟踪仪进行标定,激光跟踪仪为德国徕卡公司的 AT960-LR,空间坐标测量精度可达 $(\pm 7.5 + 3) \mu\text{m/m}$ 。

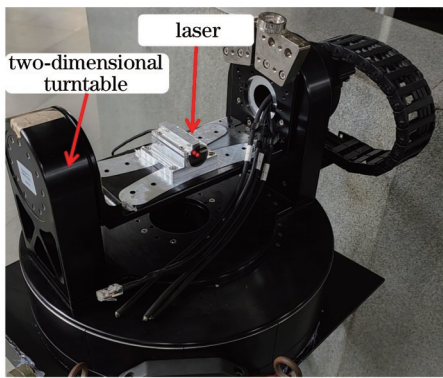


图 8 非正交轴系经纬仪样机

Fig. 8 Non-orthogonal shafting theodolite prototype

通过实验验证所提反向运动旋转角度线性模型,实验现场如图 9 所示。在实验中需要获取非正交轴系经纬仪瞄准点的高精度空间坐标。利用位置敏感探测器(PSD)采集非正交轴系经纬仪的出射激光,并结合激光跟踪仪与坐标转换技术获取相应的激光点空间坐标^[24]。在 10 m 范围空间内随机选取若干位置,驱动非

正交轴系经纬仪瞄向 PSD 靶标并通过激光跟踪仪获取入射激光的空间点坐标。同时二维转台反馈当前的高精度旋转角度,根据高精度旋转角度与轴系参数解算当前激光视准轴的空间位姿。设被瞄准目标点到激光视准轴的垂直距离为当前瞄准偏差,瞄准偏差如图 10 所示。

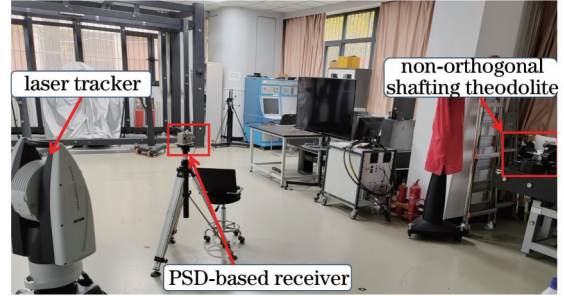


图 9 实验现场

Fig. 9 Experimental site

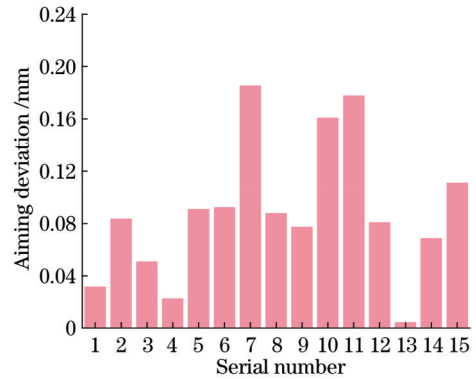


图 10 瞄准偏差

Fig. 10 Aiming deviation

瞄准偏差结果表明,基于 PSD 靶标的激光点空间坐标获取具有一定的精度。将获取的激光点坐标代入线性模型进行计算,将解算得到的旋转角度与转台输出角度进行对比。对比结果如图 11 与表 3 所示。

区别于仿真验证中的参数设置理想化,真实实验会受到其他众多因素干扰。所提反向线性模型中的关

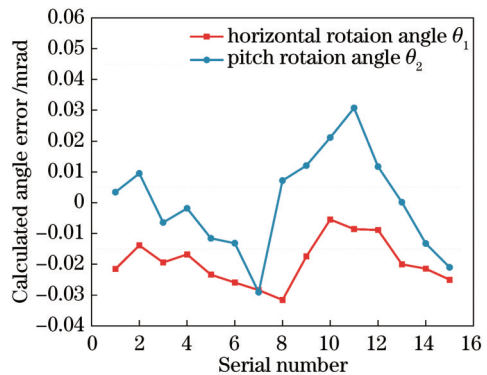


图 11 旋转角度误差实验结果

Fig. 11 Experimental results of the rotation angle error

表 3 旋转角度计算结果

Table 3 Calculation results of the rotation angles

Simulation result	θ_1	θ_2
Mean error /mrad	0.019	0.013
RMSE /mrad	0.021	0.016

键参数包括轴系参数与空间点精确坐标。当这些关键参数存在误差时,旋转角度计算结果将不可避免地出现误差。相关参数误差将在今后的工作中作进一步研究与讨论。此外,二维转台提供的旋转角度参考值也被引入转台转角误差。厂商官方资料显示,转台转角误差为 $\pm 3''$,约合 ± 0.014 mrad,解算的角度误差略高于转台转角示数误差,转角示数误差产生了相当大的影响。因此,在现有实验条件下,反向模型解算结果逼近高精度二维转台的转角示数误差,可认为其具备相当高的精度,证明了所提方法的可行性。

总体来说,实验结果证明了使用所提反向运动学线性模型可以计算对应的旋转角度,同时可以保证一定的精度水平。结合仿真与实验结果,可以验证所提反向运动学线性模型的可行性。

6 结 论

针对非正交轴系经纬仪反向运动旋转角度计算困难的问题,提出了相关的反向运动线性模型。基于李群李代数基本理论,建立了非正交轴系经纬仪正向运动模型及旋转角度误差传递理论模型。结合被测目标点与激光视准轴位姿间的约束关系,构建了旋转角度误差修正值的线性方程,实现了旋转角度的高精度计算。仿真实验验证了这一线性模型的可行性,真实实验解算得到的旋转角度平均误差为 0.019 mrad 与 0.013 mrad。由于激光点空间坐标误差、内部轴系误差以及转台示数误差等干扰因素的存在,限制了相关解算精度。但该方法解决了当前无参考末端的难题,满足误差分布模拟仿真相关研究的需要。在后期工作中,将进一步推进非正交轴系经纬仪系统相关精度理论研究,该反向运动线性模型可提供空间目标点所对应的旋转角度,便于非正交轴系经纬仪测量空间误差分布的仿真研究。此外,提升轴系参数与旋转角度的精度也将是后续研究的重点内容。

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Linear Model of Inverse Kinematics for Non-Orthogonal Shafting Laser Theodolite

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Abstract

Objective The developing modern economy and manufacturing industry have put forward a higher demand for related measuring instruments. However, the requirements for orthogonality of internal structures increase the difficulty of instrument manufacturing, and the production efficiency is reduced. To break through the limitations in the production and manufacturing of large-scale measuring instruments in China, the concept of non-orthogonal shafting measuring instruments is proposed. This kind of instrument does not require the shafting to be perpendicular or intersecting with each other, and the non-orthogonal shafting laser theodolite (N-theodolite) is a typical non-orthogonal shafting instrument. Much research has explored its measurement performance and related theories. However, due to the lack of the reference end on the laser axis for a single N-theodolite, the inverse kinematics model fails to be established accurately. The inverse kinematics model is necessary for precision theory research, which is convenient for data simulation of error spatial distribution. Besides, the guidance technology based on the inverse model can help improve the automatic measurement function of the N-theodolite. Therefore, to address the difficulty of calculating the rotation angle for the N-theodolite in inverse motion without a reference end, the linear model for the inverse motion to achieve fast and high-precision calculation of the rotation angle is proposed in this paper.

Method In this paper, the basic theory of Lie groups and Lie algebras is introduced to achieve fast and high-precision calculation of the rotation angle. First, based on the theory of Lie groups and Lie algebras, the kinematics model of the N-theodolite can be constructed. The coordinate transformation matrix is represented as the product of the exponentials formula (POE) with clear physical meanings. Second, the error model of N-theodolite can be obtained through the corresponding differential calculations, and the parts related to the rotation angle error component are preserved. Then, the constraint relationship between the spatial target point and the laser axis pose parameters is constructed, and the linear equations for solving the error correction value of the rotation angle can be established. Besides, the initial values can be quickly obtained through trigonometric functions. Finally, the high-precision rotation angle values are obtained by linear addition of the initial estimation values and the error correction values. The efficient and accurate linear inverse kinematics model of N-theodolite is established.

Results and Discussion In this paper, the simulation and real experiments are carried out to verify the proposed linear inverse kinematics model of the N-theodolite. The simulation results show that the rotation angle error calculated by the proposed method approaches 0 (Table 2 and Fig. 7). The proposed method is completely proved to be feasible in principle, and the inverse rotation angles are calculated with extremely high accuracy. However, the interference from multiple error sources is reflected in the experimental results, and the mean rotation angle error in the actual experimental

is less than 0.02 mrad (Table 3). The parameters in the proposed linear inverse kinematics model include shafting parameters and the coordinates of spatial points, which would inevitably affect the performance of the N-theodolite. The two-dimensional turntable is used to provide reference values for rotation angle, and the official data of the manufacturer shows that the angle error of the turntable is $\pm 3''$, approximately ± 0.014 mrad. The calculated angle error by the proposed method is slightly higher than the angle error of the turntable. Therefore, the results of the proposed linear inverse kinematics model approximate the angle error output by the high-precision two-dimensional turntable under the existing experimental conditions, which can prove the feasibility and accuracy of the method proposed in this paper.

Conclusions A linear inverse kinematics model is proposed to address the difficulty in the inverse motion angle calculation of N-theodolites. Based on the basic theory of Lie group and Lie algebra, the forward motion model and theoretical rotation angle error transmission model for the N-theodolite are established. By combining the constraint relationship between the measured target point and the spatial laser axis pose parameters, a linear equation for calculating the error correction value of the rotation angle is constructed, achieving high-precision calculation of the rotation angle. The feasibility of this linear inverse kinematics model has been verified through simulation experiments, and the average rotation angle error calculated from real experiments is 0.019 mrad and 0.013 mrad. Due to the influence factors such as spatial coordinate errors of laser points, internal shafting errors, and turntable angular errors, the accuracy of related calculations is limited, but the current problem of no reference end is solved by the proposed method, and the requirements of research related to error distribution are met. Further research will be carried out on the accuracy theory of N-theodolite measuring systems, and improving the accuracy of shafting parameters and rotation angles simultaneously will be a key focus.

Key words optical instruments; non-orthogonal shafting; inverse motion; linear model; error propagation