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单轴晶体中垂直光轴方向上的二维泰伯效应

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摘要 基于单轴晶体光束传输理论，从理论上推导了二维正交周期结构光场沿垂直单轴晶体光轴方向传播产生泰伯效应的公式。给出了泰伯效应的产生条件和常规泰伯距离的表达式，常规泰伯距离的大小决定于单轴晶体寻常光的折射率 n_o 与非寻常光折射率 n_e 之比和二维周期结构的 x 方向周期 p_x 和 y 方向周期 p_y 之比。模拟了二维正弦光栅和二维项链光栅的光场在单轴晶体中因各向异性衍射产生的泰伯效应。两种情况都能在更短的约化泰伯距离处产生与原光栅相同的泰伯像，在半约化泰伯距离产生与原光栅互补的像；在 1/4 约化泰伯距离和 3/4 约化泰伯距离产生的泰伯子像强度分布的周期为原光栅的 1/2，相位变化的周期与原光栅相同，但是在空间上有半个周期的移动。本研究将泰伯效应拓展到各向异性介质中的传播动力学，加深了人们对周期性光场的传输理论的理解。

关键词 衍射；泰伯效应；单轴晶体；二维正弦光栅；二维项链光栅

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1 引 言

周期性物体的衍射或者周期结构波的传播在特定距离会出现自成像现象，即所谓的泰伯效应。泰伯效应在光学、声学、电子显微术等领域都得到了广泛而深入的研究[1]。泰伯效应在现代光学中有许多重要的应用，如光学测量[2-3]、阵列照明[4]、光刻[5]、彩色数字全息术[6]以及 X 射线成像技术[7]等。泰伯效应具有重要研究价值，在国内外引起研究人员的兴趣[8-15]。尹东旭等[16]分析了球面波入射二维光栅引起的菲涅耳衍射区的光场分布，讨论了泰伯像和分数泰伯像的成像条件。韩振海[17]应用快速傅里叶变换对振幅型二维点阵光栅的泰伯效应进行了理论分析以及数值模拟，简化了二维光栅衍射问题的研究方法。Hebri 等[18]证明了二维正交不可分周期结构的泰伯距离与其周期结构的数论性质密切相关，给出了泰伯自像、负自像和子像的形成条件及其特征。

各向异性介质在许多现代设备和技术中具有重要的应用价值。许多基于非线性效应的激光技术，如二次谐波产生、光学参量放大和各种波的混频方式等，都是基于各向异性介质产生和发展的[19-22]。单轴晶体作为一种典型的各向异性光学介质，在补偿器、偏振器、振幅/相位调制器件的设计中有着广泛的应用[23-25]。光束在单轴晶体中的传播是光学和光电子领域的一个重要问题，自 1976 年 Stamnes 等[26]用平面波角谱法研究单轴晶体中光束的传输特性以来，各种类型的激光束在单轴晶体中的传播得到了广泛的研究，如艾里光束[27-34]、高斯光束[33-36]、贝塞尔光束[37]等。朱开成等[27-28]讨论了附加球面相位的 Airy 光束和附加交叉相位的 Airy-Hermite-Gaussian 光束在单轴晶体中的传输，基于求得的解析传输表达式，分析了附加相位对 Airy 和 Airy-Hermite-Gaussian 光束在单轴晶体中传播时强度分布的影响。Zhou 等[29]推导了单轴晶体中 cosh-Airy 光束的重心、束斑大小和线性矩的表达式，研究了双曲余弦-艾里光束在单轴晶体中的传输特性和单轴晶体对 cosh-Airy 光束强度和相位分布的影响。Wang 等[30]研究了 cosh-Airy 光束在单轴晶体中沿光轴正交方向传输时两个光束分量的干涉效应。然而，目前还没有关于周期结构光场在单轴晶体中传播方面的研究，特别是将泰伯效应推广到单轴晶体中去。本文通过理论推导和仿真计算，得到了二维正交周期结构光场沿垂直单轴晶体光轴方向传播产生泰伯效应的公式，确定了二维正弦光栅和二维项链光栅在单轴晶体传播时产生泰伯效应的条件，得出了两种光场在单轴晶体中传播的泰伯效应的图像并进行了讨论，进一步加深了对周期性光场的传输理论的理解。

2 二维正交周期结构的泰伯效应

一般情况下，令光束沿 z 轴正向传播，而单轴晶体的光轴沿 x 轴方向，单轴晶体介电常数张量 ε 可表示为

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$$\varepsilon=\begin{pmatrix} n_e^2 & 0 & 0 \\ 0 & n_o^2 & 0 \\ 0 & 0 & n_o^2 \end{pmatrix}, \tag{1}$$

式中：n_o和n_e分别为寻常光波的折射率与非寻常光波的折射率。

傍轴近似下，光场在垂直单轴晶体光轴方向传播时发生衍射[25]，光场的复振幅可表示为

$$E_x(x,y,z)=\frac{k_0 n_o}{2\pi i z}\exp(ik_0 n_e z)\iint dx_0 dy_0 \exp\left\{-\frac{k_0}{2izn_e}\left[n_o^2(x-x_0)^2+n_e^2(y-y_0)^2\right]\right\}E_x(x_0,y_0,0), \tag{2}$$

$$E_y(x,y,z)=\frac{k_0 n_o}{2\pi i z}\exp(ik_0 n_o z)\iint dx_0 dy_0 \exp\left\{-\frac{k_0 n_o}{2iz}\left[(x-x_0)^2+(y-y_0)^2\right]\right\}E_y(x_0,y_0,0), \tag{3}$$

式中：$k_0=2\pi/\lambda$为波数，λ为真空中单色光波长；$E_x(x_0,y_0,0)$和$E_y(x_0,y_0,0)$分别为入射面上x和y偏振的光场分布。

式(2)、(3)表明，y方向上的衍射与各向同性介质中的菲涅耳衍射相同，由于单轴晶体具有各向异性，x方向上偏振场分量发生了特殊的衍射，因而研究二维周期结构在单轴晶体中的衍射可以更好地揭示传播介质对泰伯效应的影响。现考虑x方向上的各向异性衍射，由式(2)可得，单轴晶体中x方向的点扩展函数为

$$h(x,y)=\exp(ik_0 n_e z)\frac{k_0 n_o}{2\pi i z}\exp\left(-\frac{k_0 n_o^2 x^2}{2izn_e}-\frac{k_0 n_e y^2}{2iz}\right), \tag{4}$$

忽略常定项$\exp(ik_0 n_e z)$，通过傅里叶变换获得传递函数在频域上的表示为

$$H(\xi,\eta)=\exp\left[-i\pi\lambda z n_e\left(\frac{\xi^2}{n_o^2}+\frac{\eta^2}{n_e^2}\right)\right]=\exp\left(-i\pi\lambda z\frac{n_e^2\xi^2+n_o^2\eta^2}{n_e n_o^2}\right)。 \tag{5}$$

对于二维正交周期结构，可以用二维傅里叶级数来表示其透射率函数[18]：

$$t(x,y)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\exp\left[2\pi i\left(m\frac{x}{p_x}+n\frac{y}{p_y}\right)\right], \tag{6}$$

式中：$t_{m,n}$为傅里叶系数；m、n为衍射级；p_x、p_y分别为x和y方向上的周期。对式(6)进行傅里叶变换，得到基于二维狄拉克函数$\delta(\xi,\eta)$表达的二维周期结构的空间谱[18]：

$$T(\xi,\eta)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\delta(\xi-mf_x,\eta-nf_y), \tag{7}$$

式中：f_x和f_y分别为二维周期结构在x和y方向上的基频，$f_x=1/p_x$，$f_y=1/p_y$。

利用Hebri等[18]在二维正交周期结构近场衍射的研究结果，研究二维正交周期结构在单轴晶体中的各向异性衍射。用一束x偏振的相干均匀光束照射二维正交周期结构，紧邻结构后表面的光场分布$u_0(x,y)$正比于其透射率函数$t(x,y)$。忽略常数振幅项，则光场分布的形式为

$$u_0(x,y)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\exp\left[2\pi i(mf_x x+nf_y y)\right], \tag{8}$$

光场分布$u_0(x,y)$对应的角谱表达式为

$$U_0(\xi,\eta)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\delta(\xi-mf_x,\eta-nf_y)。 \tag{9}$$

将角谱表达式$U_0(\xi,\eta)$与传递函数$H(\xi,\eta)$相乘，得到在单轴晶体中垂直光轴方向传播了任意距离z之后的x偏振光场的谱分布为

$$U_z(\xi,\eta)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\delta(\xi-mf_x,\eta-nf_y)\times \exp\left(-i\pi\lambda z\frac{n_e^2\xi^2+n_o^2\eta^2}{n_e n_o^2}\right), \tag{10}$$

对$U_z(\xi,\eta)$进行傅里叶逆变换，得到

$$u_z(x,y)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\exp\left[2\pi i(mf_x x+nf_y y)\right]\times \exp\left(-i\pi\lambda z\frac{n_e^2 m^2 f_x^2+n_o^2 n^2 f_y^2}{n_e n_o^2}\right)。 \tag{11}$$

定义x方向上的泰伯距离$z_{tx}=(2n_e n_o^2)/(\lambda f_x^2 n_e^2)$和$y$方向上的泰伯距离$z_{ty}=(2n_e n_o^2)/(\lambda f_y^2 n_o^2)$，则有

$$u_z(x,y)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\exp\left[2\pi i(mf_x x+nf_y y)\right]\times \exp\left[-2\pi i z\left(\frac{m^2}{z_{tx}}+\frac{n^2}{z_{ty}}\right)\right]。 \tag{12}$$

如果$z_{tx}/z_{ty}=[(p_x/p_y)(n_o/n_e)]^2$是一个有理数，则存在互为质数的正整数$k$和$l$满足：

$$\frac{z_{tx}}{z_{ty}}=\left(\frac{p_x}{p_y}\cdot\frac{n_o}{n_e}\right)^2=\frac{k}{l}, \tag{13}$$

则可以引入两个方向上的泰伯距离的最小公倍数z_{lcm}：

$$z_{lcm}=lz_{tx}=kz_{ty}=l\frac{2n_e n_o^2}{\lambda f_x{}^2 n_e^2}=k\frac{2n_e n_o^2}{\lambda f_y{}^2 n_o^2}, \tag{14}$$

把z_{lcm}代入式(12)，有

$$u_z(x,y)=\sum_{m,n=-\infty}^{+\infty} t_{m,n}\exp\left[2\pi i(mf_x x+nf_y y)\right]\times \exp\left[-2\pi i\left(\frac{z}{z_{lcm}}\right)(lm^2+kn^2)\right]。 \tag{15}$$

因lm^2+kn^2恒为正整数，在整数倍的z_{lcm}处，即$z=z_q=qz_{lcm}(q=1,2,3,\cdots)$，光场的复振幅完全复现出二维正交周期结构的光场，即$u_{z=z_q}(x,y)=u_0(x,y)$，实现了完美的泰伯自成像效果，所以可以将z_{lcm}作为二维

正交周期结构在单轴晶体中传播的常规泰伯距离。光场在传播整数倍常规泰伯距离后，会恢复到入射的原始光场强度分布。只要上述z_{tx}/z_{ty}为一个有理数，对于任意的二维正交周期结构光场，都可以找到相对应的常规泰伯距离z_{lcm}。当然，如果z_{tx}/z_{ty}为一个无理数，则无法定义常规泰伯距离，也就是说严格的泰伯效应不存在，但是原则上可以通过有理数的近似在一定精度下观察近似的泰伯效应。对于某些特殊的二维正交周期结构的光场（如下文所讨论的二维正弦光栅）在单轴晶体中的传播，z_{lcm}与lm^2+kn^2可能进一步约化，也就是说在更短的传播距离上就可以产生泰伯效应并观察到泰伯像。

3 数值模拟结果与分析

本节通过数值计算模拟了两种常见的二维光栅（二维正弦光栅和二维项链光栅）的光场在单轴晶体（寻常光折射率$n_o=2.616$，非寻常光折射率$n_e=2.903$）中由各向异性衍射产生的泰伯效应。具体分析了在约化泰伯距离、半约化泰伯距离、1/4 约化泰伯距离和 3/4 约化泰伯距离上的成像条件及成像特点。

3.1 二维正弦光栅

二维正弦光栅的透射率函数$t(x,y)$[18]可表示为

$$t(x,y)=\frac{1}{2}\left[1+\alpha\cos(2\pi f_x x)\cos(2\pi f_y y)\right],\quad(16)$$

式中：α为光栅参数。一个在x和y方向上具有相同周期的二维正弦光栅的光场强度分布和空间谱如图 1 所示，x和y方向上的周期$p_x=p_y=40\ \mu m$。

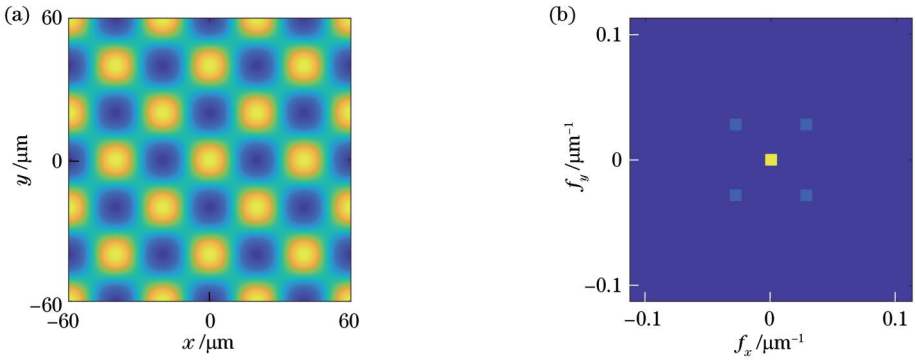

图 1 二维正弦光栅的光场强度分布和其空间谱（$p_x=p_y=40\ \mu m$）。(a)光场强度分布；(b)空间谱

Fig. 1 Intensity pattern and spatial spectrum of 2D sinusoidal grating ($p_x=p_y=40\ \mu m$). (a) Intensity distribution; (b) spatial spectrum

由于此光栅的傅里叶频谱简单，只有$m=\pm1$、$n=\pm1$的衍射级的频域成分，没有更多的高阶频域，可以将此透射率函数写成指数形式：

$$t(x,y)=t_{0,0}+\sum_{m,n=\pm1}t_{m,n}\exp\left[2\pi i\left(mf_x x+nf_y y\right)\right],\quad(17)$$

式中：$t_{0,0}=1/2$；$t_{m,n}=\alpha/8$。

进行与第 2 节类似的推导，用一束x偏振的相干均匀光束照射二维正弦光栅，紧邻光栅后表面的光场分布$u_0(x,y)$等于其透射率函数$t(x,y)$：

$$u_0(x,y)=t_{0,0}+\sum_{m,n=\pm1}t_{m,n}\exp\left[2\pi i\left(mf_x x+nf_y y\right)\right]。\quad(18)$$

由式(15)得到二维正弦光栅在单轴晶体中垂直光轴方向传播了任意距离z之后的光场分布为

$$u_z(x,y)=t_{0,0}+\exp\left[-2\pi i\left(\frac{z}{z_{lcm}}\right)(l+k)\right]\times\sum_{m,n=\pm1}t_{m,n}\exp\left[2\pi i\left(mf_x x+nf_y y\right)\right]。\quad(19)$$

当传播距离z为$z_{lcm}/(l+k)$的整数倍时，上面的$u_z(x,y)$也恢复到原始的入射光场。所以二维正弦光栅能够在小于常规泰伯距离z_{lcm}处产生泰伯效应。定义这个距离为二维正弦光栅的约化泰伯距离z_{RT}：

$$z_{RT}=\frac{z_{lcm}}{l+k}=\frac{z_{tx}z_{ty}}{z_{tx}+z_{ty}}=\frac{2n_e n_o^2}{\lambda(f_x{}^2n_e^2+f_y{}^2n_o^2)},\quad(20)$$

显然z_{RT}小于两个一维方向上的泰伯距离z_{tx}、z_{ty}和常规泰伯距离z_{lcm}，也就是说二维正弦光栅能够在更短的距离处观察到泰伯像。将在单轴晶体传播了距离z后的光场分布重新写为余弦形式：

$$u_z(x,y)=\frac{1}{2}\left\{1+\alpha\exp\left[-i2\pi\left(\frac{z}{z_{RT}}\right)\right]\times\cos\left(\frac{2\pi x}{p_x}\right)\cos\left(\frac{2\pi y}{p_y}\right)\right\}。\quad(21)$$

图 2 显示二维正弦光栅的光场沿垂直单轴晶体光轴方向传播时发生衍射后在不同约化泰伯距离产生的光场强度分布和相位分布。显然，在$z=qz_{RT}(q=1,2,3,\cdots)$处的光场分布与二维正弦光栅原来的光场分布完全相同，如图 2(a)所示，有$u_{z=z_{RT}}(x,y)=u_0(x,y)$。在两个相邻的约化泰伯距离的中间有半约化泰伯距离$z_{RHT}=(q-1/2)z_{RT}$，当$z=qz_{RHT}(q=1,2,3,\cdots)$，光场分布为

$$u_{z=z_{RHT}}(x,y)=\frac{1}{2}\left[1-\alpha\cos\left(\frac{2\pi x}{p_x}\right)\cos\left(\frac{2\pi y}{p_y}\right)\right]=u_0\left(x-\frac{p_x}{2},y\right)=u_0\left(x,y-\frac{p_y}{2}\right),\quad(22)$$

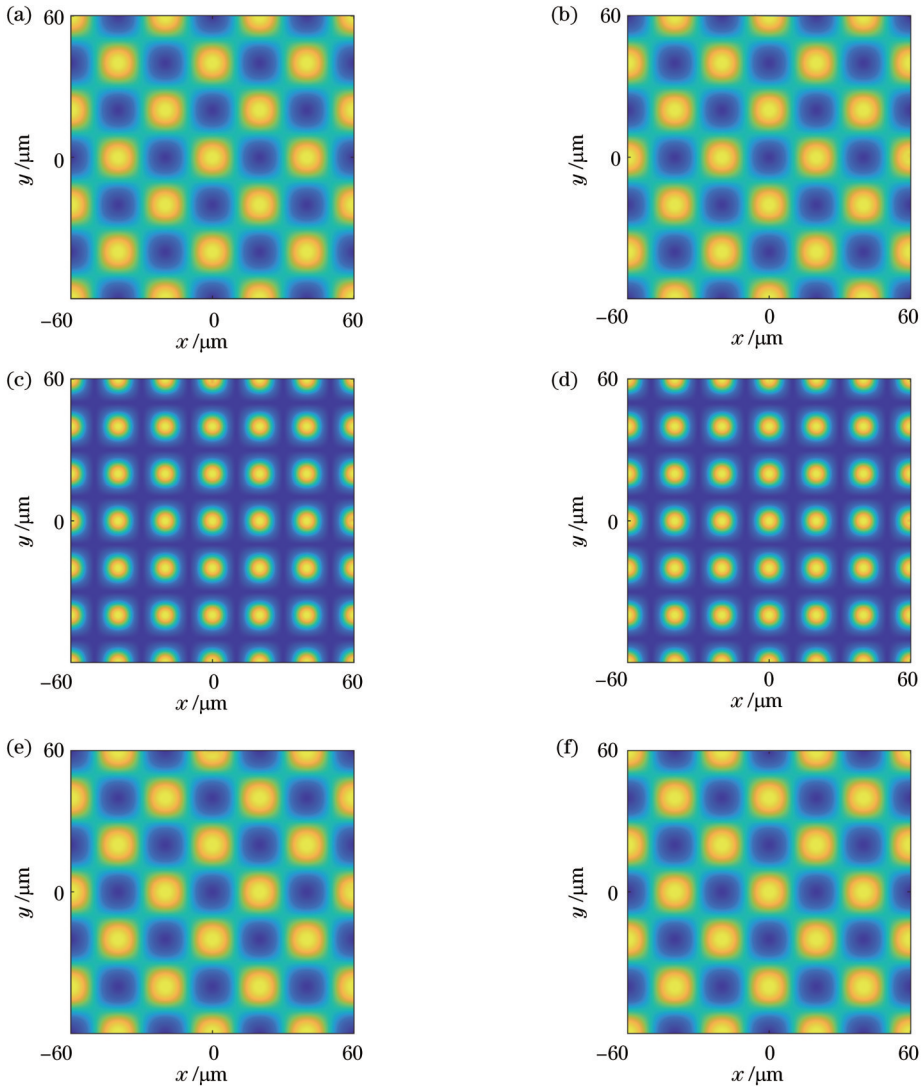

图 2 二维正弦光栅在不同位置处的光场强度分布和相位分布。(a)在 $z=z_{RT}=6.578$ mm 处的光场强度分布；(b)在 $z=z_{RT}/2$(对应于 z_{RHT})处的光场强度分布；(c)在 $z=z_{RT}/4$(对应于 q 为奇数时的 z_{RQT})处的光场强度分布；(d)在 $z=3z_{RT}/4$(对应于 q 为偶数时的 z'_{RQT})处的光场强度分布；(e)在 $z=z_{RT}/4$ 处的相位分布；(f)在 $z=3z_{RT}/4$ 处的相位分布

Fig. 2 Intensity patterns and phase patterns of 2D sinusoidal grating at different positions. (a) Intensity pattern at $z=z_{RT}=6.578$ mm; (b) intensity pattern at $z=z_{RT}/2$ (corresponding to z_{RHT}); (c) intensity pattern at $z=z_{RT}/4$ (corresponding to z_{RQT} when q is odd); (d) intensity pattern at $z=3z_{RT}/4$ (corresponding to z'_{RQT} when q is even); (e) phase pattern at $z=z_{RT}/4$; (f) phase pattern at $z=3z_{RT}/4$

与二维正弦光栅原来的光场强度分布相比，光场在 x 或 y 方向上有半周期偏移，如图 2(b)所示。在相邻的约化泰伯距离和半约化泰伯距离的中间，有 1/4 约化泰伯距离 $z_{RQT}=(q-1/2)(z_{RT}/2)(q=1,3,5,\cdots)$ 和 3/4 约化泰伯距离 $z'_{RQT}=(q-1/2)(z_{RT}/2)(q=2,4,6,\cdots)$，它们的光场分布为

$$\begin{cases} u_{z=z_{RQT}}(x,y)=\dfrac{1}{2}\left[1+\mathrm{i}(-1)^q\alpha\cos\left(\dfrac{2\pi x}{p_x}\right)\cos\left(\dfrac{2\pi y}{p_y}\right)\right] \\ u_{z=z'_{RQT}}(x,y)=\dfrac{1}{2}\left[1+\mathrm{i}(-1)^q\alpha\cos\left(\dfrac{2\pi x}{p_x}\right)\cos\left(\dfrac{2\pi y}{p_y}\right)\right] \end{cases}, \tag{23}$$

光场强度分布为

$$I_{z=z_{RQT}}(x,y)=I_{z=z'_{RQT}}(x,y)=\frac{1}{4}+\frac{\alpha}{2}\left[1+\cos\left(\frac{2\pi x}{p_x/2}\right)\right]\frac{\alpha}{2}\left[1+\cos\left(\frac{2\pi y}{p_y/2}\right)\right]。 \tag{24}$$

将在 1/4 约化泰伯距离和 3/4 约化泰伯距离处得到的光场强度分布称为泰伯子像，如图 2(c)、(d)所示。通过比较图 2(c)、(d)与图 2(a)、(b)，发现泰伯子像在 x 和 y 方向上强度分布的周期减半。图 2(e)、(f)分别为 1/4 约化泰伯距离的相位图和 3/4 约化泰伯距离的相位图，图 2(e)、(f)所示的 x 或 y 方向上相位变化的周期与二维正弦光栅的周期相同。图 2(c)、(d)的光场强度分布相同，但图 2(e)与图 2(f)在 x、y 方向上

的相位有半个周期的偏差。

根据文献[18],可以通过引入迈克耳孙强度 C_I 和相位 C_φ 对比度来分析二维正弦光栅在单轴晶体中垂直光轴方向传播的强度和相位对比度随传播距离的变化。迈克耳孙强度和相位对比度的具体形式为

$$C_I=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}},\tag{25}$$

$$C_\varphi=\frac{\varphi_{\max}-\varphi_{\min}}{\pi/2},\tag{26}$$

式中:$I_{\max}$、$I_{\min}$ 分别为平面上光场强度最大值和最小值;$\varphi_{\max}$、$\varphi_{\min}$ 分别为平面上相位的最大值和最小值。

图 3 为 x 偏振平面光照射二维正弦光栅($\alpha=0.98,0.90,0.70$)并沿垂直单轴晶体光轴方向传播的情况下在一个约化泰伯距离内的相对强度和相位对比。在约化泰伯距离 z_{RT} 和半约化泰伯距离 z_{RHT} 处光强相对最大;在 1/4 约化泰伯距离 z_{RQT} 和 3/4 约化泰伯距离 z'_{RQT} 处光强分布的频率是二维正弦光栅背面的 2 倍,因此,光场强度相对最小,相位的对比度相对最大。

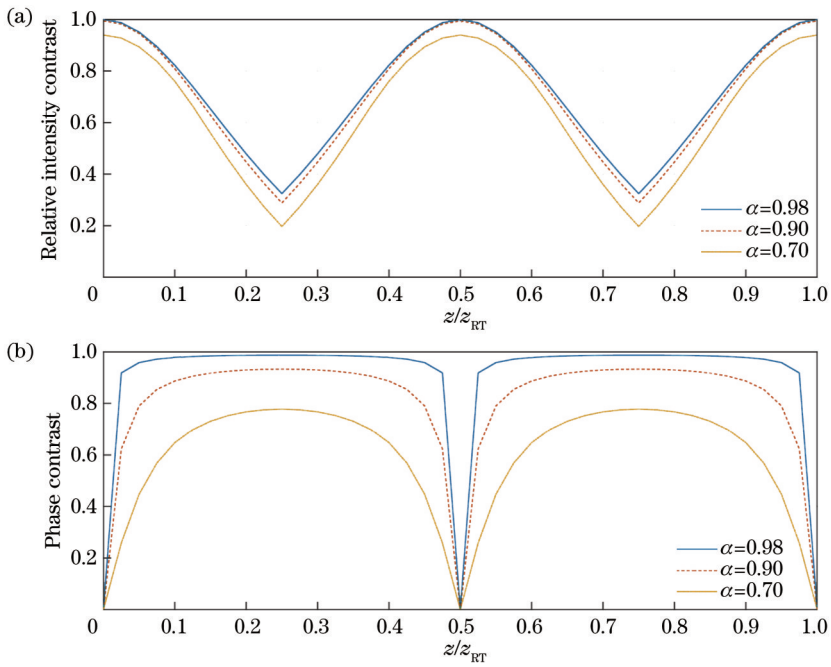

图 3 二维正弦光栅在一个约化泰伯距离内光场的相对强度和相位对比($\alpha=0.98,0.90,0.70$)。(a)相对强度对比;(b)相位对比

Fig. 3 Variation of relative intensity and phase contrasts of 2D sinusoidal grating in reduced Talbot distance ($\alpha=0.98,0.90,0.70$). (a) Relative intensity contrast; (b) phase contrast

3.2 二维项链光栅

这里通过二维项链光栅的例子来进一步说明约化泰伯距离的存在,以及在比常规泰伯距离更短的尺度上能够产生泰伯效应。二维项链光栅的透射率函数 $t'(x,y)$[18]可利用符号函数(sign 函数)表示为

$$t'(x,y)=\frac{1}{2}\left\{1+\mathrm{sign}\left[\sin\left(\frac{2\pi x}{p'_x}\right)\sin\left(\frac{2\pi y}{p'_y}\right)\right]\right\}。\tag{27}$$

一个在 x 和 y 方向具有不同周期的二维项链光栅的光场和空间谱如图 4 所示,x 方向上的周期 $p'_x=32\ \mu\mathrm{m}$,y 方向上的周期 $p'_y=95.64\ \mu\mathrm{m}$。

用一束 x 偏振的相干均匀光束照射二维项链光栅,紧邻二维项链光栅后表面的光场分布 $u'_0(x,y)$ 为

$$u'_0(x,y)=\frac{1}{2}\left\{1+\mathrm{sign}\left[\sin\left(\frac{2\pi x}{p'_x}\right)\sin\left(\frac{2\pi y}{p'_y}\right)\right]\right\}=\frac{1}{2}+\sum_{m=2s+1}^{+\infty}\sum_{n=2t+1}^{+\infty}\frac{8}{mn\pi^2}\sin\left(\frac{2\pi x}{p'_x}\right)\sin\left(\frac{2\pi y}{p'_y}\right),\tag{28}$$

式中:s、t 为任意正整数;m、n 为奇数次的衍射级。显然,不同于二维正弦光栅,这里有相当多的衍射级数。

进行与二维正弦光栅一样的推导,可得 $u'_0(x,y)$ 在单轴晶体中垂直光轴方向传播了任意距离 z 之后的光场分布 $u'_z(x,y)$ 为

$$u'_z(x,y)=\frac{1}{2}+\sum_{m=2s+1}^{+\infty}\sum_{n=2t+1}^{+\infty}\frac{8}{mn\pi^2}\exp\left[-2\pi\mathrm{i}\left(\frac{z}{z_{\mathrm{lcm}}}\right)(lm^2+kn^2)\right]\sin\left(\frac{2\pi x}{p'_x}\right)\sin\left(\frac{2\pi y}{p'_y}\right)。\tag{29}$$

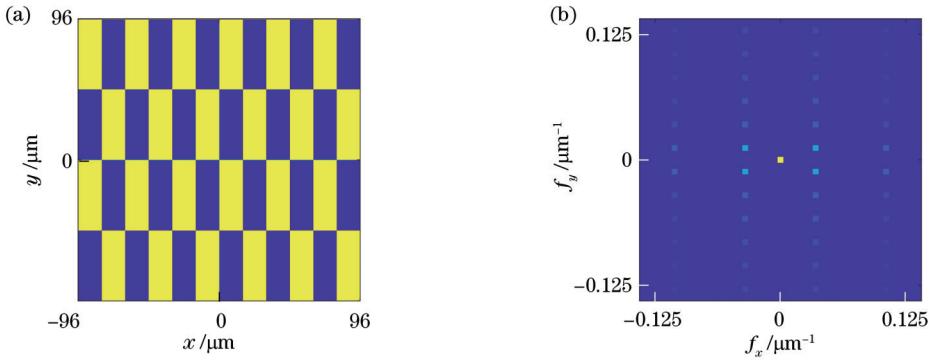

图 4 二维项链光栅的光场强度分布和空间谱(p'_x= 32 μm,p'_y= 95.64 μm)。(a)光场强度分布;(b)空间谱

Fig. 4 Intensity pattern and spatial spectrum of 2D checker grating (p'_x= 32 μm, p'_y=95.64 μm). (a) Intensity distribution; (b) spatial spectrum

从初等数论可知,奇整数的平方可以写为 $8q+1$ ($q=1,2,3,\cdots$)的形式,即 $m^2=8m'+1$, $n^2=8n'+1$。因此有

$$(lm^2+kn^2)=8(lm'+kn')+(l+k)。\tag{30}$$

当 $l+k$ 为 2、4、8 的整数倍时,lm^2+kn^2 也是 2、4、8 的整数倍,能够进一步与 z_{lcm} 约化,导致在小于 z_{lcm} 的距离发生泰伯效应,定义这个距离为二维项链光栅的约化泰伯距离 z_{T},分别对应于 $z_{\mathrm{T}}=z_{\mathrm{lcm}}/2$、$z_{\mathrm{T}}=z_{\mathrm{lcm}}/4$、$z_{\mathrm{T}}=z_{\mathrm{lcm}}/8$。显然这样的 z_{T} 小于两个一维方向上的泰伯距离 z_{tx}、z_{ty} 和常规泰伯距离 z_{lcm},所以二维项链光栅能够在更短的距离观察到泰伯像。值得强调的是,这里的常规泰伯距离 z_{lcm} 如式(14)所示,是依赖于寻常光折射率 n_{o} 和非寻常光折射率 n_{e} 的。

进行一些推导后,式(29)可以写为

$$u'_z(x,y)=\frac{1}{2}+\sum_{m=2s+1}^{+\infty}\sum_{n=2t+1}^{+\infty}\frac{8}{mn\pi^2}\times \exp\left[-2\pi\mathrm{i}\left(\frac{z}{z_{\mathrm{T}}}\right)\right]\sin\left(\frac{2\pi x}{p'_x}\right)\sin\left(\frac{2\pi y}{p'_y}\right)。\tag{31}$$

由式(13)、(14)可推导得,若二维项链光栅在 x 方向上的周期 p'_x=95.64 μm、y 方向上的周期 p'_y=95.64 μm,在寻常光折射率 n_{o}=2.616、非寻常光折射率 n_{e}=2.903 的单轴晶体中沿垂直光轴方向会发生泰伯效应。计算得,$z_{\mathrm{tx}}/z_{\mathrm{ty}}=[(p_x/p_y)(n_{\mathrm{o}}/n_{\mathrm{e}})]^2=1/11$,$z_{\mathrm{lcm}}=11z_{\mathrm{tx}}=z_{\mathrm{ty}}$,并且 $l+k=12$ 为 4 的整数倍,约化泰伯距离 $z_{\mathrm{T}}=z_{\mathrm{lcm}}/4$。

图 5 为二维项链光栅的光场沿垂直单轴晶体光轴方向传播发生衍射后在不同约化泰伯距离产生的光场强度分布和相位分布。与二维正弦光栅在单轴晶体中的衍射相似,在约化泰伯距离 z_{T} 和半约化泰伯距离 $z_{\mathrm{HT}}=(q-1/2)z_{\mathrm{T}}$($q=1,2,3,\cdots$)处的泰伯像互补,在 $z=qz_{\mathrm{T}}$ ($q=1,2,3,\cdots$)处的光场强度分布与二维项链光栅原来的光场强度分布相同,如图 5(a)、(b)所示;在 1/4 约化泰伯距离 $z_{\mathrm{QT}}=(q-1/2)(z_{\mathrm{T}}/2)$ ($q=1,3,5,\cdots$)和 3/4 约化泰伯距离 $z'_{\mathrm{QT}}=(q-1/2)(z_{\mathrm{T}}/2)$ ($q=2,4,6,\cdots$)处的光场强度分布相同,它们在 x 或 y 方向上的周期是二维项链光栅周期的一半,如图 5(c)、(d)所示;图 5(e)和图 5(f)分别为图 5(c)和图 5(d)对应的相位图,与图 5(c)、(d)不一样,在 x 或 y 方向上的图 5(e)、(f)的相位变化的周期与二维项链光栅的周期相同。与二维正弦光栅类似的是,图 5(e)与图 5(f)在 x、y 方向上的相位有半个周期的偏差。

4 结 论

基于光束在单轴晶体中的傍轴传输理论,从理论上推导了 x 偏振的相干均匀光束照射二维正交周期结构并沿垂直单轴晶体光轴方向传播产生泰伯效应的公式。给出了泰伯效应的产生条件和常规泰伯距离的表达式。由于光束在单轴晶体的传输过程中发生各向异性衍射,常规泰伯距离的大小不仅决定于二维周期结构 x 和 y 方向周期的比(p_x/p_y),还决定于单轴晶体寻常光与非寻常光折射率的比($n_{\mathrm{o}}/n_{\mathrm{e}}$)。满足特定周期的二维正交周期结构的光场在特定折射率单轴晶体传播产生了约化泰伯距离,能够在更短的传播距离上产生泰伯效应并观察到泰伯像。本文模拟了二维正弦光栅和二维项链光栅的光场在单轴晶体中发生的各向异性衍射产生的泰伯效应。两种情况都在约化泰伯距离产生与原光栅相同的泰伯像,在半约化泰伯距离产生与原光栅互补的像,在 1/4 约化泰伯距离和 3/4 约化泰伯距离产生的泰伯子像周期为原光栅的一半、相位变化的周期与原光栅的相同。对于任意线偏振入射光场,各向异性衍射导致的 x 偏振分量的泰伯距离和各向同性衍射导致的 y 偏振分量的泰伯距离不相等,所以只可能在这两个距离的公倍数对应的位置出现周期光场的重现,也就是说任意线偏振情况下的泰伯效应所需要的泰伯距离会更长。本研究将泰伯效应从各向同性介质中的传播动力学拓展到各向异性介质中的传播动力学,加深了人们对光场的传输理论,特别是周期性光场的传输理论的理解。

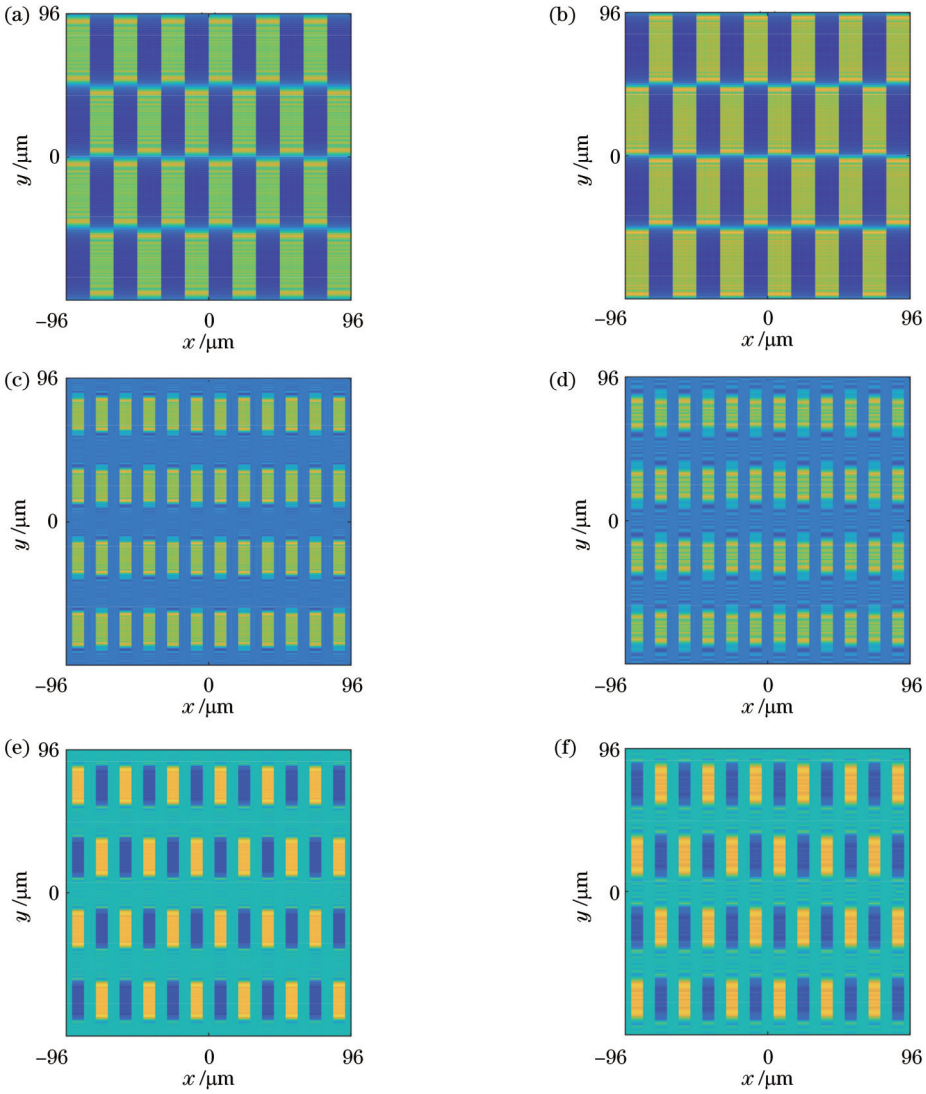

图 5 二维项链光栅在不同位置处的光场强度分布和相位分布。(a)在 $z=z_T=20.98$ mm 处的光场强度分布；(b)在 $z=z_T/2$(对应于 z_{HT})处的光场强度分布；(c)在 $z=z_T/4$(对应于 q 为奇数时的 z_{QT})处的光场强度分布；(d)在 $z=3z_T/4$(对应于 q 为偶数时的 z'_{QT})处的光场强度分布；(e)在 $z=z_T/4$ 处的相位分布；(f)在 $z=3z_T/4$ 处的相位分布

Fig. 5 Intensity patterns and phase patterns of 2D checker grating at different positions. (a) Intensity pattern at $z=z_T=20.98$ mm; (b) intensity pattern at $z=z_T/2$ (corresponding to z_{HT}); (c) intensity pattern at $z=z_T/4$ (corresponding to z_{QT} when q is odd); (d) intensity pattern at $z=3z_T/4$ (corresponding to z'_{QT} when q is even); (e) phase pattern at $z=z_T/4$; (f) phase pattern at $z=3z_T/4$

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Two-Dimensional Talbot Effect in Uniaxial Crystals Orthogonal to Optical Axis

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Abstract

Objective The Talbot effect has been studied intensively in optics, acoustics, electron microscopy, X-ray, and Bose-Einstein condensates. There are numerous important applications of the Talbot self-imaging phenomenon in modern optics, such as in optical measurement, array illumination, lithography, color digital holography, and X-ray imaging technology. Uniaxial crystal is a typical kind of anisotropic media which has been widely used in different optical devices, so the propagation of light beams in uniaxial crystals is also an important topic in the field of optics and optoelectronics. However, no research has studied the behavior of periodic light fields in uniaxial crystals, especially the Talbot effect in uniaxial crystals. Therefore, in this work, we theoretically investigate the Talbot effect in uniaxial crystals orthogonal to the optical axis to prove that the Talbot self-imaging phenomenon can also be observed in anisotropic media. Our results can extend the studies of the Talbot effect to the field of anisotropic media and may improve our understanding of the transmission properties of periodic light fields.

Methods Based on the beam transmission theory in uniaxial crystals, we successfully derive theoretical formulas to investigate the Talbot effect due to the propagation of the two-dimensional (2D) orthogonal periodic light field along the direction perpendicular to the optical axis of the uniaxial crystal (z-axis) when the optical axis of the uniaxial crystal is along the x-axis. When a 2D periodic object is illuminated by an x-polarized coherent uniform beam, the propagation is described by anisotropic diffraction. With the help of the Fourier transform and angular spectrum representation, we can derive the self-imaging conditions of the Talbot effect and the expression of the conventional Talbot distances. We have also performed numerical simulations to observe the anisotropic propagation of two special 2D orthogonal periodic optical fields (2D sinusoidal grating and 2D checker grating) in the rutile crystal. In virtue of the structures of these periodic optical fields, the Talbot self-imaging phenomenon can be observed in shorter propagation distances. The Talbot images obtained by these two kinds of light fields in the rutile crystal are presented and analyzed.

Results and Discussions We find the self-imaging conditions of the Talbot effect. The expression of the conventional Talbot distances depends on the ratio of the ordinary refractive index to the extraordinary refractive index in the uniaxial crystal (n_o/n_e) and the ratio of the period of the x direction to the period of the y direction in the 2D periodic object (p_x/p_y), as expressed by Eqs. (13) and (14). We have performed numerical simulations to observe the Talbot effect of the 2D sinusoidal grating and the 2D checker grating due to the anisotropic diffraction in the uniaxial crystal. In both cases, the Talbot images which are the same as their original gratings can be generated at reduced Talbot distances [Fig. 2 (a) and Fig. 5 (a)], and the complementary images of their original gratings can be generated at the reduced half-Talbot distance [Fig. 2(b) and Fig. 5(b)]. The Talbot sub-images with a half shift in space, whose intensity periods are halved from that of their original gratings and phase periods are the same as the period of the original gratings, are generated at the reduced quarter-Talbot distance and the reduced three-quarter-Talbot distance, as shown in Figs. 2(c)-(f) and Figs. 5(c)-(f). For the case of the 2D sinusoidal grating propagating along the z-axis (perpendicular to the optical axis of the uniaxial crystal), we have plotted the contrast variation of the intensity pattern and the phase contrast in a reduced Talbot distance (Fig. 3). The light intensity reaches its maximum value at the reduced Talbot distance and the reduced half-Talbot distance. At the reduced quarter-Talbot distance and the reduced three-quarter-Talbot distance, the light intensity reaches its minimum value. For the phase contrast, the maximum values are found at the reduced quarter-Talbot distance and the reduced three-quarter-Talbot distance.

Conclusions Based on the paraxial theory of light propagation in uniaxial crystals, we suppose that the optical axis is along the x-axis, and the propagation is along the z-axis. Furthermore, we have theoretically investigated the Talbot self-imaging phenomenon. When a 2D orthogonal periodic object is illuminated by the x-polarized coherent uniform beam, the anisotropic diffraction leads to the self-imaging conditions of the Talbot effect depending on n_o/n_e and p_x/p_y. If the 2D orthogonal periodic optical object meets certain conditions, the Talbot images can be generated in shorter propagation distances, which means that the Talbot effect can be observed at reduced Talbot distances. Finally, for an incident light

field linearly polarized at an arbitrary direction, the Talbot distance of the x-polarized component caused by the anisotropic diffraction does not equal that of the y-polarized component caused by the isotropic diffraction. Therefore, the repetition of the periodic light field can only occur at positions corresponding to the common multiples of these two distances. In other words, the Talbot distance corresponding to the arbitrarily linearly polarized case should be increased.

Key words diffraction; Talbot effect; uniaxial crystal; two-dimensional sinusoidal grating; two-dimensional checker grating