

# 光学学报

## 基于 $N$ 元正弦码元的时间相位展开

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**摘要** 提出了一种  $N$  元编码时间相位展开方法来实现彩色复杂物体的三维测量。传统的阶梯状条纹在测量整个表面反射率范围较大的物体时, 条纹数量过多会导致量化困难。利用从相移条纹中提取的  $N$  元正弦码元替换传统的量化灰度码元, 通过进制转换将  $N$  元正弦码元嵌入到所要投影的条纹中来实现条纹级次的编码。在解码时, 先通过编码条纹与  $N$  步正弦相移条纹的差异逐点计算  $N$  元量化条纹, 再根据对应的逆进制转换获取唯一的条纹级次, 最后利用截断相位消除条纹级次边缘处的错位误差。实验结果表明, 相比传统的时间相位展开方法, 所提方法有效提高了编码效率, 并在测量彩色复杂场景时表现出了较高的鲁棒性。

**关键词** 测量; 条纹投影; 时间相位展开; 条纹级次;  $N$  元编码

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### 1 引言

基于正弦条纹投影技术的光学三维测量因高精度、高分辨率和全场测量等优点被广泛应用于生物医药、工业检测, 以及虚拟现实等三维视觉相关的邻域<sup>[1-4]</sup>中。常用的条纹投影轮廓术(FPP)主要为傅里叶变换轮廓术(FTP)<sup>[5-6]</sup>和相移轮廓术(PSP)<sup>[7-8]</sup>。FTP利用带通滤波器<sup>[9]</sup>从单幅条纹图案的频谱域中提取相位, 通常适用于平滑物体的动态三维测量。相比于FTP, PSP借助多幅相移条纹图案逐点计算相位, 具有更高的分辨率和稳定性, 在测量相对静态的复杂物体时, PSP具有更高的精度和鲁棒性。然而, FTP和PSP的相位提取过程都涉及反正切运算, 这会导致获取的相位被截断在 $(-\pi, \pi]$ 或 $(\pi, -\pi]$ 范围内, 故需要采用必要的相位展开来保证物体面形的连续性。

相位展开方法主要分为两类: 空间相位展开<sup>[10]</sup>和时间相位展开<sup>[11-12]</sup>。空间相位展开利用邻点相位的可信度依次消除 $2\pi$ 歧义, 属于一种相对相位展开方法, 受限于只含单连通域的截断相位, 且难以恢复物体的绝对位置信息<sup>[13]</sup>。时间相位展开借助额外的条纹图确定条纹级次, 由于任意一点的相位都是单独展开的, 故能有效避免误差传播, 进而对于复杂物体的三维测量, 其具有较高的鲁棒性。目前, 主流的时间相位展开包括多频/多波长法<sup>[12, 14]</sup>、格雷编码法<sup>[15]</sup>和相位编码法<sup>[16]</sup>三种, 其中多频/多波长法利用分层和外差理论从一系列不同频率的相移条纹图案中获取条纹级次, 而格雷

编码法和相位编码法则是直接将条纹级次嵌入到额外的编码条纹中。通常, 多频/多波长法易受限于条纹频率的选择, 如在双频时间相位展开算法中, 高频和低频相差较大可能导致展开误差<sup>[17]</sup>。格雷编码法利用条纹的强度直接量化条纹级次, 而相位编码法利用多帧阶梯相移条纹图案在相位域量化条纹级次, 故后者较前者对表面反射率和噪声的敏感度更低。为了提高传统“全黑”和“全白”二元格雷码的编码效率, He等<sup>[18]</sup>提出了一种4元格雷编码方法, 该方法中的编码条纹包含4种不同灰度级的阶梯状码元,  $M$ 幅格雷编码条纹可最多标记 $M^4$ 个条纹级次。然而, 高频条纹或大视场测量通常需要编码大量的条纹级次, 这将压缩阶梯相移条纹中相邻条纹级次之间的量化间隔, 解码时容易产生错误的码元<sup>[19]</sup>。为了保证足够大的量化间隔, Zheng等<sup>[20]</sup>利用两组阶梯相移编码条纹对数量过多的条纹进行标记, 但同时增加了所要投影条纹的数量, 极大降低了相位编码法的效率<sup>[21]</sup>。因此, 兼备高效率和高鲁棒性优点的时间相位展开一直是条纹投影轮廓术的研究热点之一。

为了测量具有不连续面形的彩色复杂物体, 本文提出了一种基于正弦码元编码的时间相位展开方法。所提的正弦编码条纹可以突破现有的阶梯状编码条纹中量化间隔的限制且不需要进行取整运算, 通过宽松的比较运算即可逐点计算条纹级次, 故对表面的颜色和环境噪声不敏感, 具有更高的鲁棒性。对于 $N$ 步相移轮廓术,  $M$ 幅编码条纹可以标记 $N^M$ 个条纹级次, 其

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高编码效率非常适合需要大量条纹级次编码的三维测量。最后,通过一系列实验展示了所提方法的有效性和实用性。

## 2 原 理

### 2.1 *N* 步相移轮廓术

在相移轮廓术中,*N*步正弦相移条纹投影到物体表面形成相应的变形相移条纹,经相机采集后可以提取出携带物体高度信息的相位。通常,*N*步相移条纹可表示为

$$I_n(x, y) = A(x, y) + B(x, y) \cos [\varphi(x, y) + 2(n-1)\pi/N], \quad n = 1, 2, 3, 4, \quad (1)$$

式中:*A*(*x*, *y*)和*B*(*x*, *y*)分别为条纹的背景光强和调制强度; $\varphi(x, y)$ 为受到高度调制的相位,其计算公式为

$$\varphi(x, y) = \arctan \frac{\sum_{n=1}^N I_n(x, y) \sin [2(n-1)\pi/N]}{\sum_{n=1}^N I_n(x, y) \cos [2(n-1)\pi/N]}, \quad (2)$$

式(2)中的反正切运算会导致 $\varphi(x, y)$ 的范围被截断在 $(-\pi, \pi]$ 内,故 $\varphi(x, y)$ 通常被称为截断相位。为了获取连续的物体面形,需要采用相位展开算法消除不同条纹周期之间的 $2\pi$ 级次跳变,该过程可以统一地描述为

$$\phi(x, y) = \varphi(x, y) + 2\pi k(x, y), \quad (3)$$

式中: $\phi(x, y)$ 为连续分布的展开相位;*k*(*x*, *y*)为当前点所在的条纹级次。显然,相位展开的关键在于条纹级次的确定。

目前,空间相位展开主要依靠一个质量导图<sup>[22]</sup>来规划最优展开路径,用时沿着展开路径通过与前点的

差异决定当前点的条纹级次执行累加还是保持不变,故其展开相位是相对的。另外,一旦在质量降低的区域产生相位展开残余误差,该误差将传播到余下的所有展开点中。基于编码条纹的时间相位展开通过额外的*M*幅编码条纹去标记条纹级次,并辅助截断相位进行点对点的相位展开,这可以避免残余误差的传播。然而,针对大量条纹级次和被测表面复杂的反射率变化,传统编码条纹的量化间隔将被压缩,这容易导致解码时产生码元的识别错乱。虽然可以利用更多的编码条纹来增加量化间隔,但是这会降低编码的效率。为了提高编码效率并避免码元的压缩,本文提出利用一种多元正弦码元去实现条纹级次的编码。

### 2.2 所提多元编码和解码的原理

由于投影仪的分辨率为1140 pixel×912 pixel、条纹周期为32 pixel,故需要标记的条纹级次的数量为36。以4步相移条纹轮廓术为例,图1显示了所提的*N*元编码的原理,此时*N*=4,编码条纹的数量为*M*=3,故可标记64个条纹级次。由于条纹级次数量较多,故图1只显示了[12, 21]范围内条纹级次的编码。与传统的阶梯状码元不同,本文的码元直接从4步相移正弦条纹中提取,根据十进制-四进制的转换规律将4元码元依次嵌入到3幅编码条纹 $[C_1(x, y), C_2(x, y), C_3(x, y)]$ 的不同周期中。在编码时,码元的周期与截断相位的周期相等,且相邻码元的边界与截断相位的 $2\pi$ 跳变点需要依次重合,第*m*幅编码条纹可以描述为

$$C_m(x, y_{k-1}: y_k) = I_n(x, y_{k-1}: y_k), \quad k \leq 36, \quad (4)$$

式中:*y*为条纹相移方向;*y*<sub>*k*-1</sub>和*y*<sub>*k*</sub>为第*k*个编码周期*P*<sub>*k*</sub>的起始坐标和终止坐标。图2(a)~(c)为本文所提的4元正弦编码条纹、传统的4元格雷编码条纹和相位

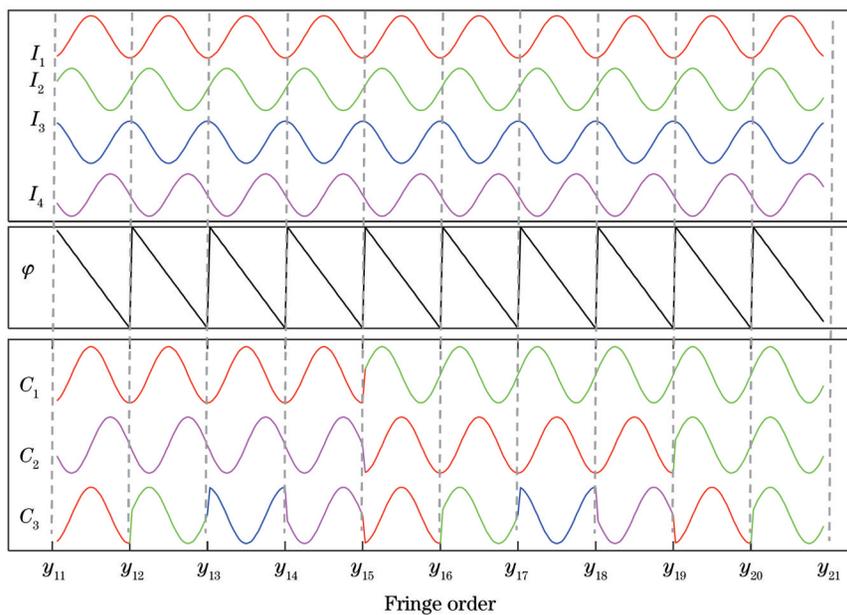


图1 4元编码原理

Fig. 1 Principle of quaternary encoding

编码条纹。在传统的阶梯状编码条纹中,量化所引起的截断性通常对物体表面的反射率比较敏感,尤其是物体整个表面反射率变化范围较大或条纹数量过多时,需要取整运算的传统编码条纹中容易出现解码误

差。不同于上述的两种编码条纹,本文所采用的  $N$  元正弦编码条纹源于相移正弦条纹,没有引入其他的灰度码元,属于一种自编码方法,故对物体的反射率具有较高的自适应性。

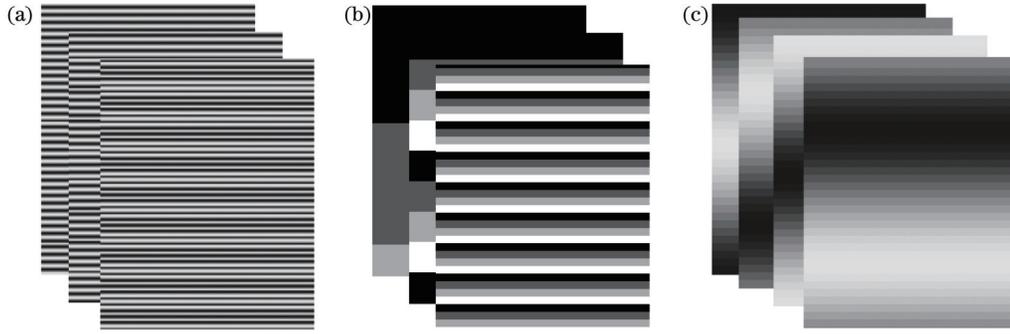


图 2 编码条纹。(a)提出的4元编码条纹;(b)4元格雷编码条纹;(c)相位编码条纹

Fig. 2 Coded patterns. (a) Proposed quaternary coded patterns; (b) quaternary Gray-coded patterns; (c) phase-coded patterns

在解码时,3幅正弦编码条纹  $[C_1(x, y), C_2(x, y), C_3(x, y)]$  需要转换成由元素 0、1、2、3 组成的量化条纹  $[Q_1(x, y), Q_2(x, y), Q_3(x, y)]$ 。从图 1 可知,通过比较当前编码条纹与 4 幅相移条纹之间的接近程度即可简单地确定对应的量化值,该过程可以描述为

$$Q_m(x, y) = \begin{cases} 0, & \text{if } D_{\min}(x, y) = d_{m1}(x, y) \\ 1, & \text{if } D_{\min}(x, y) = d_{m2}(x, y) \\ 2, & \text{if } D_{\min}(x, y) = d_{m3}(x, y) \\ 3, & \text{if } D_{\min}(x, y) = d_{m4}(x, y) \end{cases} \quad (5)$$

其中

$$D_{\min}(x, y) = \min [d_{m1}(x, y), d_{m2}(x, y), d_{m3}(x, y), d_{m4}(x, y)], \quad (6)$$

$$d_{mn}(x, y) = \text{abs} [C_m(x, y) - I_n(x, y)], \quad (7)$$

式中:  $\min(\bullet)$  为取最小值函数;  $\text{abs}(\bullet)$  为取绝对值函数;  $d_{mn}(x, y)$  为编码条纹  $C_m(x, y)$  与相移条纹  $I_n(x, y)$  之间的绝对差。图 3 为所提方法的解码过程,其中图 3 (a) 是根据式 (4) ~ (6) 计算的 3 幅量化条纹  $[Q_1(x, y), Q_2(x, y), Q_3(x, y)]$ 。在获取所有的量化条纹后,经四进制-十进制转换关系逐点计算条纹级次  $k(x, y)$ , 可表示为

$$k(x, y) = \sum_{m=1}^M Q_m(x, y) \times N^{M-m}. \quad (8)$$

图 3(b) 为解码后的条纹级次,所有周期内截断相位附加的整数倍  $2\pi$  都得到了精准的确定。最后,利用式(3)即可将  $\varphi(x, y)$  展开为绝对相位  $\phi(x, y)$ 。

### 2.3 相位高度关系

为了获取真实世界坐标系下的物体三维形貌,通常需要对测量系统进行标定,利用系统参数将从条纹中提取的相位差  $\Delta\phi(x, y)$  转换成物体的高度信息

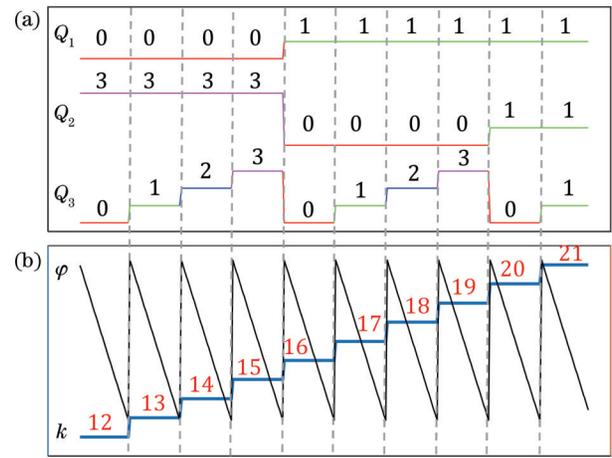


图 3 所提方法的解码过程。(a)量化条纹;(b)解码结果

Fig. 3 Decoding process of proposed method. (a) Quantized patterns; (b) decoding result

$h(x, y)$ 。传统的显式标定法<sup>[23-24]</sup>因涉及的标定参数较多,标定过程相对复杂,容易引入额外的误差。相比之下,目前被广泛使用的隐式标定法<sup>[25-27]</sup>不需要获取系统的内外参数,且将系统的畸变和非线性都纳入标定过程中,直接利用几个已知高度通过数据拟合出相位与高度之间的关系。本文采用 Li 等<sup>[26]</sup>提出的相位-高度映射模型,其中  $1/h(x, y)$  与  $1/\Delta\phi(x, y)$  之间的关系为

$$\frac{1}{h(x, y)} = \alpha(x, y) + \frac{\beta(x, y)}{\Delta\phi(x, y)} + \frac{\gamma(x, y)}{[\Delta\phi(x, y)]^2}, \quad (9)$$

式中:  $\alpha(x, y)$ 、 $\beta(x, y)$ 、 $\gamma(x, y)$  为三个拟合参数,可通过测量几个已知高度平面时,利用最小二乘法求解。此外,增加拟合的阶数可以进一步提高映射的精度。

## 3 实验

### 3.1 投影系统

图 4(a) 为一个典型的条纹投影系统,计算机生成

的条纹图案由数字光投影仪(DLP)投影后,经物体高度调制形成变形条纹,最后利用相机进行采集。本文采用的主要实验设备如图 4(b)所示:DLP的型号为 Light Crafter 4500,分辨率为 1140 pixel×912 pixel;分

辨率为 1236 pixel×1624 pixel 的单色相机 IMPERX GEV-B1610M-SC000 用于测量条纹数量过多的大视场场景;分辨率为 768 pixel×1024 pixel 的 RGB 相机用于测量反射率丰富的彩色物体。

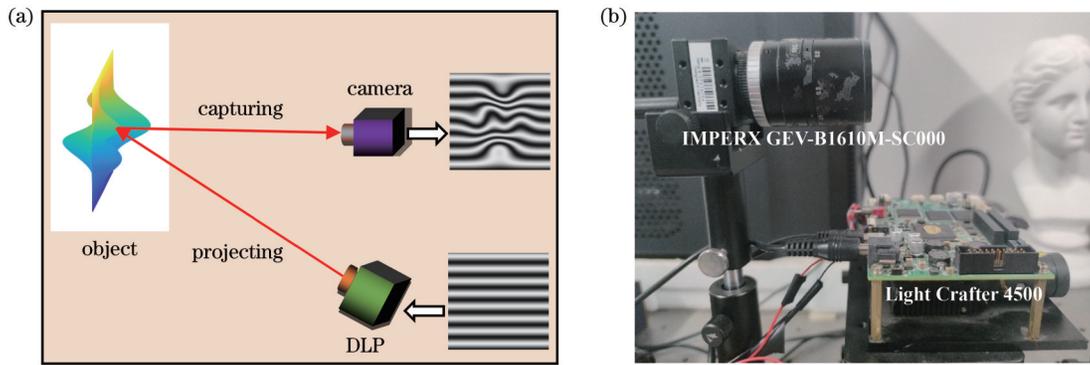


图 4 条纹投影系统。(a)投影示意图;(b)实验设备

Fig. 4 Fringe projection system. (a) Projecting diagram; (b) experimental devices

### 3.2 编码条纹量化和精度分析

首先,通过测量由不同高度平面的阶梯和一个彩色杯子组成的物体来验证所提方法的可行性,如图 5(a)所示。图 5(b)~(e)为用于计算截断相位  $\varphi(x, y)$  的 4 幅相移条纹  $[I_1^c(x, y), I_2^c(x, y), I_3^c(x, y),$

$I_4^c(x, y)]$ ,图 5(f)~(h)是 3 幅携带条纹级次线索的编码条纹  $[C_1^c(x, y), C_2^c(x, y), C_3^c(x, y)]$ 。如 2.2 节所述,计算条纹级次之前,需要对采集的 3 幅编码条纹进行逐一量化。理论上,编码越密,量化难度越高,故主要以量化最密集的第 3 幅编码条纹为例来体现所提解

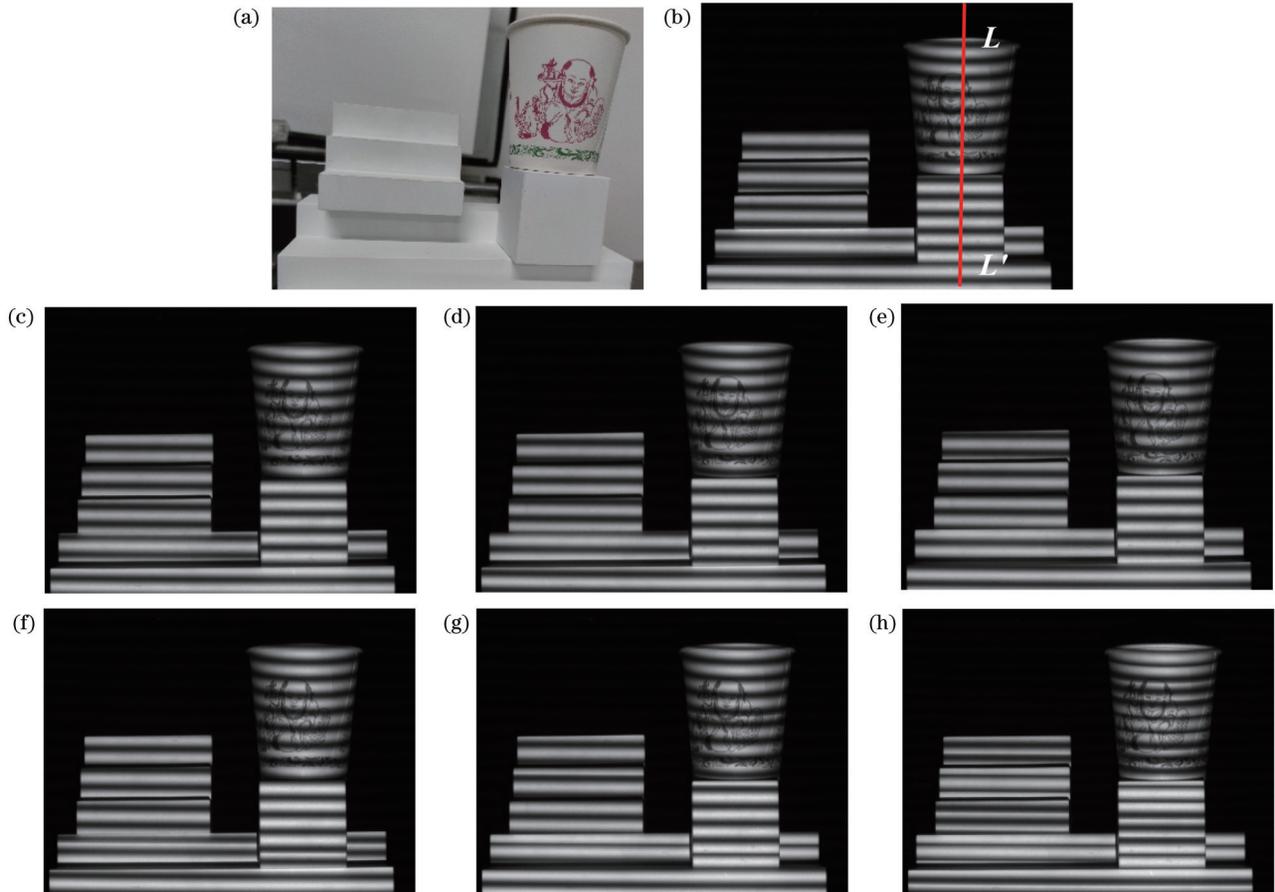


图 5 相机采集的图像。(a)待测物体;(b)~(e)四幅相移条纹;(f)~(h)所提方法得到的三幅编码条纹

Fig. 5 Images captured by camera. (a) Object to be measured; (b)~(e) four phase-shifting patterns; (f)~(h) three coded patterns obtained by proposed method

码方法的有效性,如图 6 所示。图 6(a)依次显示了第 3 幅编码条纹与 4 幅相移条纹的剖面对比图,其中剖面图的位置如图 5(b)中标记的  $LL'$  所示。从图 6(a)可以直观地看出,编码条纹  $C_3^c(x,y)$  与 4 幅相移条纹在不同周期内交替近似重合,即可通过直接比较条纹强度识别当前点编码的码元。在理想情况下,  $C_3^c(x,y)$  对应的量化条纹  $Q_3^{(0)}(x,y)$  如图 6(b)所示,  $Q_3^{(0)}(x,y)$  不仅精确地识别出了编码条纹与 4 幅相移条纹交点处的歧义,在码元边缘与截断相位的  $2\pi$  跳变点之间还没有

发生错位。然而,受到不可避免的图像数字化和投影系统噪声等因素的影响,编码条纹和其他相移条纹的交点处或码元边缘处的差异可能小于其与对应的相移条纹在该点的重合度。如图 6(c)所示,实际的 3 幅量化条纹  $[Q_1^c(x,y), Q_2^c(x,y), Q_3^c(x,y)]$  包含不同程度的刺状误差。值得注意的是,该类刺状误差通常只占单个像素,且相互之间距离足够大,故可直接利用中值滤波器进行消除。图 6(d)为滤波后的 3 幅量化条纹  $[Q_1^f(x,y), Q_2^f(x,y), Q_3^f(x,y)]$ , 本文所用滤波器的尺寸为  $3 \text{ pixel} \times 3 \text{ pixel}$ 。

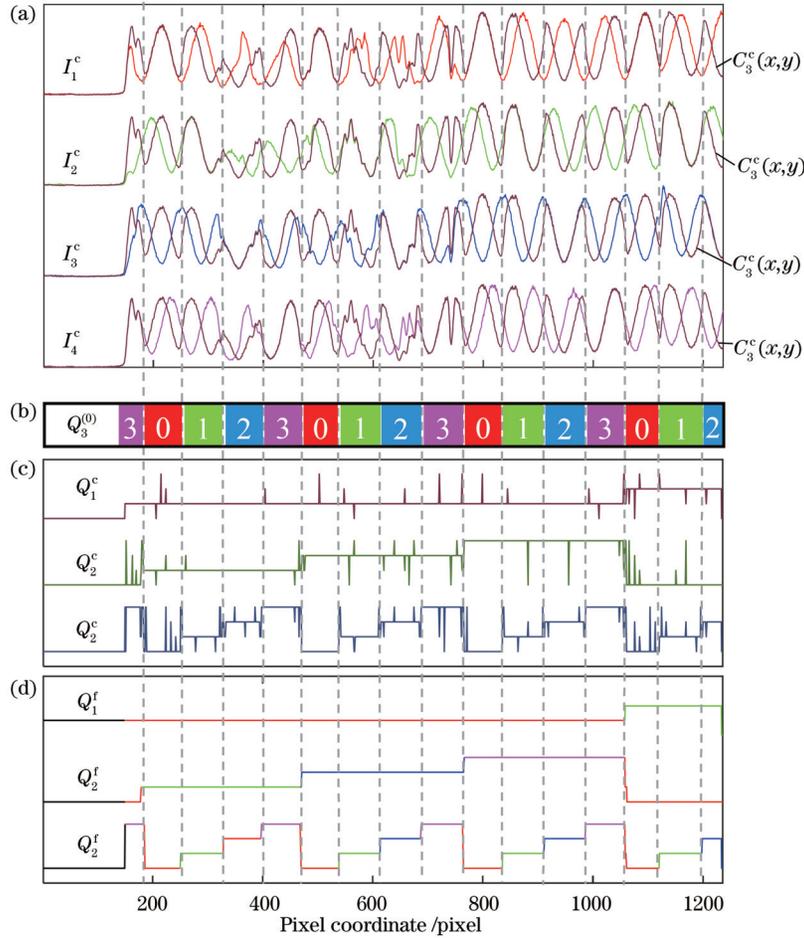


图 6 编码条纹的量化。(a)第 3 幅编码条纹与 4 幅相移条纹的剖面图;(b)第 3 幅编码条纹对应的理想量化条纹;(c)带有刺状误差的实际量化条纹;(d)滤波后的量化条纹

Fig. 6 Quantization of coded patterns. (a) Sectional view between 3rd coded pattern and 4 phase-shifting patterns; (b) ideal quantized patterns corresponding to 3rd coded pattern; (c) practical quantized patterns with spiny error; (d) quantized patterns after filtering

此外,受到投影系统散焦的影响,解码的条纹级次  $k(x,y)$  的边缘和截断相位  $\varphi(x,y)$  的  $2\pi$  跳变存在低于 3 pixel 的错位,如图 7(a)所示,其中右侧图为左侧图阴

影区域的局部放大图。由于边缘错位的间隙比条纹周期小太多,故可通过定位截断相位的  $2\pi$  跳变逐行进行自校正,该过程可表示为

$$k_c(x,y) = \begin{cases} k(x-r:x+r,y), & \text{if } \text{abs}[\varphi(x,y) - \varphi(x-1,y)] = 2\pi \\ k(x,y), & \text{others} \end{cases}, \quad (10)$$

式中:  $x$  为需要校正的相移方向;  $r$  为校正半径,本文令  $r = 5 \text{ pixel}$ 。从图 7(a)中校正前的条纹级次  $k(x,y)$

和校正后的条纹级次  $k_c(x,y)$  的对比可知,截断相位和条纹级次之间的错位被有效地消除了。在经过式

(3) 逐点获取绝对相位和式(9)所示的相位-高度映射后,得到的重构结果如图 7(b)所示。可以清晰地看

出,所提方法可以消除尖锐的阶梯跳变和阴影的影响,有效地重构物体的三维形貌。

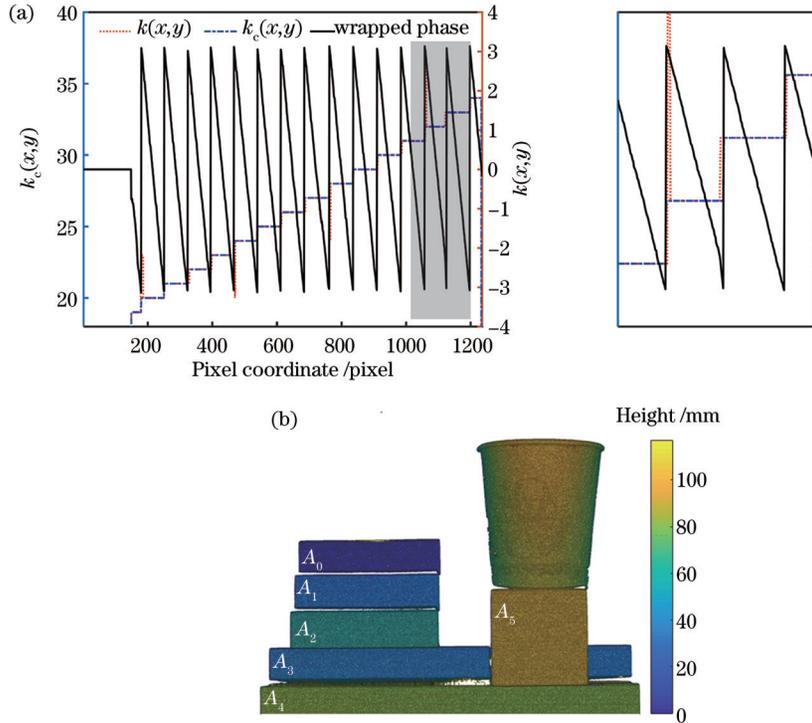


图 7 所提方法的测量结果。(a)校正前后的条纹级次;(b)重构结果

Fig. 7 Measuring result obtained by proposed method. (a) Fringe orders before and after correction; (b) reconstructed result

最后,通过对图 7(b)所示的 5 个重构平面  $A_1 \sim A_5$  进行定量分析来验证所提方法的测量精度,如表 1 所示,其中: $h_0$ 表示用常规直尺测量的标准平面高度;重构平面的平均高度  $h_a$ 与标准高度  $h_0$ 之间的最大绝对差  $\Delta h$ 用来体现整体的测量精度和局部出现的最大偏离程度。由表 1 可知,不同高度的重构平面与对应的标准平面在数值上都非常接近,说明所提的时间相位展开方法可以精确获取截断相位对应的条纹级次。

表 1 所提方法对平面  $A_1 \sim A_5$  的重构结果

Parameter	method				
	$A_1$	$A_2$	$A_3$	$A_4$	$A_5$
$h_0$	40.00	60.00	40.00	80.00	90.00
$h_a$	39.94	59.97	39.98	80.01	80.04
$\Delta h$	0.10	0.13	0.15	0.11	0.09

### 3.3 彩色复杂物体的测量

为了体现所提方法的实用性,用 RGB 相机替换单色相机后进一步测量了如图 8(a)~(c)所示的三组具有复杂面形的彩色物体。显然,这些待测物纹理各异,具有较大范围的表面反射率。图 8(d)~(f)分别为三个彩色场景对应的第 3 幅 RGB 正弦编码条纹,在将所有 RGB 条纹灰度化后,重复 3.2 节论述的步骤以提取截断相位和条纹级次,最后得到图 8(g)~(i)所示的三个重构结果。可以看出,条纹在不同颜色的物体表面

反射的强度存在较大差异,考虑式(4)是采用宽松的比较运算量化编码条纹的,故对反射率具有较高的自适应性。无论是图 8(a)中“叮当猫”脸部的高反光区域和图 8(b)、(c)中较暗的彩色块,还是各物体之间错综复杂的阴影,所提方法都可以保证其较高的鲁棒性进而实现高精度的三维重构。图 8(g)~(i)中重构的“叮当猫”的胡须、手关节起伏等细节证明了所提方法为复杂彩色物体的三维测量提供了一种有效的途径。

## 4 讨 论

为了进一步说明  $N$  元正弦编码的效率和稳定性,在同一时间采集了图 5(a)所示的物体对应的 4 元格雷编码条纹和相位编码条纹,如图 9(a)、(b)所示。在两种传统的编码方法中,一系列编码条纹被直接投影在待测物的表面上,对于级次信息嵌入强度的这类编码条纹而言,容易出现高阶编码条纹投影到低反射率表面后采集的变形条纹强度小于低阶编码条纹投影到高反射率表面后采集的条纹强度,从而造成解码错误。相位编码法将条纹级次信息嵌入到相位域中,与求解截断相位类似,可得到如图 9(c)所示的阶梯相位,最后通过取整运算获得相应的条纹级次。相位编码法对反射率的敏感度更低,故其比格雷编码法更适用于测量彩色物体。然而,随着条纹级次数量的增加,如需要高频条纹对大视场物体进行高精度重构时,相邻阶梯

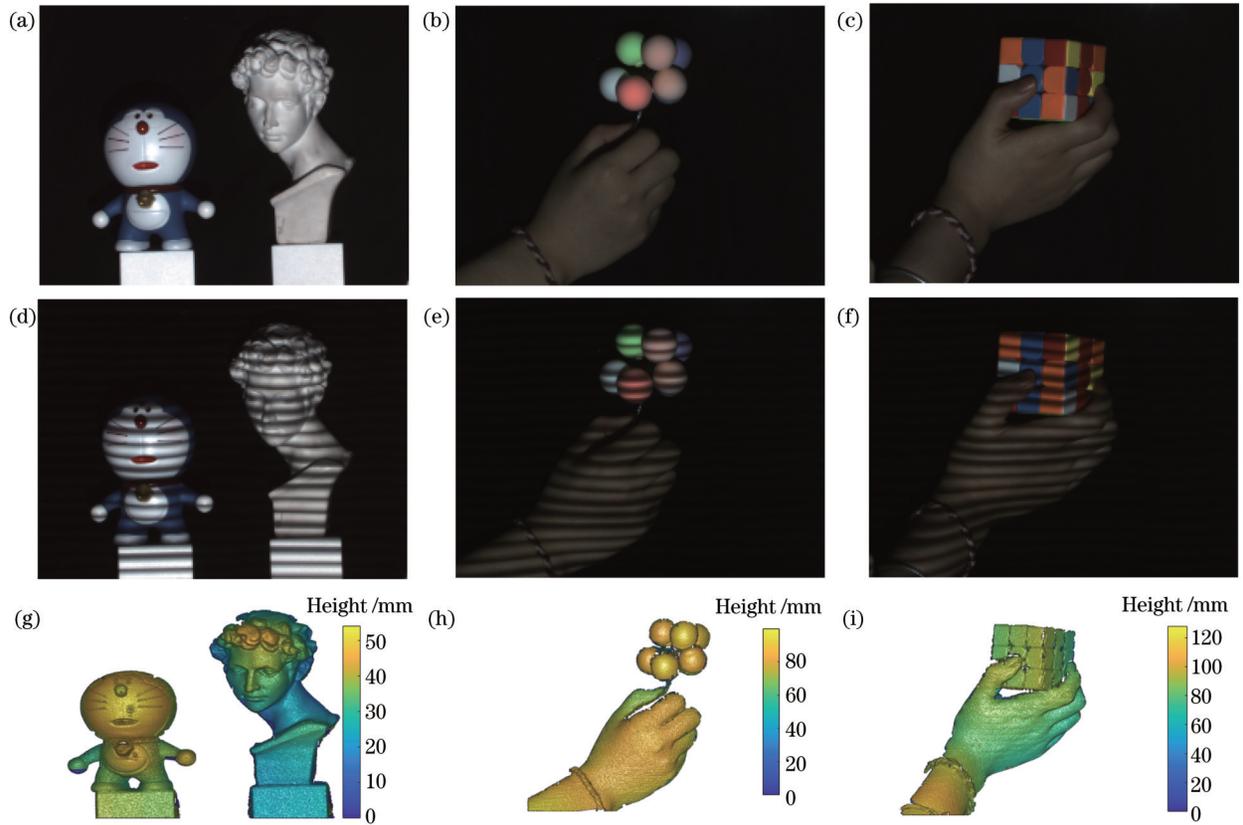


图 8 彩色物体的测量。(a)~(c)彩色待测场景;(d)~(f)第3幅编码条纹;(g)~(i)重构结果

Fig. 8 Colorful object measurement. (a)~(c) Colorful scenes to be measured; (d)~(f) 3rd coded patterns; (g)~(i) reconstructed results

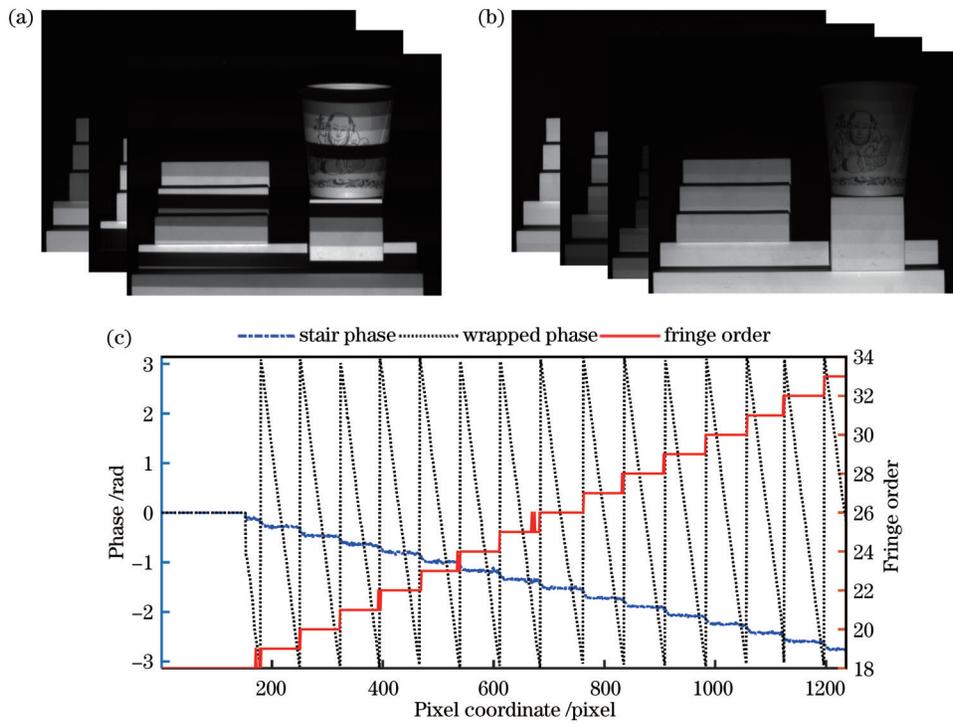


图 9 传统阶梯状编码条纹。(a) 4元格雷编码条纹;(b)相移编码条纹;(c)相移编码法计算的条纹级次

Fig. 9 Traditional stair-shaped coded patterns. (a) Quaternary Gray-coded patterns; (b) phase-coded patterns; (c) fringe orders calculated by phase-coded method

相位之间的边界逐渐模糊,甚至压缩至难以区分的程度。相反,所提正弦码元在反射率的响应上始终与某一幅相移条纹保持一致,只需要通过简单的比较运算即可识别当前点的条纹级次线索。此外,所提方法的编码效率与相移步数直接相关,对于需要高精度的多步相移轮廓术而言, $M$ 幅编码条纹可以标记的条纹级次总数 $K$ 为

$$K = N^M, \quad (11)$$

换句话说,当条纹级次较少时,在保持条纹数量不变的情况下,本文采用的4幅相移条纹+3幅编码条纹的模型(标记总数为64)可以转换成5幅相移条纹+2幅编码条纹的模型(标记总数为25)。因此,所提的 $N$ 元编码时间相位展开算法可以通过选择相移条纹和编码条纹的数量来平衡测量的视场和精度需求。

## 5 结 论

系统地介绍了一种基于 $N$ 元正弦码元的时间相位展开算法,该方法具有高编码效率和高鲁棒性等优势,为大数量条纹的编码提供了一种灵活有效的途径。相比直接投影阶梯状量化条纹,所提正弦编码条纹通过相对宽松的比较运算逐点量化条纹级次,对表面的反射率具有较高的自适应性,适用于彩色复杂物体的三维重构。此外,提出了一种条纹级次校正方法来消除截断相位的 $2\pi$ 跳变和条纹级次边缘之间的相位误差。最后,通过重构的物体细节证明了所提方法适用于复杂场景的三维测量。

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## Temporal Phase Unwrapping Based on $N$ -Ary Sinusoidal Codewords

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### Abstract

**Objective** Phase-shifting profilometry (PSP) has been widely used in various three-dimensional (3D) scenes due to its high accuracy and robustness. In fringe projection profilometry (FPP), unwrapping the phase map in  $(-\pi, \pi]$  is an inevitable consideration. Phase unwrapping algorithms are commonly divided into two types: spatial phase unwrapping algorithms (SPUAs) and temporal phase unwrapping algorithms (TPUAs). Conventionally, TPUAs are more suitable for measuring discontinuous objects as they can identify fringe orders pixel by pixel. TPUAs are also employed in real-time 3D measurement because of the development of hardware and defocusing systems. Numerous TPUAs have developed at a fast pace in the past few decades, mainly including multi-frequency, intensity-code, and phase-code ones. Multi-frequency methods suffer the low accuracy of the low-frequency patterns and the complicated selection among different frequencies. Intensity-code methods, mainly  $N$ -ary Gray codes with concision and high efficiency, directly use intensity information to generate the fringe order map, but they can barely measure colorful objects. In phase-code methods, massive codewords are coded into the phase domain with a depth of only  $2\pi$ , and the difference between adjacent quantized phases may be too small to ensure the correct decoded codewords for a large number of fringe orders. Essentially, the proposed sinusoidal codewords are directly extracted from  $N$ -step phase-shifting patterns to replace additional stair-shaped codewords in intensity-code and phase-code methods. By contrast, the proposed fringe-order encoding method based on  $N$ -ary sinusoidal codewords performs outstanding coding flexibility and efficiency while breaking through the limitations of the number of fringe orders in phase-code methods and overcoming the sensitivity to reflectivity in Gray-code methods.

**Methods** A temporal phase unwrapping method based on  $N$ -ary coding is proposed to realize the 3D measurement of colorfully complex objects. When measuring an object with a large range of overall surface reflectivity, traditional stair-shaped patterns face the difficulty of quantization of an excessive number of fringe orders. During encoding,  $N$ -ary sinusoidal codewords are successively extracted from sinusoidal phase-shifting patterns to replace traditional quantized gray codewords. It is worth noting that the edges of the extracted  $N$ -ary codewords coincide with the  $2\pi$  discontinuities of wrapped phases to reduce the mismatch. By numeral system conversion,  $N$ -ary sinusoidal codewords are embedded into different periods of projected patterns to achieve the encoding of fringe orders. During decoding, the differences between coded patterns and  $N$ -step sinusoidal phase-shifting patterns can be first used to calculate  $N$ -ary quantized patterns by a loose operation [Eq. (5)], and then a unique fringe order can be obtained by reverse numeral system conversion. To remove the mismatches caused by the defocusing and noise of the system, a fringe-order self-correction method [Eq. (10)] is used to correct the jump errors around the  $2\pi$  discontinuities of wrapped phases. Finally, the absolute phase can be obtained by the collation of the corrected fringe orders and wrapped phases. In this paper, an object [Fig. 5 (a)] with several planes of known height is measured to verify the feasibility of the proposed method, and some colorfully complex scenes [Figs. 8 (a)–(c)] in our daily life are further measured to demonstrate its high performance.

**Results and Discussions** For  $N$ -step PSP,  $M$ -coded (in this paper,  $M=3$ ) patterns can mark  $N^M$  fringe orders, which can effectively improve the number of fringe orders. In this paper, four conventional sinusoidal phase-shifting patterns provide a high-quality wrapped phase, and three quasi-sinusoidal coded patterns can be used to extract the corresponding fringe order map independently. The results (Table 1) of several planes with known heights show that the reconstructed planes with different heights are very close to the corresponding standard planes, which verifies that the proposed temporal phase unwrapping method can accurately obtain the corresponding fringe orders of wrapped phases. To obtain fringe orders, both Gray-code and phase-code methods require rounding operation, which generates round-off errors. Overall, the existing stair-shaped codes have high sensitivity and great difficulty in determining numerous fringe orders due to complex shapes or

texture variations. Facing complex surfaces with a large range of reflectivity, the proposed  $N$ -ary sinusoidal codewords are considerably self-adaptable to non-uniform reflectivity due to the loose quantification processing. The analysis of theory and experimental results indicates that the proposed coding method has higher coding efficiency and robustness than intensity-code methods, and is more flexible and concise than phase-code methods while breaking the limitation of the total number of fringe orders to recover the absolute phase more accurately. The validity and superiority of the proposed method are confirmed by experiments.

**Conclusions** A temporal phase unwrapping method based on  $N$ -ary coding is proposed. By extracting the codewords from phase-shifting patterns to replace additional stair-shaped codewords in the existing TPUAs, the proposed method makes encoding and decoding more flexible and efficient. The experimental results demonstrate that the proposed self-coding method features high robustness for measuring sharply discontinuous and colorful objects in practice.

**Key words** measurement; fringe projection; temporal phase unwrapping; fringe order;  $N$ -ary coding