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平均场框架下两分量玻色-爱因斯坦凝聚中集体激发的阻尼

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摘要 研究两分量玻色-爱因斯坦凝聚中集体激发的阻尼，该阻尼包括朗道和巴利耶夫两种机制。采用哈特利-福特-波戈留波夫平均场理论，基于单分量系统理论，构建了两分量系统的理论框架，严格按照单分量系统理论推导公式，并且分析两分量系统和单分量系统在理论框架构建上的主要差别。以理想的能级连续的均匀系统朗道阻尼为例进行半经典近似计算，展示粒子相互作用细节，说明所提理论的物理意义和应用方法。在计算过程中，通过无量纲阻尼函数分析朗道阻尼与温度的依赖关系，分析对阻尼有贡献的准粒子跃迁，包括同类型准粒子之间和不同类型准粒子之间的各种情况，并通过误差函数分析不同能量范围的准粒子跃迁对阻尼的贡献。

关键词 量子光学；元激发；无量纲阻尼函数；误差函数；波戈留波夫变换；傅里叶变换

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1 引 言

玻色-爱因斯坦凝聚（BEC）中的元激发属于统计物理学和凝聚态物理学的基础性课题[1-11]，随着超冷原子气体中费希巴赫共振（Feshbach resonance）技术的发展，利用高度可控的超冷量子系统的相关研究取得显著进展[12-32]。

集体激发振幅的衰减称为阻尼，产生于粒子间相互作用。阻尼是BEC实验中低能集体激发的重要特征[33-37]，阻尼的精确计算对于理解量子多体物理本质十分重要。文献[33-36]研究了磁囚禁中的轴对称BEC系统，文献[37]研究了箱囚禁中的均匀BEC系统。在高温和高密度、粒子发生碰撞的情况下，阻尼机制为耗散型，可用二流体理论描述[38-41]；在低温和低密度、粒子碰撞起次要作用的情况下，阻尼与热化过程无关而与激发之间的耦合有关，一般用平均场理论描述[42-60]。

基于平均场理论的研究工作相对较多，但方法也各有不同，有微扰方法[42-45]、格林函数方法[46-47]、二阶量子场理论方法[48]和哈特利-福特-波戈留波夫（HFB）近似方法[49-60]，其中HFB近似方法的使用最广泛，而文献[49-50]最先构建HFB平均场理论框架。

文献[44-51]研究了均匀BEC系统，其能级是连续的，各个准粒子跃迁对集体激发阻尼的贡献通过积分来计算，这种方法称为半经典近似方法；文献[42-43，52-60]研究了轴对称囚禁BEC系统，其能级是分立的，各个准粒子跃迁对集体激发阻尼的贡献通过求和进行计算。以能级连续的均匀BEC系统为例的计算工作，是研究集体激发阻尼的基础工作，通过半经典近似计算介绍理论的物理意义和应用方法，在此基础上，进一步研究轴对称囚禁BEC系统，这是因为轴对称囚禁BEC系统远比均匀BEC系统复杂、研究难度大。

值得一提的是，在研究能级分立的轴对称囚禁BEC系统的工作中，除文献[42-43]的工作外，其余工作都由本团队完成[52-60]。文献[52-54]采用文献[42]引入的洛伦兹宽度得到计算阻尼的方法；文献[55-56]采用文献[43]的迭代计算方法。在此基础上，文献[57-60]对文献[42-43]的方法进行了发展和改进：在集体激发摄动本征频率关系中，考虑元激发弛豫及其正交关系，得到计算集体激发阻尼包括频移（频率的改变量）的迭代公式，计算结果[57-59]分别与相关实验结果[35-36]一致。

以上介绍的是单分量BEC（1BEC）中集体激发阻尼的工作。两分量BEC（2BECs）是BEC研究的一个重要方向，很多学者研究了2BECs元激发的理论工

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作[61-72],但由于问题的复杂性较高,关于2BECs中集体激发阻尼的理论研究工作还很少,目前只有文献[73-74]进行了初步探索。文献[73]研究的2BECs系统由2种不同精细结构的同种原子组成,而文献[74]研究的2BECs系统由两种不同种原子组成。文献[73-74]分别构建了各自研究系统的HFB平均场理论框架,但都缺少以均匀系统为例的半经典近似计算,没有对其理论的物理意义和应用方法加以说明。

本文基于HFB平均场理论研究2BECs系统中集体激发的阻尼。首先,构建2BECs中集体激发阻尼的HFB平均场理论框架;然后,给出均匀系统的半经典近似计算方法,对理论的物理意义和应用方法加以说明,并讨论2BECs系统和1BEC系统在HFB平均场理论构建上的主要差别,分析文献[73-74]在HFB平均场理论构建中存在的问题。

2 2BECs中集体激发阻尼的HFB平均场理论

本节给出2BECs中集体激发阻尼的HFB平均场理论,其构建方法与文献[49-50]的1BEC中集体激发阻尼HFB平均场理论相同,表述参照文献[49-50]自然清晰的方式,但比它们更为详细。

2.1 HFB平均场理论近似和波戈留波夫变换与巨正则哈密顿和运动方程

由2种原子(分别标记为1和2)组成的2BECs系统的巨正则哈密顿为

$$K=H-\mu_1N_1-\mu_2N_2=\int\psi_1^\dagger(\boldsymbol{r},t)H_1\psi_1(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r}+\int\psi_2^\dagger(\boldsymbol{r},t)H_2\psi_2(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r}+\frac{g_1}{2}\int\psi_1^\dagger(\boldsymbol{r},t)\psi_1^\dagger(\boldsymbol{r},t)\psi_1(\boldsymbol{r},t)\psi_1(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r}+\frac{g_2}{2}\int\psi_2^\dagger(\boldsymbol{r},t)\psi_2^\dagger(\boldsymbol{r},t)\psi_2(\boldsymbol{r},t)\psi_2(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r}+g_{12}\int\psi_1^\dagger(\boldsymbol{r},t)\psi_2^\dagger(\boldsymbol{r},t)\psi_1(\boldsymbol{r},t)\psi_2(\boldsymbol{r},t)\mathrm{d}\boldsymbol{r},\tag{1}$$

式中:$\boldsymbol{r}$为位矢;t为时间;H为系统的哈密顿;$\mu_{1(2)}$为单粒子化学势;$H_{1(2)}$为第1(2)种粒子的单粒子哈密顿;$\psi_{1(2)}^\dagger$和$\psi_{1(2)}$分别为第1(2)种粒子的产生和湮灭玻色场算符,满足对易关系$\left[\psi_{1(2)}(\boldsymbol{r},t),\psi_{1(2)}^\dagger(\boldsymbol{r}',t)\right]=\delta(\boldsymbol{r}-\boldsymbol{r}')$、$\left[\psi_{1(2)},\psi_{1(2)}\right]=0$、$\left[\psi_1,\psi_2\right]=0$、$\left[\psi_1^\dagger,\psi_2\right]=0$,其中$\int\psi_{1(2)}^\dagger\psi_{1(2)}\mathrm{d}\boldsymbol{r}=N_{1(2)}$,而$N_{1(2)}$为第1(2)种粒子的数量,$\delta(\boldsymbol{r}-\boldsymbol{r}')$为狄拉克函数;$g_{1(2)}$为第1(2)种粒子间相互作用的耦合常数;$g_{12}$为两种粒子间相互作用的耦合常数。

$$H_{1(2)}=-\frac{\hbar^2\nabla^2}{2m_{1(2)}}+V_{1(2)}-\mu_{1(2)},\tag{2}$$

式中:$\hbar$、$m_{1(2)}$、$V_{1(2)}$分别为约化普朗克常数、第1(2)种粒子的质量、粒子在外场中的势能。

场算符$\psi_{1(2)}(\boldsymbol{r},t)$的运动方程可写为

$$\mathrm{i}\hbar\frac{\partial\psi_{1(2)}}{\partial t}=\left[\psi_{1(2)},K\right]=H_{1(2)0}\psi_{1(2)}+g_{1(2)}\psi_{1(2)}^\dagger\psi_{1(2)}\psi_{1(2)}+g_{12}\psi_{1(2)}\psi_{2(1)}^\dagger\psi_{2(1)},\tag{3}$$

式中:i为虚数单位。

场算符分为凝聚和非凝聚部分,即凝聚波函数$\varPhi(\boldsymbol{r},t)=\left\langle\psi(\boldsymbol{r},t)\right\rangle$和非凝聚算符$\tilde{\psi}(\boldsymbol{r},t)$,表示为

$$\psi_{1(2)}=\varPhi_{1(2)}+\tilde{\psi}_{1(2)},\tag{4}$$

$$\left\langle\tilde{\psi}(\boldsymbol{r},t)\right\rangle=0,\tag{5}$$

式中:$\langle\cdot\rangle$表示非平衡平均,与时间有关。后文中的$\langle\cdot\rangle_0$表示平衡平均,与时间无关。

将式(4)代入式(3),式(3)中场算符$\psi_{1(2)}$的立方项变为$\psi_{1(2)}^\dagger\psi_{1(2)}\psi_{1(2)}=\left|\varPhi_{1(2)}\right|^2\varPhi_{1(2)}+2\left|\varPhi_{1(2)}\right|^2\tilde{\psi}_{1(2)}+\varPhi_{1(2)}^2\tilde{\psi}_{1(2)}^\dagger+\varPhi_{1(2)}\tilde{\psi}_{1(2)}^\dagger\tilde{\psi}_{1(2)}+\varPhi_{1(2)}^*\tilde{\psi}_{1(2)}\tilde{\psi}_{1(2)}+\tilde{\psi}_{1(2)}^\dagger\tilde{\psi}_{1(2)}\tilde{\psi}_{1(2)}$,$\psi_{1(2)}^\dagger\psi_{2(1)}\psi_{2(1)}=\varPhi_{1(2)}^*\varPhi_{2(1)}^2+2\varPhi_{1(2)}^*\varPhi_{2(1)}\tilde{\psi}_{2(1)}+\varPhi_{2(1)}^2\tilde{\psi}_{1(2)}^\dagger+\varPhi_{1(2)}^*\tilde{\psi}_{2(1)}\tilde{\psi}_{2(1)}+\varPhi_{2(1)}\tilde{\psi}_{1(2)}^\dagger\tilde{\psi}_{2(1)}+\tilde{\psi}_{1(2)}^\dagger\tilde{\psi}_{2(1)}\tilde{\psi}_{2(1)}$,其中包含非凝聚算符$\tilde{\psi}_{1(2)}$的立方项,取近似为

$$\left\langle\tilde{\psi}_{1(2)}^\dagger\tilde{\psi}_{2(1)}\tilde{\psi}_{2(1)}\right\rangle=0。\tag{6}$$

这样,将式(4)代入式(3)后再取非平衡平均,得到凝聚波函数的运动方程:

$$\mathrm{i}\hbar\frac{\partial\varPhi_{1(2)}}{\partial t}=H_{1(2)0}\varPhi_{1(2)}+g_{1(2)}\left|\varPhi_{1(2)}\right|^2\varPhi_{1(2)}+g_{12}\varPhi_{1(2)}\left|\varPhi_{2(1)}\right|^2+2g_{1(2)}\varPhi_{1(2)}\tilde{n}_{1(2)}+g_{1(2)}\varPhi_{1(2)}^*\tilde{m}_{1(2)}+g_{12}\varPhi_{1(2)}\tilde{n}_{2(1)}+g_{12}\varPhi_{2(1)}^*\tilde{m}_{12(21)}+g_{12}\varPhi_{2(1)}\tilde{n}_{21(12)}。\tag{7}$$

上述推导过程中,除了用到式(5)和式(6),对于非凝聚部分,定义了正常准粒子密度和反常准粒子密度:

$$\begin{cases}\tilde{n}_{1(2)}(\boldsymbol{r},t)=\left\langle\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}_{1(2)}\right\rangle\\ \tilde{n}_{12(21)}(\boldsymbol{r},t)=\left\langle\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}_{2(1)}\right\rangle\\ \tilde{m}_{1(2)}(\boldsymbol{r},t)=\left\langle\tilde{\psi}_{1(2)}\tilde{\psi}_{1(2)}\right\rangle\\ \tilde{m}_{12}(\boldsymbol{r},t)=\tilde{m}_{21}(\boldsymbol{r},t)=\left\langle\tilde{\psi}_{1}\tilde{\psi}_{2}\right\rangle\end{cases}。\tag{8}$$

凝聚部分又分为静态和激发部分，即基态 $\Phi_{1(2)0}=\left\langle\Phi_{1(2)}\right\rangle_0$ 和集体激发 $\delta\Phi_{1(2)}$。

$$\Phi_1(\boldsymbol{r},t)=\Phi_{10}(\boldsymbol{r})+\delta\Phi_1(\boldsymbol{r},t),\tag{9}$$

$$\Phi_2(\boldsymbol{r},t)=\Phi_{20}(\boldsymbol{r})+\delta\Phi_2(\boldsymbol{r},t),\tag{10}$$

式中：$\Phi_{1(2)0}$ 为一个实数函数；$\delta\Phi_{1(2)}$ 为微小变量。因此，凝聚静态部分的粒子数密度 $n_{1(2)0}=\left|\Phi_{1(2)0}\right|^2=\Phi^2_{1(2)0}=\Phi^{*2}_{1(2)0}$。

集体激发产生于外驱，其本征频率与外驱频率 ω_0 相同。在能级分立的实际轴对称囚禁 2BECs 系统中，两个分量中具有相同本征频率的集体激发属于特殊情况，外界驱动一般一次只能集体激发一个分量。为不失一般性，设集体激发为 $\delta\Phi_1\neq0$、$\delta\Phi_2=0$。

定义正常和反常准粒子静态密度 $\tilde{n}^{(0)}_{1(2)}=\left\langle\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}_{1(2)}\right\rangle_0$、$\tilde{n}^{(0)}_{12(21)}=\left\langle\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}_{2(1)}\right\rangle_0$ 和 $\tilde{m}^{(0)}_{1(2)}=\left\langle\tilde{\psi}_{1(2)}\tilde{\psi}_{1(2)}\right\rangle_0$、$\tilde{m}^{(0)}_{12}=\tilde{m}^{(0)}_{21}=\left\langle\tilde{\psi}_1\tilde{\psi}_2\right\rangle_0$。把正常和反常准粒子密度也分为静态和激发部分，表示为

$$\begin{cases}\tilde{n}_{1(2)}(\boldsymbol{r},t)=\tilde{n}^{(0)}_{1(2)}(\boldsymbol{r})+\delta\tilde{n}_{1(2)}(\boldsymbol{r},t)\\ \tilde{n}_{12(21)}(\boldsymbol{r},t)=\tilde{n}^{(0)}_{12(21)}(\boldsymbol{r})+\delta\tilde{n}_{12(21)}(\boldsymbol{r},t)\\ \tilde{m}_{1(2)}(\boldsymbol{r},t)=\tilde{m}^{(0)}_{1(2)}(\boldsymbol{r})+\delta\tilde{m}_{1(2)}(\boldsymbol{r},t)\\ \tilde{m}_{12}(\boldsymbol{r},t)=\tilde{m}_{21}(\boldsymbol{r},t)=\tilde{m}^{(0)}_{12}(\boldsymbol{r})+\delta\tilde{m}_{12}(\boldsymbol{r},t)\end{cases}。\tag{11}$$

将式(9)和式(11)代入式(7)，可得到基态的格罗斯-皮塔耶夫斯基(GP)方程：

$$H_{1(2)0}\Phi_{1(2)0}+g_{1(2)}n_{1(2)0}\Phi_{1(2)0}+g_{12}n_{2(1)0}\Phi_{1(2)0}+2g_{1(2)}\Phi_{1(2)0}\tilde{n}^{(0)}_{1(2)}+g_{1(2)}\Phi_{1(2)0}\tilde{m}^{(0)}_{1(2)}+g_{12}\Phi_{1(2)0}\tilde{n}^{(0)}_{2(1)}+g_{12}\Phi_{2(1)0}\tilde{m}^{(0)}_{12}+g_{12}\Phi_{2(1)0}\tilde{n}^{(0)}_{21(12)}=0。\tag{12}$$

集体激发的运动方程为

$$\mathrm{i}\hbar\frac{\partial\delta\Phi_1}{\partial t}=L_0\delta\Phi_1+\left(g_1n_{10}+g_1\tilde{m}^{(0)}_1\right)\delta\Phi^*_1+R_1,\tag{13}$$

$$\mathcal{L}_0=H_{10}+2g_1n_{10}+g_{12}n_{20}+2g_1\tilde{n}^{(0)}_1+g_{12}\tilde{n}^{(0)}_2,\tag{14}$$

$$R_1=2g_1\Phi_{10}\delta\tilde{n}_1+g_1\Phi_{10}\delta\tilde{m}_1+g_{12}\Phi_{10}\delta\tilde{n}_2+g_{12}\Phi_{20}\delta\tilde{m}_{12}+g_{12}\Phi_{20}\delta\tilde{n}_{21}。\tag{15}$$

采用波戈留波夫变换

$$\begin{cases}\tilde{\psi}_{1(2)}(\boldsymbol{r},t)=\sum_j\left[u_{1(2)j}(\boldsymbol{r})\alpha_{1(2)j}(t)+v^*_{1(2)j}(\boldsymbol{r})\alpha^{\dagger}_{1(2)j}(t)\right]\\ \tilde{\psi}^{\dagger}_{1(2)}(\boldsymbol{r},t)=\sum_j\left[u^*_{1(2)j}(\boldsymbol{r})\alpha^{\dagger}_{1(2)j}(t)+v_{1(2)j}(\boldsymbol{r})\alpha_{1(2)j}(t)\right]\end{cases},\tag{16}$$

式中：准粒子算符 $\alpha_{1(2)}$、$\alpha^{\dagger}_{1(2)}$ 满足 $\left[\alpha_{1(2)i},\alpha^{\dagger}_{1(2)j}\right]=\delta_{ij}$、$\left[\alpha_{1(2)i},\alpha_{1(2)j}\right]=0$、$\left[\alpha_{1(2)i},\alpha^{\dagger}_{2(1)j}\right]=0$、$\left[\alpha_{1(2)i},\alpha_{2(1)j}\right]=0$，其中 δ 为克罗内克符号，当 $i=j$ 时，$\delta_{ij}=1$，而当 $i\neq j$ 时，$\delta_{ij}=0$；准粒子振幅 $u_{1(2)}$、$u^*_{1(2)}$、$v_{1(2)}$、$v^*_{1(2)}$ 满足 $\int\left[u^*_{1(2)i}u_{1(2)j}-v^*_{1(2)i}v_{1(2)j}\right]\mathrm{d}\boldsymbol{r}=\delta_{ij}$、$\int\left[u^*_{1(2)i}v_{1(2)j}-v^*_{1(2)i}u_{1(2)j}\right]\mathrm{d}\boldsymbol{r}=0$。

将式(16)代入式(8)，并根据式(11)得到：

$$\begin{cases}\tilde{n}^{(0)}_{1(2)}=\sum_i\left|v_{1(2)i}\right|^2+\sum_i f^{(0)}_{1(2)i}\left(\left|u_{1(2)i}\right|^2+\left|v_{1(2)i}\right|^2\right),\tilde{n}^{(0)}_{12(21)}=0\\ \tilde{m}^{(0)}_{1(2)}=\sum_i u_{1(2)i}v^*_{1(2)i}\left(1+2f^{(0)}_{1(2)i}\right),\tilde{m}^{(0)}_{12}=\tilde{m}^{(0)}_{21}=0\end{cases},\tag{17}$$

$$\begin{cases}\delta\tilde{n}_{1(2)}=\sum_{ij}\left[\left(u^*_{1(2)i}u_{1(2)j}+v^*_{1(2)i}v_{1j}\right)f_{1(2)ij}+u_{1(2)i}v_{1(2)j}g_{1(2)ij}+u^*_{1(2)i}v^*_{1(2)j}g^*_{1(2)ij}\right]\\ \delta\tilde{n}_{12(21)}=\sum_{ij}\left(u^*_{1(2)i}u_{2(1)j}f_{12(21)ij}+v_{1(2)i}v^*_{2(1)j}f_{21(12)ji}+v_{1(2)i}u_{2(1)j}g_{12(21)ij}+u^*_{1(2)i}v^*_{2(1)j}g^*_{12(21)ij}\right)\\ \delta\tilde{m}_{1(2)}=\sum_{ij}\left(2u_{1(2)j}v^*_{1(2)i}f_{1(2)ij}+u_{1(2)i}u_{1(2)j}g_{1(2)ij}+v^*_{1(2)i}v^*_{1(2)j}g^*_{1(2)ij}\right)\\ \delta\tilde{m}_{12}=\delta\tilde{m}_{21}=\sum_{ij}\left(u_{1i}v^*_{2j}f_{21ji}+v^*_{1i}u_{2j}f_{12ij}+u_{1i}u_{2j}g_{12ij}+v^*_{1i}v^*_{2j}g^*_{12ij}\right)\end{cases},\tag{18}$$

式中：$f_{1(2)ij}$、$f_{12(21)ij}$ 为正常准粒子分布函数；$g_{1(2)ij}g_{12(21)ij}$ 为反常准粒子分布函数。

$$\begin{cases} f_{1(2)ij}(t)=\left\langle \alpha^{\dagger}_{1(2)i}(t)\alpha_{1(2)j}(t)\right\rangle - f^{0}_{1(2)i}\delta_{ij} \\ f_{12(21)ij}(t)=\left\langle \alpha^{\dagger}_{1(2)i}(t)\alpha_{2(1)j}(t)\right\rangle \\ g_{1(2)ij}(t)=\left\langle \alpha_{1(2)i}(t)\alpha_{1(2)j}(t)\right\rangle \\ g_{12ij}(t)=g_{21ji}(t)=\left\langle \alpha_{1i}(t)\alpha_{2j}(t)\right\rangle \end{cases}, \tag{19}$$

式中：$f^{(0)}_{1(2)i}=\left\langle \alpha^{\dagger}_{1(2)i}(t),\alpha_{1(2)i}(t)\right\rangle_{0}=\dfrac{1}{\exp\left[\dfrac{\varepsilon_{1(2)i}}{k_{\mathrm{B}}T}\right]-1}$为准粒子平衡密度，其中$\varepsilon_{1(2)i}$、$k_{\mathrm{B}}$、$T$分别为准粒子能量、玻尔兹曼常量和温度。

根据式(17)，将式(12)简化为

$$H_{1(2)0}\Phi_{1(2)0}+g_{1(2)}n_{1(2)0}\Phi_{1(2)0}+g_{12}n_{2(1)0}\Phi_{1(2)0}+ 2g_{1(2)}\Phi_{1(2)0}\tilde{n}^{(0)}_{1(2)}+g_{1(2)}\Phi_{1(2)0}\tilde{m}^{0}_{1(2)}+g_{12}\Phi_{1(2)0}\tilde{n}^{(0)}_{2(1)}=0。\tag{20}$$

根据式(18)，将式(15)改写为

$$R_1=2g_1\Phi_{10}\sum_{ij}\left[\left(u^{*}_{1i}u_{1j}+v^{*}_{1i}v_{1j}\right)f_{1ij}+u_{1i}v_{1j}g_{1ij}+u^{*}_{1i}v^{*}_{1j}g^{*}_{1ij}\right]+ g_1\Phi_{10}\sum_{ij}\left(2v^{*}_{1i}u_{1j}f_{1ij}+u_{1i}u_{1j}g_{1ij}+v^{*}_{1i}v^{*}_{1j}g^{*}_{1ij}\right)+g_{12}\Phi_{10}\sum_{ij}\left[\left(u^{*}_{2i}u_{2j}+v^{*}_{2i}v_{2j}\right)f_{2ij}+u_{2i}v_{2j}g_{2ij}+u^{*}_{2i}v^{*}_{2j}g^{*}_{2ij}\right]+ g_{12}\Phi_{20}\sum_{ij}\left(v^{*}_{1i}u_{2j}f_{12ij}+u_{1i}v^{*}_{2j}f_{21ji}+u_{1i}u_{2j}g_{12ij}+v^{*}_{1i}v^{*}_{2j}g^{*}_{12ij}\right)+g_{12}\Phi_{20}\sum_{ij}\left(u^{*}_{2i}u_{1j}f_{21ij}+v_{2i}v^{*}_{1j}f_{12ji}+v_{2i}u_{1j}g_{21ij}+u^{*}_{2i}v^{*}_{1j}g^{*}_{21ij}\right)。\tag{21}$$

反映非凝聚激发$\delta\tilde{n}_{1(2)}$、$\delta\tilde{n}_{12(21)}$、$\delta\tilde{m}_{1(2)}$、$\delta\tilde{m}_{12}$随时间演化的运动方程为

$$\begin{cases} \mathrm{i}\hbar\dfrac{\partial f_{1(2)ij}}{\partial t}=\left\langle\left[\alpha^{\dagger}_{1(2)i}\alpha_{1(2)j},K\right]\right\rangle \\ \mathrm{i}\hbar\dfrac{\partial f_{12(21)ij}}{\partial t}=\left\langle\left[\alpha^{\dagger}_{1(2)i}\alpha_{2(1)j},K\right]\right\rangle \\ \mathrm{i}\hbar\dfrac{\partial}{\partial t}g_{1(2)ij}=\left\langle\left[\alpha_{1(2)i}\alpha_{1(2)j},K\right]\right\rangle \\ \mathrm{i}\hbar\dfrac{\partial}{\partial t}g_{12(21)ij}=\left\langle\left[\alpha_{1(2)i}\alpha_{2(1)j},K\right]\right\rangle \end{cases}。\tag{22}$$

为了进行式(22)的对易计算，把式(4)和式(9)代入式(1)，则系统巨正则哈密顿分解为非凝聚算符$\tilde{\psi}$的零次项、一次项、平方项、立方项、四次项。其中：零次项仅与不含时的凝聚静态部分有关，对上述对易计算结果没有贡献；由式(5)和式(6)可知，在取非平衡平均后，一次项和立方项为0，可忽略不计；平方项$K_2=K^{(0)}_2+K^{(1)}_2$，且

$$K^{(0)}_2=\int\left[\tilde{\psi}^{\dagger}_1\left(H_{10}+2g_1n_{10}+g_{12}n_{20}\right)\tilde{\psi}_1+\tilde{\psi}^{\dagger}_2\left(H_{20}+2g_2n_{20}+g_{12}n_{10}\right)\tilde{\psi}_2+ \frac{g_1}{2}n_{10}\left(\tilde{\psi}_1\tilde{\psi}_1+\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_1\right)+\frac{g_2}{2}n_{20}\left(\tilde{\psi}_2\tilde{\psi}_2+\tilde{\psi}^{\dagger}_2\tilde{\psi}^{\dagger}_2\right)+g_{12}\sqrt{n_{10}n_{20}}\left(\tilde{\psi}^{\dagger}_1\tilde{\psi}_2+\tilde{\psi}^{\dagger}_2\tilde{\psi}_1+\tilde{\psi}_1\tilde{\psi}_2+\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_2\right)\right]\mathrm{d}\boldsymbol{r},\tag{23}$$

$$K^{(1)}_2=\int\Phi_{10}\left[\left(\delta\Phi^{*}_1+\delta\Phi_1\right)\left(2g_1\tilde{\psi}^{\dagger}_1\tilde{\psi}_1+g_{12}\tilde{\psi}^{\dagger}_2\tilde{\psi}_2\right)+g_1\left(\delta\Phi^{*}_1\tilde{\psi}_1\tilde{\psi}_1+\delta\Phi_1\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_1\right)\right]\mathrm{d}\boldsymbol{r}+ g_{12}\int\Phi_{20}\delta\Phi^{*}_1\left[\left(\tilde{\psi}_1\tilde{\psi}_2+\tilde{\psi}_1\tilde{\psi}^{\dagger}_2\right)+\delta\Phi_1\left(\tilde{\psi}^{\dagger}_1\tilde{\psi}_2+\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_2\right)\mathrm{d}\boldsymbol{r}\right.。\tag{24}$$

采用非平衡平均近似$\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}_{1(2)}\tilde{\psi}_{1(2)}=4\tilde{n}_{1(2)}\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}_{1(2)}+\tilde{m}_{1(2)}\tilde{\psi}^{\dagger}_{1(2)}\tilde{\psi}^{\dagger}_{1(2)}+\tilde{m}^{*}_{1(2)}\tilde{\psi}_{1(2)}\tilde{\psi}_{1(2)}$，$\tilde{\psi}^{\dagger}_1\tilde{\psi}_1\tilde{\psi}^{\dagger}_2\tilde{\psi}_2=\tilde{n}_2\tilde{\psi}^{\dagger}_1\tilde{\psi}_1+\tilde{m}_{12}\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_2+\tilde{n}_{21}\tilde{\psi}^{\dagger}_1\tilde{\psi}_2+\tilde{n}_{12}\tilde{\psi}_1\tilde{\psi}^{\dagger}_2+\tilde{m}^{*}_{12}\tilde{\psi}_1\tilde{\psi}_2+\tilde{n}_1\tilde{\psi}^{\dagger}_2\tilde{\psi}_2$，并根据式(11)和式(17)得到四次项$K_4=K^{(0)}_4+K^{(1)}_4$，其中

$$K^{(0)}_4=\int\left[\left(2g_1\tilde{n}^{(0)}_1+g_{12}\tilde{n}^{(0)}_2\right)\tilde{\psi}^{\dagger}_1\tilde{\psi}_1+\frac{1}{2}g_1\tilde{m}^{(0)}_1\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_1+\frac{1}{2}g_1\tilde{m}^{(0)*}_1\tilde{\psi}_1\tilde{\psi}_1+ \left(2g_2\tilde{n}^{(0)}_2+g_{12}\tilde{n}^{(0)}_1\right)\tilde{\psi}^{\dagger}_2\tilde{\psi}_2+\frac{1}{2}g_2\tilde{m}^{(0)}_2\tilde{\psi}^{\dagger}_2\tilde{\psi}^{\dagger}_2+\frac{1}{2}g_2\tilde{m}^{(0)*}_2\tilde{\psi}_2\tilde{\psi}_2\right]\mathrm{d}\boldsymbol{r},\tag{25}$$

$$K^{(1)}_4=\int\left[\frac{g_1}{2}\left(4\delta\tilde{n}_1\tilde{\psi}^{\dagger}_1\tilde{\psi}_1+\delta\tilde{m}_1\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_1+\delta\tilde{m}^{*}_1\tilde{\psi}_1\tilde{\psi}_1\right)+\frac{g_2}{2}\left(4\delta\tilde{n}_2\tilde{\psi}^{\dagger}_2\tilde{\psi}_2+\delta\tilde{m}_2\tilde{\psi}^{\dagger}_2\tilde{\psi}^{\dagger}_2+\delta\tilde{m}^{*}_2\tilde{\psi}_2\tilde{\psi}_2\right)+ g_{12}\left(\delta\tilde{n}_1\tilde{\psi}^{\dagger}_2\tilde{\psi}_2+\delta\tilde{n}_2\tilde{\psi}^{\dagger}_1\tilde{\psi}_1+\delta\tilde{n}_{12}\tilde{\psi}_1\tilde{\psi}^{\dagger}_2+\delta\tilde{n}_{21}\tilde{\psi}^{\dagger}_1\tilde{\psi}_2+\delta\tilde{m}_{12}\tilde{\psi}^{\dagger}_1\tilde{\psi}^{\dagger}_2+\delta\tilde{m}^{*}_{12}\tilde{\psi}_1\tilde{\psi}_2\right)\right]\mathrm{d}\boldsymbol{r}。\tag{26}$$

因此，巨正则哈密顿对式(22)有贡献的项为

$$K_2+K_4=K^{(0)}+K^{(1)}_2,\tag{27}$$

式中：$K^{(0)}=K^{(0)}_2+K^{(0)}_4$，而$K^{(1)}_4$相对$K^{(1)}_2$较小，可被忽略，这是因为低温下$\Phi^2\gg\tilde{n},\tilde{m}$且$\Phi\delta\Phi\gg\delta\tilde{n},\delta\tilde{m}$。

2.2 巨正则哈密顿的对角化与非凝聚准粒子的BdG方程

为了对角化$K^{(0)}$，取近似

$$\begin{cases}\tilde{\psi}_1\tilde{\psi}_2=\dfrac{1}{2}\left(\tilde{\psi}_1\tilde{\psi}_1+\tilde{\psi}_2\tilde{\psi}_2\right)\\ \tilde{\psi}_1^{\dagger}\tilde{\psi}_2^{\dagger}=\dfrac{1}{2}\left(\tilde{\psi}_1^{\dagger}\tilde{\psi}_1^{\dagger}+\tilde{\psi}_2^{\dagger}\tilde{\psi}_2^{\dagger}\right)\\ \tilde{\psi}_1^{\dagger}\tilde{\psi}_2+\tilde{\psi}_2^{\dagger}\tilde{\psi}_1=\tilde{\psi}_1^{\dagger}\tilde{\psi}_1+\tilde{\psi}_2^{\dagger}\tilde{\psi}_2\end{cases},\tag{28}$$

可将式(23)改写为

$$K_2^{(0)}=\int\Big[\tilde{\psi}_1^{\dagger}\left(H_{10}+2g_1n_{10}+g_{12}n_{20}+g_{12}\sqrt{n_{10}n_{20}}\right)\tilde{\psi}_1+\tilde{\psi}_2^{\dagger}\left(H_{20}+2g_2n_{20}+g_{12}n_{10}+g_{12}\sqrt{n_{10}n_{20}}\right)\tilde{\psi}_2+\frac{1}{2}\left(g_1n_{10}+g_{12}\sqrt{n_{10}n_{20}}\right)\left(\tilde{\psi}_1\tilde{\psi}_1+\tilde{\psi}_1^{\dagger}\tilde{\psi}_1^{\dagger}\right)+\frac{1}{2}\left(g_2n_{20}+g_{12}\sqrt{n_{10}n_{20}}\right)\left(\tilde{\psi}_2\tilde{\psi}_2+\tilde{\psi}_2^{\dagger}\tilde{\psi}_2^{\dagger}\right)\Big]\mathrm{d}\boldsymbol{r}。\tag{29}$$

由式(25)和式(29)可得

$$K^{(0)}=K_2^{(0)}+K_4^{(0)}=\int\Big[\tilde{\psi}_1^{\dagger}L_1\tilde{\psi}_1+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)\tilde{\psi}_1\tilde{\psi}_1+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)\tilde{\psi}_1^{\dagger}\tilde{\psi}_1^{\dagger}\Big]\mathrm{d}\boldsymbol{r}+\int\Big[\tilde{\psi}_2^{\dagger}L_2\tilde{\psi}_2+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)\tilde{\psi}_2\tilde{\psi}_2+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)\tilde{\psi}_2^{\dagger}\tilde{\psi}_2^{\dagger}\Big]\mathrm{d}\boldsymbol{r},\tag{30}$$

其中

$$\mathcal{L}_{1(2)}=H_{10}+2g_{1(2)}n_{1(2)0}+g_{12}n_{2(1)0}+g_{12}\sqrt{n_{10}n_{20}}+2g_{1(2)}\tilde{n}_{1(2)}^{(0)}+g_{12}\tilde{n}_{2(1)}^{(0)}。\tag{31}$$

将式(16)代入式(30)可得：

$$\begin{aligned}K^{(0)}=&\sum_{jj'}\int\Big\{\Big[u_{1j}^{*}\mathcal{L}_1u_{1j'}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{1j}^{*}v_{1j'}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{1j}^{*}u_{1j'}\Big]\alpha_{1j}^{\dagger}\alpha_{1j'}+\\&\Big[u_{1j}^{*}\mathcal{L}_1v_{1j'}^{*}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{ij}^{*}v_{1j'}^{*}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{1j}^{*}u_{1j'}^{*}\Big]\alpha_{1j}^{\dagger}\alpha_{1j'}^{\dagger}+\\&\Big[v_{1j}\mathcal{L}_1u_{1j'}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{1j}u_{1j'}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{1j'}v_{1j'}\Big]\alpha_{1j}\alpha_{1j'}+\\&\Big[v_{1j}\mathcal{L}_1v_{1j'}^{*}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{1j}u_{1j'}^{*}+\frac{1}{2}\left(g_1n_{10}+g_1\tilde{m}_1^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{1j}v_{1j'}^{*}\Big]\alpha_{1j}\alpha_{1j'}^{\dagger}\Big\}\mathrm{d}\boldsymbol{r}+\\&\sum_{jj'}\int\Big\{\Big[u_{2j}^{*}\mathcal{L}_2u_{2j'}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{2j}^{*}v_{2j'}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{2j}^{*}u_{2j'}\Big]\alpha_{2j}^{\dagger}\alpha_{2j'}+\\&\Big[u_{2j}^{*}\mathcal{L}_2v_{2j'}^{*}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{2j}^{*}v_{2j'}^{*}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{2j}^{*}u_{2j'}^{*}\Big]\alpha_{2j}^{\dagger}\alpha_{1j'}^{\dagger}+\\&\Big[v_{2j}\mathcal{L}_2u_{2j'}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{2j}u_{2j'}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{2j'}v_{2j'}\Big]\alpha_{2j}\alpha_{2j'}+\\&\Big[v_{2j}\mathcal{L}_2v_{2j'}^{*}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{2j}u_{2j'}^{*}+\frac{1}{2}\left(g_2n_{20}+g_2\tilde{m}_2^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{2j}v_{2j'}^{*}\Big]\alpha_{2j}\alpha_{2j'}^{\dagger}\Big\}\mathrm{d}\boldsymbol{r}。\end{aligned}\tag{32}$$

如果准粒子振幅 $u_{1(2)j}$ 和 $v_{1(2)j}$ 满足非凝聚准粒子波戈留波夫-德热纳(BdG)方程

$$\begin{cases}\mathcal{L}_{1(2)}u_{1(2)j}+\left(g_{1(2)}n_{1(2)0}+g_{1(2)}\tilde{m}_{1(2)}^{(0)}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{1(2)j}=\varepsilon_{1(2)j}u_{1(2)j}\\ \mathcal{L}_{1(2)}v_{1(2)j}+\left(g_{1(2)}n_{1(2)0}+g_{1(2)}\tilde{m}_{1(2)}^{(0)*}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{1(2)j}=-\varepsilon_{1(2)j}v_{1(2)j}\end{cases},\tag{33}$$

及其归一化条件 $\int\left[u_{1(2)i}^{*}u_{1(2)j}-v_{1(2)i}^{*}v_{1(2)j}\right]\mathrm{d}\boldsymbol{r}=\delta_{ij}$，可将式(32)对角化为

$$K^{(0)}=-\sum_j\int\varepsilon_{1j}\left|v_{1j}\right|^2\mathrm{d}\boldsymbol{r}+\sum_j\varepsilon_{1j}\alpha_{1j}^{\dagger}\alpha_{1j}-\sum_j\int\varepsilon_{2j}\left|v_{2j}\right|^2\mathrm{d}\boldsymbol{r}+\sum_j\varepsilon_{2j}\alpha_{2j}^{\dagger}\alpha_{2j}。\tag{34}$$

2.3 正常和反常准粒子分布函数运动方程的对易计算和傅里叶变换与三模耦合矩阵元

将式(16)代入式(24)可得：

$$
\begin{aligned}
K_2^{(1)}=&\sum_{l,m}\int g_1\Phi_{10}\left[2\left(\delta\Phi_1+\delta\Phi_1^*\right)u_{1l}^*u_{1m}+\delta\Phi_1u_{1l}^*v_{1m}+\delta\Phi_1^*v_{1l}^*u_{1m}\right]\alpha_{1l}^\dagger\alpha_{1m}\,\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_1\Phi_{10}\left[2\left(\delta\Phi_1+\delta\Phi_1^*\right)u_{1l}^*v_{1m}^*+\delta\Phi_1u_{1l}^*u_{1m}^*+\delta\Phi_1^*v_{1l}^*v_{1m}^*\right]\alpha_{1l}^\dagger\alpha_{1m}^\dagger\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_1\Phi_{10}\left[2\left(\delta\Phi_1+\delta\Phi_1^*\right)v_{1l}u_{1m}+\delta\Phi_1v_{1l}v_{1m}+\delta\Phi_1^*u_{1l}u_{1m}\right]\alpha_{1l}\alpha_{1m}\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_1\Phi_{10}\left[2\left(\delta\Phi_1+\delta\Phi_1^*\right)+\delta\Phi_1v_{1l}u_{1m}^*+\delta\Phi_1^*u_{1l}v_{1m}^*\right]\alpha_{1l}\alpha_{1m}^\dagger\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_{12}\Phi_{10}\left[\left(\delta\Phi_1+\delta\Phi_1^*\right)u_{2l}^*u_{2m}\right]\alpha_{2l}^\dagger\alpha_{2m}\,\mathrm{d}\boldsymbol{r}+\sum_{l,m}\int g_{12}\Phi_{10}\left[\left(\delta\Phi_1+\delta\Phi_1^*\right)u_{2l}^*v_{2m}^*\right]\alpha_{2l}^\dagger\alpha_{2m}^\dagger\,\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_{12}\left[\Phi_{10}\left(\delta\Phi_1+\delta\Phi_1^*\right)\right]u_{2l}u_{2m}\alpha_{2l}\alpha_{2m}\,\mathrm{d}\boldsymbol{r}+\sum_{l,m}\int g_{12}\left[\Phi_{10}\left(\delta\Phi_1+\delta\Phi_1^*\right)\right]v_{2l}v_{2m}^*\alpha_{2l}\alpha_{2m}^\dagger\,\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_{12}\Phi_{20}\left[\delta\Phi_1\left(u_{1l}^*u_{2m}+u_{1l}^*v_{2m}\right)+\delta\Phi_1^*\left(v_{1l}^*u_{2m}+v_{1l}^*v_{2m}\right)\right]\alpha_{1l}^\dagger\alpha_{2m}\,\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_{12}\Phi_{20}\left[\delta\Phi_1\left(u_{1l}^*v_{2m}^*+v_{1l}^*u_{2m}^*\right)+\delta\Phi_1^*\left(v_{1l}^*v_{2m}^*+v_{1l}^*u_{2m}^*\right)\right]\alpha_{1l}^\dagger\alpha_{2m}^\dagger\,\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_{12}\Phi_{20}\left[\delta\Phi_1\left(v_{1l}u_{2m}+v_{1l}v_{2m}\right)+\delta\Phi_1^*\left(u_{1l}u_{2m}+u_{1l}v_{2m}\right)\right]\alpha_{1l}\alpha_{2m}\,\mathrm{d}\boldsymbol{r}+\\
&\sum_{l,m}\int g_{12}\Phi_{20}\left[\delta\Phi_1\left(v_{1l}v_{2m}^*+v_{1l}u_{2m}^*\right)+\delta\Phi_1^*\left(u_{1l}v_{2m}^*+u_{1l}u_{2m}^*\right)\right]\alpha_{1l}\alpha_{2m}^\dagger\,\mathrm{d}\boldsymbol{r}。
\end{aligned}\tag{35}
$$

将式(34)和式(35)代入式(22)进行对易计算，取非平衡平均，并采用零级近似$\left\langle\alpha_{1(2)i}^\dagger\alpha_{1(2)j}\right\rangle\approx\left\langle\alpha_{1(2)i}^\dagger\alpha_{1(2)j}\right\rangle_0$、$\left\langle\alpha_{1(2)i}\alpha_{1(2)j}\right\rangle\approx\left\langle\alpha_{1(2)i}\alpha_{1(2)j}\right\rangle_0$、$\left\langle\alpha_{1(2)i}^\dagger\alpha_{2(1)j}\right\rangle\approx\left\langle\alpha_{1(2)i}^\dagger\alpha_{2(1)j}\right\rangle_0$、$\left\langle\alpha_{1i}\alpha_{2j}\right\rangle\approx\left\langle\alpha_{1i}\alpha_{2j}\right\rangle_0$（对角化哈密顿量$K^{(0)}$后，$\left\langle\alpha_{1(2)i}^\dagger\alpha_{1(2)j}\right\rangle_0=f_{1(2)i}^{(0)}\delta_{ij}$、$\left\langle\alpha_{1(2)i}\alpha_{1(2)j}\right\rangle_0=0$、$\left\langle\alpha_{1(2)i}^\dagger\alpha_{2(1)j}\right\rangle_0=0$、$\left\langle\alpha_{1i}\alpha_{2j}\right\rangle_0=0$），式(22)变为

$$
\begin{cases}
\mathrm{i}\hbar\dfrac{\partial f_{1ij}}{\partial t}=\left(\varepsilon_{1j}-\varepsilon_{1i}\right)f_{1ij}+2g_1\left(f_{1i}^{(0)}-f_{1j}^{(0)}\right)\displaystyle\int\Phi_{10}\left[\delta\Phi_1\left(u_{1i}u_{1j}^*+v_{1i}v_{1j}^*+v_{1i}u_{1j}^*\right)+\delta\Phi_1^*\left(u_{1i}u_{1j}^*+v_{1i}v_{1j}^*+u_{1i}v_{1j}^*\right)\right]\mathrm{d}\boldsymbol{r}\\
\mathrm{i}\hbar\dfrac{\partial f_{2ij}}{\partial t}=\left(\varepsilon_{2j}-\varepsilon_{2i}\right)f_{2ij}+g_{12}\left(f_{2i}^{(0)}-f_{2j}^{(0)}\right)\displaystyle\int\Phi_{10}\left(\delta\Phi_1+\delta\Phi_1^*\right)\left(u_{2i}u_{2j}^*+v_{2i}v_{2j}^*\right)\mathrm{d}\boldsymbol{r}\\
\mathrm{i}\hbar\dfrac{\partial f_{12ij}}{\partial t}=\left(\varepsilon_{2j}-\varepsilon_{1i}\right)f_{12ij}+g_{12}\left(f_{1i}^{(0)}-f_{2j}^{(0)}\right)\displaystyle\int\Phi_{20}\left[\delta\Phi_1\left(v_{1i}u_{2j}^*+v_{1i}v_{2j}^*\right)+\delta\Phi_1^*\left(u_{1i}u_{2j}^*+u_{1i}v_{2j}^*\right)\right]\mathrm{d}\boldsymbol{r}\\
\mathrm{i}\hbar\dfrac{\partial f_{21ij}}{\partial t}=\left(\varepsilon_{1j}-\varepsilon_{2i}\right)f_{21ij}+g_{12}\left(f_{2i}^{(0)}-f_{1j}^{(0)}\right)\displaystyle\int\Phi_{20}\left[\delta\Phi_1\left(u_{2i}u_{1j}^*+v_{2i}u_{1j}^*\right)+\delta\Phi_1^*\left(u_{2i}v_{1j}^*+v_{2i}v_{1j}^*\right)\right]\mathrm{d}\boldsymbol{r}
\end{cases},\tag{36}
$$

$$
\begin{cases}
\mathrm{i}\hbar\dfrac{\partial}{\partial t}g_{1ij}=\left(\varepsilon_{1i}+\varepsilon_{1j}\right)g_{1ij}+\\
2g_1\left(1+f_{1i}^{(0)}+f_{1j}^{(0)}\right)\displaystyle\int\Phi_{10}\left[\delta\Phi_1\left(u_{1i}^*v_{1j}^*+v_{1i}^*u_{1j}^*+v_{1i}^*v_{1j}^*\right)+\delta\Phi_1^*\left(u_{1i}^*u_{1j}^*+u_{1i}^*v_{1j}^*+v_{1i}^*u_{1j}^*\right)\right]\mathrm{d}\boldsymbol{r}\\
\mathrm{i}\hbar\dfrac{\partial}{\partial t}g_{2ij}=\left(\varepsilon_{2i}+\varepsilon_{2j}\right)g_{2ij}+\\
2g_{12}\left(1+f_{2i}^{(0)}+f_{2j}^{(0)}\right)\displaystyle\int\Phi_{10}\left(\delta\Phi_1+\delta\Phi_1^*\right)\left(u_{2i}^*v_{2j}^*+v_{2i}^*u_{2j}^*\right)\mathrm{d}\boldsymbol{r}\\
\mathrm{i}\hbar\dfrac{\partial g_{12ij}}{\partial t}=\mathrm{i}\hbar\dfrac{\partial g_{21ji}}{\partial t}=\left(\varepsilon_{1i}+\varepsilon_{2j}\right)g_{12ij}+\\
g_{12}\left(1+f_{1i}^{(0)}+f_{2j}^{(0)}\right)\displaystyle\int\Phi_{20}\left[\delta\Phi_1\left(u_{1i}^*u_{2j}^*+u_{1i}^*v_{2j}^*\right)+\delta\Phi_1^*\left(v_{1i}^*u_{2j}^*+v_{1i}^*v_{2j}^*\right)\right]\mathrm{d}\boldsymbol{r}
\end{cases}。\tag{37}
$$

关注外界驱动（如前所述，其频率为ω_0）取消后集体激发本征频率ω和振幅$\delta\Phi_1$的变化，设$\delta\Phi_1(\boldsymbol{r},t)=\delta\Phi_{11}(\boldsymbol{r})\exp(-\mathrm{i}\omega t)$、$\delta\Phi_1^*(\boldsymbol{r},t)=\delta\Phi_{12}(\boldsymbol{r})\exp(-\mathrm{i}\omega t)$，进行如下傅里叶变换

$$\begin{cases}\delta\Phi_1(\boldsymbol{r},t)=\int_{-\infty}^{\infty}\delta\Phi_{11}(\boldsymbol{r},\omega)\exp(-\mathrm{i}\omega t)\mathrm{d}\omega,\ \delta\Phi_1^*(x,t)=\int_{-\infty}^{\infty}\delta\Phi_{12}(\boldsymbol{r},\omega)\exp(-\mathrm{i}\omega t)\mathrm{d}\omega\\ R_1(\boldsymbol{r},t)=\int_{-\infty}^{\infty}R_{11}(\boldsymbol{r},\omega)\exp(-\mathrm{i}\omega t)\mathrm{d}\omega,\ R_1^*(\boldsymbol{r},t)=\int_{-\infty}^{\infty}R_{12}(\boldsymbol{r},\omega)\exp(-\mathrm{i}\omega t)\mathrm{d}\omega\\ f_{ij}(t)=\int_{-\infty}^{\infty}\exp(-\mathrm{i}\omega t)f_{ij}^{(1)}(\omega)\mathrm{d}\omega,\ f_{ij}^*(t)=\int_{-\infty}^{\infty}\exp(-\mathrm{i}\omega t)f_{ij}^{(2)}(\omega)\mathrm{d}\omega\\ g_{ij}(t)=\int_{-\infty}^{\infty}\exp(-\mathrm{i}\omega t)g_{ij}^{(1)}(\omega)\mathrm{d}\omega,\ g_{ij}^*(t)=\int_{-\infty}^{\infty}\exp(-\mathrm{i}\omega t)g_{ij}^{(2)}(\omega)\mathrm{d}\omega\end{cases},\tag{38}$$

式中:$f_{ij}^{(1)}(\omega)$、$f_{ij}^{(2)}(\omega)$分别为$f_{ij}(t)$、$f_{ij}^*(t)$的傅里叶变换结果;$g_{ij}^{(1)}(\omega)$、$g_{ij}^{(2)}(\omega)$分别为$g_{ij}(t)$、$g_{ij}^*(t)$的傅里叶变换结果。

进行级数展开,并且只保留零级项和一级项

$$\begin{cases}\omega=\omega_0+\omega'\\ \delta\Phi_{11}(\omega)=\delta\Phi_{11}^{(0)}(\omega_0)+\delta\Phi'_{11},\ \delta\Phi_{12}(\omega)=\delta\Phi_{12}^{(0)}(\omega_0)+\delta\Phi'_{12}\\ R_{11}(\boldsymbol{r},\omega)=R_{11}(\boldsymbol{r},\omega_0)+\cdots,\ R_{12}(\boldsymbol{r},\omega)=R_{12}(\boldsymbol{r},\omega_0)+\cdots\\ f_{ij}^{(1)}(\omega)=f_{ij}^{(1)}(\omega_0)+\cdots,\ f_{ij}^{(2)}(\omega)=f_{ij}^{(2)}(\omega_0)+\cdots\\ g_{ij}^{(1)}(\omega)=g_{ij}^{(1)}(\omega_0)+\cdots,\ g_{ij}^{(2)}(\omega)=g_{ij}^{(2)}(\omega_0)+\cdots\end{cases},\tag{39}$$

等号右边ω'、$\delta\Phi'_{11}$、$\delta\Phi'_{11}$为一级项,其余为零级项。

将式(38)和式(39)代入式(36)和它的共轭方程并取零级近似,得到

$$\begin{cases}f_{1ij}^{(1)}(\omega_0)=f_{1ji}^{(2)}(\omega_0)=2g_1\dfrac{f_{1i}^{(0)}-f_{1j}^{(0)}}{\hbar\omega_0+\varepsilon_{1i}-\varepsilon_{1j}}A_{1ij}\\ f_{2ij}^{(1)}(\omega_0)=f_{2ji}^{(2)}(\omega_0)=2g_{12}\dfrac{f_{2i}^{(0)}-f_{2j}^{(0)}}{\hbar\omega_0+\varepsilon_{2i}-\varepsilon_{2j}}A_{2ij}\\ f_{12(21)ij}^{(1)}(\omega_0)=f_{21(12)ji}^{(2)}(\omega_0)=2g_{12}\dfrac{f_{1(2)i}^{(0)}-f_{2(1)j}^{(0)}}{\hbar\omega_0+\varepsilon_{1(2)i}-\varepsilon_{2(1)j}}A_{12(21)ij}\end{cases},\tag{40}$$

其中

$$\begin{cases}A_{1ij}=\int\Phi_{10}\left[\delta\Phi_{11}^{(0)}\left(u_{1i}u_{1j}^*+v_{1i}u_{1j}^*+v_{1i}v_{1j}^*\right)+\delta\Phi_{12}^{(0)}\left(u_{1i}u_{1j}^*+u_{1i}v_{1j}^*+v_{1i}v_{1j}^*\right)\right]\mathrm{d}\boldsymbol{r}\\ A_{2ij}=\dfrac{1}{2}\int\Phi_{10}\left[\left(\delta\Phi_{11}^{(0)}+\delta\Phi_{12}^{(0)}\right)\left(u_{2i}u_{2j}^*+v_{2i}v_{2j}^*\right)\right]\mathrm{d}\boldsymbol{r}\\ A_{12ij}=\dfrac{1}{2}\int\Phi_{20}\left[\delta\Phi_{11}^{(0)}\left(v_{1i}u_{2j}^*+v_{1i}v_{2j}^*\right)+\delta\Phi_{12}^{(0)}\left(u_{1i}u_{2j}^*+u_{1i}v_{2j}^*\right)\right]\mathrm{d}\boldsymbol{r}\\ \mathrm{A}_{21ij}=\dfrac{1}{2}\int\Phi_{20}\left[\delta\Phi_{11}^{(0)}\left(u_{2i}u_{1j}^*+v_{2i}u_{1j}^*\right)+\delta\Phi_{12}^{(0)}\left(u_{2i}v_{1j}^*+v_{2i}v_{1j}^*\right)\right]\mathrm{d}\boldsymbol{r}\end{cases},\tag{41}$$

是朗道(Landau)机制的三模耦合矩阵元。

将式(38)和式(39)代入式(37)并取零级近似,得到

$$\begin{cases}g_{1ij}^{(1)}(\omega_0)=2g_1\dfrac{1+f_{1i}^{(0)}+f_{1j}^{(0)}}{\hbar\omega_0-\left(\varepsilon_{1i}+\varepsilon_{1j}\right)}B_{1ij}\\ g_{2ij}^{(1)}(\omega_0)=2g_{12}\dfrac{1+f_{2fi}^{(0)}+f_{2j}^{(0)}}{\hbar\omega_0-\left(\varepsilon_{2i}+\varepsilon_{2j}\right)}B_{2ij}\\ g_{12(21)ij}^{(1)}(\omega_0)=g_{21(12)ji}^{(1)}(\omega_0)=2g_{12}\dfrac{1+f_{1(2)i}^{(0)}+f_{2(1)j}^{(0)}}{\hbar\omega_0-\left(\varepsilon_{1(2)i}+\varepsilon_{2(1)j}\right)}B_{12(21)ij}\end{cases},\tag{42}$$

其中

$$\begin{cases} B_{1ij}=\int \Phi_{10}\left[\delta\Phi_{11}^{(0)}\left(u_{1i}^{*}u_{1j}^{*}+u_{1i}^{*}v_{1j}^{*}+v_{1i}^{*}u_{1j}^{*}\right)+\delta\Phi_{12}^{(0)}\left(u_{1i}^{*}v_{1j}^{*}+v_{1i}^{*}u_{1j}^{*}+v_{1i}^{*}v_{1j}^{*}\right)\right]\mathrm{d}\boldsymbol{r} \\ B_{2ij}=\dfrac{1}{2}\int \Phi_{10}\left(\delta\Phi_{11}^{(0)}+\delta\Phi_{12}^{(0)}\right)\left(u_{2i}^{*}v_{2j}^{*}+v_{2i}^{*}u_{2j}^{*}\right)\mathrm{d}\boldsymbol{r} \\ B_{12ij}=B_{21ji}=\dfrac{1}{2}\int \Phi_{20}\left[\left(u_{1i}^{*}u_{2j}^{*}+u_{1i}^{*}v_{2j}^{*}\right)\delta\Phi_{11}^{(0)}+\left(v_{1i}^{*}u_{2j}^{*}+v_{1i}^{*}v_{2j}^{*}\right)\delta\Phi_{12}^{(0)}\right]\mathrm{d}\boldsymbol{r} \end{cases}, \tag{43}$$

是巴利耶夫(Beliaev)机制的三模耦合矩阵元。

将式(38)和式(39)代入式(37)的共轭方程并取零级近似,得到

$$\begin{cases} g_{1ij}^{(2)}(\omega_0)=-2g_1\dfrac{1+f_{1i}^{(0)}+f_{1j}^{(0)}}{\hbar\omega_0+\varepsilon_{1i}+\varepsilon_{1j}}\tilde{B}_{1ij} \\ g_{2ij}^{(2)}(\omega_0)=-2g_{12}\dfrac{1+f_{2i}^{(0)}+f_{2j}^{(0)}}{\hbar\omega_0+\varepsilon_{2i}+\varepsilon_{2j}}\tilde{B}_{2ij} \\ g_{12(21)ij}^{(2)}(\omega_0)=g_{21(12)ji}^{(2)}(\omega_0)=-2g_{12}\dfrac{1+f_{1(2)i}^{(0)}+f_{2j(1)}^{(0)}}{\hbar\omega_0+\varepsilon_{1(2)i}+\varepsilon_{2(1)j}}\tilde{B}_{12(21)ij} \end{cases}, \tag{44}$$

其中

$$\begin{cases} \tilde{B}_{1ij}=\int \Phi_{10}\left[\delta\Phi_{12}^{(0)}\left(u_{1i}v_{1j}+v_{1i}u_{1j}+v_{1i}v_{1j}\right)+\delta\Phi_{11}^{(0)}\left(u_{1i}u_{1j}+u_{1i}v_{1j}+v_{1i}u_{1j}\right)\right]\mathrm{d}\boldsymbol{r} \\ \tilde{B}_{2ij}=\dfrac{1}{2}\int \Phi_{10}\left(\delta\Phi_{11}^{(0)}+\delta\Phi_{12}^{(0)}\right)\left(u_{2i}v_{2j}+v_{2i}u_{2j}\right)\mathrm{d}\boldsymbol{r} \\ \tilde{B}_{12ij}=\tilde{B}_{21ji}=\dfrac{1}{2}\int \Phi_{20}\left[\delta\Phi_{12}^{(0)}\left(u_{1i}u_{2j}+u_{1i}v_{2j}\right)+\delta\Phi_{11}^{(0)}\left(v_{1i}u_{2j}+v_{1i}v_{2j}\right)\right]\mathrm{d}\boldsymbol{r} \end{cases}。 \tag{45}$$

2.4 集体激发运动方程的傅里叶变换与集体激发摄动本征频率关系和阻尼系数

将式(38)和式(39)代入式(13)和式(21),得到

$$\left(\hbar\omega_0+\hbar\omega'\right)\left(\delta\Phi_{11}^{(0)}+\delta\Phi'_{11}\right)=L_0\left(\delta\Phi_{11}^{(0)}+\delta\Phi'_{11}\right)+g_1\left(n_{10}+\tilde{m}_1^{(0)}\right)\left(\delta\Phi_{12}^{(0)}+\delta\Phi'_{12}\right)+R_{11}, \tag{46}$$

其中:

$$R_{11}=2g_1\Phi_{10}\sum_{ij}\left[\left(u_{1i}^{*}u_{1j}+v_{1i}^{*}v_{1j}\right)f_{1ij}^{(1)}+u_{1i}v_{1j}g_{1ij}^{(1)}+u_{1i}^{*}v_{1j}^{*}g_{1ij}^{(2)}\right]+g_1\Phi_{10}\sum_{ij}\left(2u_{1j}v_{1i}^{*}f_{1ij}^{(1)}+u_{1i}u_{1j}g_{1ij}^{(1)}+v_{1i}^{*}v_{1j}^{*}g_{1ij}^{(2)}\right)+$$
$$g_{12}\Phi_{10}\sum_{ij}\left[\left(u_{2i}^{*}u_{2j}+v_{2i}^{*}v_{2j}\right)f_{2ij}^{(1)}+u_{2i}v_{2j}g_{2ij}^{(1)}+u_{2i}^{*}v_{2j}^{*}g_{2ij}^{(2)}\right]+g_{12}\Phi_{20}\sum_{ij}\left(u_{1i}v_{2j}^{*}f_{21ji}^{(1)}+v_{1i}^{*}u_{2j}f_{12ij}^{(1)}+u_{1i}u_{2j}g_{12ij}^{(1)}+v_{1i}^{*}v_{2j}^{*}g_{12ij}^{(2)}\right)+$$
$$g_{12}\Phi_{20}\sum_{ij}\left(u_{2i}^{*}u_{1j}f_{21ij}^{(1)}+v_{2i}v_{1j}^{*}f_{12ji}^{(1)}+u_{2i}^{*}v_{1j}^{*}g_{21ij}^{(2)}+v_{2i}u_{1j}g_{21ij}^{(1)}\right)。 \tag{47}$$

将式(38)和式(39)代入式(13)的共轭方程和式(21)的共轭方程,得到

$$-\left(\hbar\omega_0+\hbar\omega'\right)\left(\delta\Phi_{12}^{(0)}+\delta\Phi'_{12}\right)=\mathcal{L}_1\left(\delta\Phi_{12}^{(0)}+\delta\Phi'_{12}\right)+g_1\left(n_{10}+\tilde{m}_1^{(0)*}\right)\left(\delta\Phi_{11}^{(0)}+\delta\Phi'_{11}\right)+R_{12}, \tag{48}$$

其中

$$R_{12}=2g_1\Phi_{10}\sum_{ij}\left[\left(u_{1i}u_{1j}^{*}+v_{1i}v_{1j}^{*}\right)f_{1ij}^{(2)}+u_{1i}^{*}v_{1j}^{*}g_{1ij}^{(2)}+u_{1i}v_{1j}g_{1ij}^{(1)}+\frac{1}{2}\left(2u_{1j}^{*}v_{1i}f_{1ij}^{(2)}+u_{1i}^{*}u_{1j}^{*}g_{1ij}^{(2)}+v_{1i}v_{1j}g_{1ij}^{(1)}\right)\right]+$$
$$g_{12}\Phi_{10}\sum_{ij}\left[\left(u_{2i}u_{2j}^{*}+v_{2i}v_{2j}^{*}\right)f_{2ij}^{(2)}+u_{2i}^{*}v_{2j}^{*}g_{2ij}^{(2)}+u_{2i}v_{2j}g_{2ij}^{(1)}\right]+g_{12}\Phi_{20}\sum_{ij}\left(u_{1i}^{*}v_{2j}f_{21ji}^{(2)}+v_{1i}u_{2j}^{*}f_{12ij}^{(2)}+u_{1i}^{*}u_{2j}^{*}g_{12ij}^{(2)}+v_{1i}v_{2j}g_{12ij}^{(1)}\right)+$$
$$g_{12}\Phi_{20}\sum_{ij}\left(u_{2i}u_{1j}^{*}f_{21ij}^{(2)}+v_{2i}^{*}v_{1j}f_{12ji}^{(2)}+u_{2i}v_{1j}g_{21ij}^{(1)}+v_{2i}^{*}u_{1j}^{*}g_{21ij}^{(2)}\right)。 \tag{49}$$

取式(46)和式(48)的零级项,得到集体激发的BdG方程:

$$\begin{cases} \hbar\omega_0\delta\Phi_{11}^{(0)}=\mathcal{L}_0\delta\Phi_{11}^{(0)}+g_1\left(n_{10}+\tilde{m}_1^{(0)}\right)\delta\Phi_{12}^{(0)} \\ -\hbar\omega_0\delta\Phi_{12}^{(0)}=\mathcal{L}_0\delta\Phi_{12}^{(0)}+g_1\left(n_{10}+\tilde{m}_1^{(0)*}\right)\delta\Phi_{11}^{(0)} \end{cases}。 \tag{50}$$

取式(46)和式(48)的一级项,并取归一化条件 $\int\left(\delta\Phi_{11}^{(0)}\delta\Phi_{11}^{(0)}-\delta\Phi_{12}^{(0)}\delta\Phi_{12}^{(0)}\right)\mathrm{d}\boldsymbol{r}=1$、$\int\left(\delta\Phi_{11}^{(0)}\delta\Phi'_{12}-\delta\Phi_{12}^{(0)}\delta\Phi'_{11}\right)\mathrm{d}\boldsymbol{r}=0$、$\int\left(\delta\Phi_{12}^{(0)}\delta\Phi'_{11}-\delta\Phi_{11}^{(0)}\delta\Phi'_{12}\right)\mathrm{d}\boldsymbol{r}=0$,得到:

$$\hbar\omega'=\hbar\omega-\hbar\omega_0=\int\delta\Phi_{11}^{(0)*}R_{11}\mathrm{d}\boldsymbol{r}+\int\delta\Phi_{12}^{(0)*}R_{12}\mathrm{d}\boldsymbol{r}。 \tag{51}$$

先将式(40)、式(42)、式(44)代入式(47)、式(49),然后再将式(47)、式(49)代入式(51),得集体激发的摄动本

征频率关系：

$$\hbar\omega-\hbar\omega_0=4g_1^{(2)}\sum_{ij}\frac{f_{1i}^{(0)}-f_{1j}^{(0)}}{\hbar\omega_0+\left(\varepsilon_{1i}-\varepsilon_{1j}\right)+\mathrm{i}_0}\left|A_{1ij}\right|^2+4g_{12}^2\sum_{ij}\frac{f_{2i}^{(0)}-f_{2j}^{(0)}}{\hbar\omega_0+\left(\varepsilon_{2i}-\varepsilon_{2j}\right)+\mathrm{i}_0}\left|A_{2ij}\right|^2+$$
$$4g_{12}^{(2)}\sum_{ij}\frac{f_{1i}^{(0)}-f_{2j}^{(0)}}{\hbar\omega_0+\left(\varepsilon_{1i}-\varepsilon_{2j}\right)+\mathrm{i}_0}\left|A_{12ij}\right|^2+4g_{12}^{(2)}\sum_{ij}\frac{f_{2i}^{(0)}-f_{1j}^{(0)}}{\hbar\omega_0+\left(\varepsilon_{2i}-\varepsilon_{1j}\right)+\mathrm{i}_0}\left|A_{21ij}\right|^2+$$
$$2g_1^{(2)}\sum_{ij}\frac{1+f_{1i}^{(0)}+f_{1j}^{(0)}}{\hbar\omega_0-\left(\varepsilon_{1i}+\varepsilon_{1j}\right)+\mathrm{i}_0}\left|B_{1ij}\right|^2+2g_{12}^{(2)}\sum_{ij}\frac{1+f_{2i}^{(0)}+f_{2j}^{(0)}}{\hbar\omega_0-\left(\varepsilon_{2i}+\varepsilon_{2j}\right)+\mathrm{i}_0}\left|B_{2ij}\right|^2+2g_{12}^{(2)}\sum_{ij}\frac{1+f_{1i}^{(0)}+f_{2j}^{(0)}}{\hbar\omega_0-\left(\varepsilon_{1i}+\varepsilon_{2j}\right)+\mathrm{i}_0}\left|B_{12ij}\right|^2-$$
$$2g_1^{(2)}\sum_{ij}\frac{1+f_{1i}^{(0)}+f_{1j}^{(0)}}{\hbar\omega_0+\left(\varepsilon_{1i}+\varepsilon_{1j}\right)+\mathrm{i}_0}\left|\tilde{B}_{1ij}\right|^2-2g_{12}^{(2)}\sum_{ij}\frac{1+f_{2i}^{(0)}+f_{2j}^{(0)}}{\hbar\omega_0+\left(\varepsilon_{2i}+\varepsilon_{2j}\right)+\mathrm{i}_0}\left|\tilde{B}_{2ij}\right|^2-2g_{12}^{(2)}\sum_{ij}\frac{1+f_{1i}^{(0)}+f_{2j}^{(0)}}{\hbar\omega_0+\left(\varepsilon_{1i}+\varepsilon_{2j}\right)+\mathrm{i}_0}\left|\tilde{B}_{12ij}\right|^2。\tag{52}$$

考虑到集体激发的弛豫，在式(52)中引入无穷小的虚数 i_0，式(52)等号右边相应分为实部和虚部，分别记为 $\Delta\omega$、$-\mathrm{i}\gamma$，得到一个唯象的公式 $\omega=\omega_0+\Delta\omega-\mathrm{i}\gamma$。不考虑其虚部，$\Delta\omega=\omega-\omega_0$ 恰好是集体激发的频移；不考虑其实部，$\omega=\omega_0-\mathrm{i}\gamma$，代入集体激发的时间因子，得到 $\exp(-\mathrm{i}\omega t)=\exp(-\gamma t)\times\exp(-\mathrm{i}\omega_0 t)$。可以看出，$\gamma$ 是集体激发振幅衰减因子(阻尼系数)。

式(52)等号右边的第 1～4 项的虚部给出朗道机制的阻尼系数 $\gamma_{\mathrm{L}}=\gamma_{\mathrm{L1}}+\gamma_{\mathrm{L2}}+\gamma_{\mathrm{L12}}+\gamma_{\mathrm{L21}}$，第 5～7 项的虚部给出巴利耶夫机制的阻尼系数 $\gamma_{\mathrm{B}}=\gamma_{\mathrm{B1}}+\gamma_{\mathrm{B2}}+\gamma_{\mathrm{B12}}$，其中：

$$\begin{cases}\gamma_{\mathrm{L1}}=4\pi g_1^{(2)}\sum_{ij}|A_{1ij}|^2\left(f_{1i}^{(0)}-f_{1j}^{(0)}\right)\delta\left(\hbar\omega_0+\varepsilon_{1i}-\varepsilon_{1j}\right)\\ \gamma_{\mathrm{L2}}=4\pi g_{12}^{(2)}\sum_{ij}|A_{2ij}|^2\left(f_{2i}^{(0)}-f_{2j}^{(0)}\right)\delta\left(\hbar\omega_0+\varepsilon_{2i}-\varepsilon_{2j}\right)\\ \gamma_{\mathrm{L12}}=4\pi g_{12}^{(2)}\sum_{ij}|A_{12ij}|^2\left(f_{1i}^{(0)}-f_{2j}^{(0)}\right)\delta\left(\hbar\omega_0+\varepsilon_{1i}-\varepsilon_{2j}\right)\\ \gamma_{\mathrm{L21}}=4\pi g_{12}^{(2)}\sum_{ij}|A_{21ij}|^2\left(f_{2i}^{(0)}-f_{1j}^{(0)}\right)\delta\left(\hbar\omega_0+\varepsilon_{2i}-\varepsilon_{1j}\right)\\ \gamma_{\mathrm{B1}}=2\pi g_1^{(2)}\sum_{ij}|B_{1ij}|^2\left(1+f_{1i}^{(0)}+f_{1j}^{(0)}\right)\delta\left(\hbar\omega_0-\varepsilon_{1i}-\varepsilon_{1j}\right)\\ \gamma_{\mathrm{B2}}=2\pi g_{12}^{(2)}\sum_{ij}|B_{2ij}|^2\left(1+f_{2i}^{(0)}+f_{2j}^{(0)}\right)\delta\left(\hbar\omega_0-\varepsilon_{2i}-\varepsilon_{2j}\right)\\ \gamma_{\mathrm{B12}}=2\pi g_{12}^{(2)}\sum_{ij}|B_{12ij}|^2\left(1+f_{1i}^{(0)}+f_{2j}^{(0)}\right)\delta\left(\hbar\omega_0-\varepsilon_{1i}-\varepsilon_{2j}\right)\end{cases}。\tag{53}$$

朗道阻尼和巴利耶夫阻尼分别产生于满足共振条件 $\hbar\omega_0+\varepsilon_i=\varepsilon_j$(一个准粒子吸收一个集体激发变为另一个准粒子)、$\hbar\omega_0=\varepsilon_i+\varepsilon_j$(一个集体激发变为两个准粒子)的跃迁。$\gamma_{\mathrm{L1}}$、$\gamma_{\mathrm{L2}}$、$\gamma_{\mathrm{B1}}$、$\gamma_{\mathrm{B2}}$ 产生于同种准粒子之间的跃迁，γ_{L12}、γ_{L21}、γ_{B12} 产生于不同种准粒子之间的跃迁。

由于满足共振条件 $\hbar\omega_0+\varepsilon_i+\varepsilon_j=0$ 的跃迁不发生，式(52)等号右边的第 8～10 项不予考虑。

3 计算和讨论

本节以 2BECs 均匀系统中集体激发的朗道阻尼为例，通过半经典近似计算，简要说明所构建理论的物理意义和应用方法，并对理论构建中关键的近似问题进行对比讨论。

3.1 2BECs 均匀系统中集体激发的朗道阻尼

为了进行解析计算，低温下进一步采用近似 $\tilde{m}_{1(2)}^{(0)}=0$、$\tilde{n}_{1(2)}^{(0)}=0$，则式(33)变为

$$\begin{cases}L_{1(2)}u_{1(2)j}+\left(g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{1(2)j}=\\ \quad\varepsilon_{1(2)j}u_{1(2)j}\\ L_{1(2)}v_{1(2)j}+\left(g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{1(2)j}=\\ \quad-\varepsilon_{1(2)j}v_{1(2)j}\end{cases},\tag{54}$$

式(31)变为

$$\mathcal{L}_{1(2)}=H_{1(2)0}+2g_{1(2)}n_{1(2)0}+g_{12}n_{2(1)0}+g_{12}\sqrt{n_{10}n_{20}},\tag{55}$$

式(50)变为

$$\begin{cases}\mathcal{L}_0\delta\Phi_{11}^{(0)}+g_1n_{10}\delta\Phi_{12}^{(0)}=\varepsilon_q\delta\Phi_{11}^{(0)}\\ \mathcal{L}_0\delta\Phi_{12}^{(0)}+g_1n_{10}\delta\Phi_{11}^{(0)}=-\varepsilon_q\delta\Phi_{12}^{(0)}\end{cases}。\tag{56}$$

将 $\hbar\omega_0$ 改写为 ε_q(ε_q 为集体激发能量)，则式(14)变为

$$\mathcal{L}_0=H_{10}+2g_1n_{10}+g_{12}n_{20}。\tag{57}$$

研究囚禁势 $V_{1(2)}=0$ 的均匀系统，其凝聚体基态波函数 $\phi_{1(2)0}=\sqrt{n_{1(2)0}}=\sqrt{\frac{N_{1(2)0}}{V}}$(其中 $N_{1(2)0}$ 为凝聚部分的粒子数，V 为系统体积)在整个空间是一个常数，低温下忽略粒子动能和非凝聚部分粒子数，系统能量近似为 $E=\frac{N_{10}(N_{10}-1)g_1}{2V}+\frac{N_{20}(N_{20}-1)g_2}{2V}+\frac{N_{10}N_{20}g_{12}}{V}$，单粒子化学式 $\mu_{1(2)}=\frac{\partial E}{\partial N_{1(2)0}}=n_{1(2)0}g_{1(2)}+n_{2(1)0}g_{12}$，根据式(2)和 $V_{1(2)}=0$，将式(55)和式(57)分别改写为

$$\mathcal{L}_{1(2)}=-\frac{\hbar^2\nabla_{1(2)}^2}{2m_{1(2)}}+g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}},\tag{58}$$

$$\mathcal{L}_0=-\frac{\hbar^2\nabla_1^2}{2m_1}+g_1n_{10}。\tag{59}$$

均匀系统的准粒子激发和集体激发可以用平面波函数分别表示为

$$\begin{cases}u_{1(2)\boldsymbol{p}}(\boldsymbol{r})=\dfrac{u_{1(2)}}{\sqrt{V}}\exp\left(\mathrm{i}\dfrac{\boldsymbol{p}\cdot\boldsymbol{r}}{\hbar}\right)\\ v_{1(2)\boldsymbol{p}}(\boldsymbol{r})=\dfrac{v_{1(2)}}{\sqrt{V}}\exp\left(\mathrm{i}\dfrac{\boldsymbol{p}\cdot\boldsymbol{r}}{\hbar}\right)\end{cases},\tag{60}$$

$$\begin{cases}\delta\phi_{11}^{(0)}(\boldsymbol{r})=\dfrac{u_q}{\sqrt{V}}\exp\left(\mathrm{i}\dfrac{\boldsymbol{q}\cdot\boldsymbol{r}}{\hbar}\right)\\ \delta\phi_{12}^{(0)}(\boldsymbol{r})=\dfrac{v_q}{\sqrt{V}}\exp\left(\mathrm{i}\dfrac{\boldsymbol{q}\cdot\boldsymbol{r}}{\hbar}\right)\end{cases},\tag{61}$$

式中：$\boldsymbol{p}$ 和 $\boldsymbol{q}$ 分别为准粒子和集体激发的动量。

将式(60)代入式(54)可得

$$\begin{cases}\left(\xi_{1(2)}-\varepsilon_{1(2)}\right)u_{1(2)}+\left(g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)v_{1(2)}=0\\ \left(\xi_{1(2)}+\varepsilon_{1(2)}\right)v_{1(2)}+\left(g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)u_{1(2)}=0\end{cases},\tag{62}$$

式中：$\xi_{1(2)}=\dfrac{p^2}{2m_{1(2)}}+g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}$，其中 p 为动量 $\boldsymbol{p}$ 的大小。

将式(61)代入式(56)可得

$$\begin{cases}\left(\xi_0-\varepsilon_q\right)u_0+g_1n_{10}v_0=0\\ \left(\xi_0+\varepsilon_q\right)v_0+g_1n_{10}u_0=0\end{cases},\tag{63}$$

式中：$\xi_0=\dfrac{q^2}{2m_1}+g_1n_{10}$，其中 q 为动量 $\boldsymbol{q}$ 的大小。

根据式(62)可以得到准粒子激发的能量本征值：

$$\varepsilon_{1(2)}=\sqrt{\left(\frac{p^2}{2m_{1(2)}}+g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)^2-\left(g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)^2}。\tag{64}$$

根据式(63)可以得到集体激发的能量本征值：

$$\varepsilon_q=\sqrt{\left(\frac{q^2}{2m_1}+g_1n_{10}\right)^2-\left(g_1n_{10}\right)^2}。\tag{65}$$

再根据波戈留波夫归一化条件 $u_{1(2)}^2-v_{1(2)}^2=1$，得到准粒子激发的波戈留波夫振幅：

$$\begin{cases}u_{1(2)}(\varepsilon)=\sqrt{\dfrac{1}{2}\left[\sqrt{1+\dfrac{\left(g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)^2}{\varepsilon^2}}+1\right]}\\ v_{1(2)}(\varepsilon)=-\sqrt{\dfrac{1}{2}\left[\sqrt{1+\dfrac{\left(g_{1(2)}n_{1(2)0}+g_{12}\sqrt{n_{10}n_{20}}\right)^2}{\varepsilon^2}}-1\right]}\end{cases}。\tag{66}$$

最后根据波戈留波夫归一化条件 $u_0^2-v_0^2=1$，得到集体激发的波戈留波夫振幅：

$$\begin{cases}u_q=\sqrt{\dfrac{1}{2}\left[\sqrt{1+\dfrac{\left(g_1n_{10}\right)^2}{\varepsilon_q^2}}+1\right]}\\ v_q=-\sqrt{\dfrac{1}{2}\left[\sqrt{1+\dfrac{\left(g_1n_{10}\right)^2}{\varepsilon_q^2}}-1\right]}\end{cases}。\tag{67}$$

对于平面波，首先采用玻恩-卡曼(Born Kaman)条件 $\exp\left(\dfrac{\mathrm{i}}{\hbar}p_\zeta\zeta\right)=\exp\left[\dfrac{\mathrm{i}}{\hbar}p_\zeta(\zeta+L)\right]$，则有 $p_\zeta=n_\zeta\dfrac{h}{L}$，其中 $n_\zeta=0,1,2,\cdots$，$\zeta=x,y,z$，h 为普朗克常数，L 为系统的尺寸，且有 $L^3=V$，故在 $p\to p+\mathrm{d}p$ 内准粒子状态数为 $\dfrac{4\pi p^2\mathrm{d}p}{(h/L)^3}$，可将求和运算 $\sum_{ij}$ 替换为积分运算 $\iint\dfrac{V4\pi p^2\mathrm{d}p}{h^3}\dfrac{V4\pi p'^2\mathrm{d}p'}{h^3}$ 以改写式(53)，改写后用两个准粒子的动量 p 和 p' 代替 i 和 j 来标识阻尼机制中的 2 个准粒子；其次，根据式(64)给出的 p 和 ε 的关系，进一步改写式(53)，改写后用两个准粒子的能量 ε 和 ε' 代替 p 和 p' 来标识准粒子；然后，利用 δ 函数归一化的性质，进一步改写式(53)，改写后用 ε 和 $\varepsilon+\varepsilon_q$ 代替 ε 和 ε' 来标识准粒子；最后，引入 $x=\varepsilon/(kT)$、$x_q=\varepsilon_q/(kT)$、$\tau=kT/(n_{10}g_1)$，令 $\lambda_1=m_2/m_1$、$\lambda_2=n_{20}/n_{10}$、$\lambda_3=g_2/g_1$、$\lambda_4=g_{12}/g_1$，进一步改写式(53)，得到阻尼率

$$\frac{\gamma_{\mathrm{L}}}{\varepsilon_q}=\frac{128\pi^3V}{h^6}m_1^3n_{10}^2g_1^3F_{\mathrm{L}}(\tau),\tag{68}$$

式中：$F_{\mathrm{L}}(\tau)=F_{\mathrm{L1}}(\tau)+F_{\mathrm{L2}}(\tau)+F_{\mathrm{L12}}(\tau)+F_{\mathrm{L21}}(\tau)$。

$$\begin{cases}F_{\mathrm{L1}}(\tau)=\tau\int\left|M_{1x(x+x_q)}\right|^2\left(f_x^{(0)}-f_{x+x_q}^{(0)}\right)\rho_{1x}\,\mathrm{d}x\\ F_{\mathrm{L2}}(\tau)=\lambda_1^3\lambda_4^2\tau\int\left|M_{2x(x+x_q)}\right|^2\left(f_x^{(0)}-f_{x+x_q}^{(0)}\right)\rho_{2x}\,\mathrm{d}x\\ F_{\mathrm{L12}}(\tau)=\lambda_1^{\frac{3}{2}}\lambda_2\lambda_4^2\tau\int\left|M_{12x(x+x_q)}\right|^2\left(f_x^{(0)}-f_{x+x_q}^{(0)}\right)\rho_{12x}\,\mathrm{d}x\\ F_{\mathrm{L21}}(\tau)=\lambda_1^{\frac{3}{2}}\lambda_2\lambda_4^2\tau\int\left|M_{21x(x+x_q)}\right|^2\left(f_x^{(0)}-f_{x+x_q}^{(0)}\right)\rho_{21x}\,\mathrm{d}x\end{cases},\tag{69}$$

式中：$F_{\mathrm{L1}}(\tau)$、$F_{\mathrm{L2}}(\tau)$、$F_{\mathrm{L12}}(\tau)$、$F_{\mathrm{L21}}(\tau)$ 为无量纲阻尼函数。用 x 和 $x+x_q$ 代替 ε 和 $\varepsilon+\varepsilon_q$ 来标识阻尼机制中

的2个准粒子。

$$\begin{cases} \rho_{1x} = \dfrac{\rho_{x_q}\rho_1(x)\rho_1(x+x_q)}{\rho_1'(x)\rho_1'(x+x_q)} \\ \rho_{2x} = \dfrac{\rho_{x_q}\rho_2(x)\rho_2(x+x_q)}{\rho_2'(x)\rho_2'(x+x_q)} \\ \rho_{12x} = \dfrac{\rho_{x_q}\rho_1(x)\rho_2(x+x_q)}{\rho_1'(x)\rho_2'(x+x_q)} \\ \rho_{21x} = \dfrac{\rho_{x_q}\rho_2(x)\rho_1(x+x_q)}{\rho_2'(x)\rho_1'(x+x_q)} \end{cases}, \tag{70}$$

$$\begin{cases} f_x^0 = \dfrac{1}{e^x - 1} \\ f_{x+x_q}^0 = \dfrac{1}{\exp(x+x_q) - 1} \end{cases}。 \tag{71}$$

$$\begin{cases} M_{1x(x+x_q)} = u_{x_q}\left[u_1(x)u_1(x+x_q) + v_1(x)u_1(x+x_q) + v_1(x)v_1(x+x_q)\right] + \\ \qquad v_{x_q}\left[u_1(x)u_1(x+x_q) + u_1(x)v_1(x+x_q) + v_1(x)v_1(x+x_q)\right] \\ M_{2x(x+x_q)} = \dfrac{1}{2}\left(u_{x_q} + v_{x_q}\right)\left[u_2(x)u_2(x+x_q) + v_2(x)v_2(x+x_q)\right] \\ M_{12x(x+x_q)} = \dfrac{1}{2}u_{x_q}\left[v_1(x)u_2(x+x_q) + v_1(x)v_2(x+x_q)\right] + v_{x_q}\left[u_1(x)u_2(x+x_q) + u_1(x)v_2(x+x_q)\right] \\ M_{21x(x+x_q)} = \dfrac{1}{2}u_{x_q}\left[u_2(x)u_1(x+x_q) + v_2(x)u_1(x+x_q)\right] + v_{x_q}\left[u_2(x)v_1(x+x_q) + v_2(x)v_1(x+x_q)\right] \end{cases}, \tag{72}$$

式中：$\rho_{x_q} = \sqrt{\dfrac{x}{x_q}\left(\dfrac{x}{x_q}+1\right)}$；$\rho_1(x) = \sqrt{\sqrt{1+\dfrac{\left(1+\lambda_4\lambda_2^{1/2}\right)^2}{\tau^2x^2}} - \dfrac{1+\lambda_4\lambda_2^{1/2}}{\tau x}}$；$\rho_2(x) = \sqrt{\sqrt{1+\dfrac{\left(\lambda_2\lambda_3+\lambda_4\lambda_2^{1/2}\right)^2}{\tau^2x^2}} - \dfrac{\lambda_2\lambda_3+\lambda_4\lambda_2^{1/2}}{\tau x}}$；$\rho_1'(x) = \sqrt{1+\dfrac{\left(1+\lambda_4\lambda_2^{1/2}\right)}{\tau^2x^2}}$；$\rho_2'(x) = \sqrt{1+\dfrac{\left(\lambda_2\lambda_3+\lambda_4\lambda_2^{1/2}\right)^2}{\tau^2x^2}}$；$u_{x_q} = \sqrt{\dfrac{1}{2}\left(\sqrt{1+\dfrac{1}{\tau^2x_q^2}}+1\right)}$；$v_{x_q} = -\sqrt{\dfrac{1}{2}\left(\sqrt{1+\dfrac{1}{\tau^2x_q^2}}-1\right)}$；$u_1(x) = \sqrt{\dfrac{1}{2}\left(\sqrt{1+\dfrac{\left(1+\lambda_4\lambda_2^{1/2}\right)^2}{\tau^2x^2}}+1\right)}$；$v_1(x) = -\sqrt{\dfrac{1}{2}\left(\sqrt{1+\dfrac{\left(1+\lambda_4\lambda_2^{1/2}\right)^2}{\tau^2x^2}}-1\right)}$；$u_2(x) = \sqrt{\dfrac{1}{2}\left(\sqrt{1+\dfrac{\left(\lambda_2\lambda_3+\lambda_4\lambda_2^{1/2}\right)^2}{\tau^2x^2}}+1\right)}$；$v_2(x) = -\sqrt{\dfrac{1}{2}\left(\sqrt{1+\dfrac{\left(\lambda_2\lambda_3+\lambda_4\lambda_2^{1/2}\right)^2}{\tau^2x^2}}-1\right)}$。

考虑^{87}Rb原子（$m_1 = 87\times1.67\times10^{-27}$ kg，$a_1 = 90a_0$[1]，其中 $a_0 = 5.29\times10^{-11}$ m，$g_1 = 4\pi\hbar^2 a_1/m_1 = 4.583\times10^{-51}$ J·m^3）和^{39}K原子（$m_2 = 39\times1.67\times10^{-27}$ kg，$a_2 = 140a_0$[1]，$g_2 = 4\pi\hbar^2 a_2/m_2 = 1.59\times10^{-50}$ J·m^3）组成的两分量系统，并取 $g_{12} = 4.269\times10^{-51}$，满足2个分量相容的条件 $g_{12} < \sqrt{g_1 g_2} = 8.538\times10^{-51}$。

实际实验中的系统粒子数密度 $n \approx 10^{19} - 10^{21}\,\mathrm{m}^{-3}$[1]，集体激发能量 $\varepsilon_q = \hbar\omega \approx 10^{-30} - 10^{-31}$ J，其中 $\omega = 126 - 1444$ Hz[33-36]，取其中间值进行计算。

图1所示为系统粒子数密度 $n_1 = n_2 = 10^{19}\ \mathrm{m}^{-3}$ 的情况下，能量 ε_q 为 10^{-31} J 的集体激发无量纲阻尼函数 $F_{L1}(\tau)$、$F_{L2}(\tau)$、$F_{L12}(\tau)$、$F_{L21}(\tau)$ 随无量纲温度 τ 变化的规律。从图1可以看出，阻尼随温度的增加而增大，4种不同的跃迁对集体激发阻尼的贡献大小不同，温度小时差别大，温度大时差别小。

无量纲温度 $\tau = kT/\left(n_{10}g_1\right)$ 与温度 T 的关系比较复杂，凝聚粒子数密度 $n_{10} = n\left[1-\left(T/T_{c1}\right)^{3/2}\right]$[1]。根

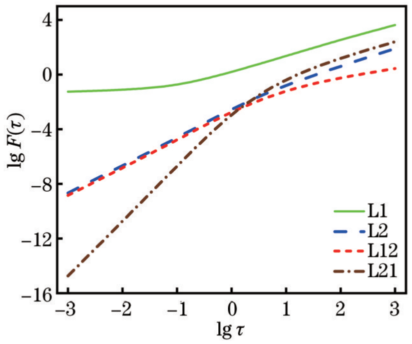

图 1 粒子数 $n_1=n_2=10^{19}\ \mathrm{m^{-3}}$、集体激发能量 $\varepsilon_q=10^{-31}$ J时，$F_{\mathrm{L1}}(\tau)$、$F_{\mathrm{L2}}(\tau)$、$F_{\mathrm{L12}}(\tau)$、$F_{\mathrm{L21}}(\tau)$随τ的变化

Fig. 1 $F_{\mathrm{L1}}(\tau)$, $F_{\mathrm{L2}}(\tau)$, $F_{\mathrm{L12}}(\tau)$, and $F_{\mathrm{L21}}(\tau)$ as a function of τ when the number of particle $n_1=n_2=10^{19}\ \mathrm{m^{-3}}$ and the collective excitation energy $\varepsilon_q=10^{-31}$ J

据相变临界温度公式 $T_{\mathrm{c}}=\dfrac{2\pi}{(2.612)^{2/3}}\dfrac{\hbar^2}{mk}n^{2/3}$[1]，2BECs系统的相变临界温度分别为 $T_{\mathrm{c1}}=8.536\times10^{-8}$ K、$T_{\mathrm{c2}}=1.904\times10^{-7}$ K。当 $\tau\to0$ 时，$n_{10}\to n_1$，$T\to0$；当 $\tau\to\infty$ 时，$n_{10}\to0$，$T\to T_{\mathrm{c1}}$。

以下通过计算展示不同能量范围的粒子跃迁对阻尼的贡献。

将式(69)中无量纲阻尼函数改写为 $F_{\mathrm{L1}}(\tau)=\int_0^\infty f_{\mathrm{L1}}(x)\mathrm{d}x$、$F_{\mathrm{L2}}(\tau)=\int_0^\infty f_{\mathrm{L2}}(x)\mathrm{d}x$、$F_{\mathrm{L12}}(\tau)=\int_0^\infty f_{\mathrm{L12}}(x)\mathrm{d}x$、$F_{\mathrm{L21}}(\tau)=\int_0^\infty f_{\mathrm{L21}}(x)\mathrm{d}x$，其中

$$\begin{cases}f_{\mathrm{L1}}(x)=\tau\left|M_{1x(x+x_q)}\right|^2\left(f_x^0-f_{x+x_q}^0\right)\rho_{1x}\\ f_{\mathrm{L2}}(x)=\lambda_1^3\lambda_4^2\tau\left|M_{2x(x+x_q)}\right|^2\left(f_x^0-f_{x+x_q}^0\right)\rho_{2x}\\ f_{\mathrm{L12}}(x)=\lambda_1^{3/2}\lambda_2\lambda_4^2\tau\left|M_{12x(x+x_q)}\right|^2\left(f_x^{(0)}-f_{x+x_q}^{(0)}\right)\rho_{12x}\\ f_{\mathrm{L21}}(x)=\lambda_1^{3/2}\lambda_2\lambda_4^2\tau\left|M_{21x(x+x_q)}\right|^2\left(f_x^{(0)}-f_{x+x_q}^{(0)}\right)\rho_{21x}\end{cases},\tag{73}$$

从而有 $F_{\mathrm{L}}(\tau)=\int_0^\infty f_{\mathrm{L}}(x)\mathrm{d}x$，其中 $f_{\mathrm{L}}(x)=f_{\mathrm{L1}}(x)+f_{\mathrm{L2}}(x)+f_{\mathrm{L12}}(x)+f_{\mathrm{L21}}(x)$，据此定义误差函数 $\mathrm{erf}(x,\tau)=\int_0^x f_{\mathrm{L}}(x)\mathrm{d}x$。

图 2所示为 $\tau=0.1,1,10$ 时 $\mathrm{erf}(x,\tau)$ 随 x 的变化规律。$x=\varepsilon/(kT)$ 为准粒子的无量纲能量，$x_q=\varepsilon_q/(kT)$ 为集体激发的无量纲能量，则 x/x_q 为准粒子能量相对集体激发能量的大小，误差函数 $\mathrm{erf}(x,\tau)=\int_0^x f(x)\mathrm{d}x$ 是无量纲能量小于 x 的准粒子跃迁对阻尼的贡献。其中，系统粒子数密度取 $n_1=n_2=10^{19}\ \mathrm{m^{-3}}$、集体激发能量取 $\varepsilon_q=10^{-31}$ J。从图 2可以看出：当 x/x_q 增大到星号(*)所示值时，$\mathrm{erf}(x,\tau)$ 达到最大值，最大值就是 $F_{\mathrm{L}}(\tau)$，是所有准粒子跃迁对阻尼的贡献，因此对阻尼的贡献来自 x/x_q 小于星号所示值的准粒子跃迁；温度越大，跃迁对阻尼有贡献的准粒子 x/x_q 越大。

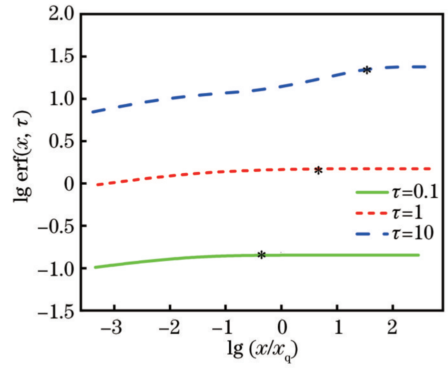

图 2 粒子数 $n_1=n_2=10^{19}\ \mathrm{m^{-3}}$、集体激发能量 $\varepsilon_q=10^{-31}$ J时，$\mathrm{erf}(x,\tau)$随x的变化

Fig. 2 $\mathrm{erf}(x,\tau)$ as a function of x when the number of particle $n_1=n_2=10^{19}\ \mathrm{m^{-3}}$ and the collective excitation energy $\varepsilon_q=10^{-31}$ J

3.2 在 HFB 平均场理论构建中 2BECs 和 1BEC 系统的主要差别

在 2BECs 平均场理论构建中，采用 2 种不同的产生和湮灭准粒子算符 $\alpha_{1(2)j}^\dagger$、$\alpha_{1(2)j}$，分别对 2 个分量的非凝聚部分算符 $\tilde{\psi}_{1(2)}$ 进行波戈留波夫变换；在 1BEC 平均场理论构建中，只需要一种产生和湮灭准粒子算符。

2BECs 系统的组成包括两种不同精细结构的同种原子、同一元素 2 种不同同位素的原子、2 种不同种原子这 3 种情况，其中前 2 种情况是比较简单的特殊情况，本文构建的是针对第 3 种情况的一般理论，可以比较容易地推广到前 2 种情况中。

上述 3 种情况的 HFB 平均场理论构建中，由于 2BECs 系统的 2 种粒子不完全相同，在进行波戈留波夫变换时，都需要 2 种不同的产生和湮灭准粒子算符，但在研究 2 种不同精细结构的同种原子组成的 2BECs 系统的文献[73]中，只用了一种产生和湮灭准粒子算符。

在本文理论的构建中，完全按照文献[49-50]的方法进行公式推导，唯一不同的是引入了由式(28)给出的近似，这是对式(23)最后一项 $\int g_{12}\sqrt{n_{10}n_{20}}\left(\tilde{\psi}_1^\dagger\tilde{\psi}_2+\tilde{\psi}_2^\dagger\tilde{\psi}_1+\tilde{\psi}_1\tilde{\psi}_2+\tilde{\psi}_1^\dagger\tilde{\psi}_2^\dagger\right)\mathrm{d}\boldsymbol{r}$ 进行的近似，这一项是 2 个分量非凝聚算符的交叉项。由于分别采用 2 种不同的产生和湮灭准粒子算符，对 2 个分量的非凝聚部分算符进行波戈留波夫变换后，产生 2 种不同的产生和湮灭准粒子算符的交叉项，巨正则哈密顿无法对角化。引入由式(28)给出的近似，上述 2 个分量的非凝聚部分算符交叉项变为正交项 $g_{12}\sqrt{n_{10}n_{20}}\left(\tilde{\psi}_1^\dagger\tilde{\psi}_1+\tilde{\psi}_2^\dagger\tilde{\psi}_2+\frac{1}{2}\tilde{\psi}_1\tilde{\psi}_1+\frac{1}{2}\tilde{\psi}_2\tilde{\psi}_2+\frac{1}{2}\tilde{\psi}_1^\dagger\tilde{\psi}_1^\dagger+\frac{1}{2}\tilde{\psi}_2^\dagger\tilde{\psi}_2^\dagger\right)\mathrm{d}\boldsymbol{r}$，经波戈留波夫变换后，不产生 2 种不同的产生和湮灭准粒子算符的交叉项，巨正则哈密顿得到

对角化。

如前所述，文献[74]也构建了由2种不同种原子组成的2BECs平均场理论，但它缺少以均匀系统为例的半经典近似计算。除此之外，它完全忽略了上述2个分量非凝聚部分算符的交叉项。本文对文献[74]的工作进行改进，无论是文献[74]忽略上述2个分量非凝聚部分算符的交叉项，还是本文对此交叉项作近似，目的都是将巨正则哈密顿对角化，以得到准粒子的BdG方程，但本文的方法更为合理。

4 总　结

采用HFB近似方法构建研究2BECs中集体激发阻尼的平均场理论框架。在公式推导过程中，沿用已有的单分量相关理论构建方法，并且严格按照单分量相关理论文献进行表述。研究表明，尽管单分量理论框架的构建非常复杂，但是两分量理论框架的构建更为复杂且需要引入更多的近似。本文把研究BEC中集体激发阻尼的HFB平均场理论成功地从单分量系统推广到两分量系统，对原有的平均场理论适用性进行了推广。以均匀系统为例进行集体激发朗道阻尼的半经典近似计算，分析了各种跃迁对阻尼系数的贡献及其与温度的依赖关系，分析了不同能量范围的准粒子对阻尼系数的贡献，分别用无量纲阻尼函数和误差函数展示粒子相互作用细节。由于集体激发的阻尼是研究量子多体物理长期且重要的课题之一，并且基于2BECs系统具有丰富物理性质的事实，本文关于HFB平均场理论应用的拓展，预期可以为相关研究工作的顺利开展提供思路。在早期的磁囚禁实验中，集体激发阻尼作为BEC实现的证据，其在激发之间的耦合相互作用受到了广泛关注，因此开展两分量系统集体激发阻尼的实验研究也对理解量子多体物理本质具有重要作用。由于磁囚禁中的轴对称系统非常复杂，而箱囚禁中的均匀系统在早期的1BEC集体激发阻尼的理论研究中作为一个容易计算的简单、理想的例子，目前箱囚禁中的1BEC集体激发的阻尼已有实验研究，随着冷原子技术的进一步发展，箱囚禁中的2BECs集体激发阻尼的实验研究工作也会开展。希望本文的理论研究能对有关2BECs实验工作的开展起到一定的推动作用。

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Damping of Collective Excitations in Two-Component Bose-Einstein Condensates Using Mean-Field Description

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Abstract

Objective The damping of collective excitations in two-component Bose-Einstein condensates (BECs) is studied. The damping includes Landau and Baliaev mechanisms. Elementary excitation in BECs is the basic subject of statistical physics and condensed matter physics. With the development of Feshbach resonance technology in ultra-cold atomic gases and the use of highly controllable ultra-cold quantum systems, significant progress has been made in related research. The attenuation of collective excitation amplitude is called damping, which is generated by the interaction between particles. Damping is an important feature of low-energy collective excitation in the BEC experiment. Accurate calculation of damping is very important for understanding the essence of quantum multi-body physics. Since the damping of collective excitation is one of the long-term and important topics in the study of quantum multi-body physics, and the two-component BEC system has rich physical properties, the application of Hartree-Fock-Bogoliubov (HFB) mean-field theory in this paper is extended, so as to provide ideas for the smooth development of related research work.

Methods HFB mean-field theory is used, and the theoretical framework of the two-component system is constructed based on the original work of the one-component system theory. The collective excitation damping formula is derived strictly according to the original work method, including the Bogoliubov-de Gennes equations of non-condensed quasi-particles obtained by diagonalizing the giant canonical Hamiltonian. The three-mode coupling matrix element describing the interaction between particles is obtained by commutation calculation and Fourier transform of normal and abnormal quasi-particle distribution function motion equations, and the relation for the perturbed eigenfrequency and the damping rate of collective excitation are obtained by Fourier transform of collective excitation motion equations. Landau damping of collective excitation in a continuous two-component BEC homogeneous system with an ideal energy level is taken as an example, and a semi-classical approximate calculation is carried out to show the details of particle interaction. In addition, the physical significance and application method of the theory are explained.

Results and Discussions The main differences between the two-component system and the one-component system in the construction of the theoretical framework are analyzed. The two-component system includes three cases: two kinds of atoms of the same kind with different fine structures, two kinds of atoms of different isotopes of the same element, and two kinds of atoms of different kinds. The first two cases are not exactly the same. In the construction of the mean-field theory of two-component BECs, two different quasi-particle generation and annihilation operators are used respectively, and the Bogoliubov transform is applied to the non-condensed part operators of the two components, respectively. Although the construction of the one-component theoretical framework is complex, that of the two-component theoretical framework is more complicated. Since two different quasi-particle generation and annihilation operators are used respectively, the cross terms of two different quasi-particle generation and annihilation operators are generated after the Bogoliubov transform of the non-condensed part operators of the two components, and more approximations than the construction of the one-component theoretical framework are needed to introduce and thus diagonalize the giant canonical Hamiltonian. In the semi-classical approximate calculation process of Landau damping of collective excitation in a two-component BEC homogeneous system, the dependence of Landau damping on temperature is analyzed, and the quasi-particle transitions that contribute to damping are analyzed, including various cases between the same type of quasi-particles and between different types of quasi-particles, which are expressed by dimensionless damping functions. The contribution of quasi-particle transitions in different energy ranges to damping is analyzed and expressed by error function.

Conclusions In this paper, HFB mean-field theory for studying collective excitation damping in BEC is successfully extended from a one-component system to a two-component system, and the application of the original mean-field theory is extended. The observation of collective excitation damping is the evidence for the realization of BECs in early magnetic trap experiments, and the theory of related one-component systems has been used to carry out more in-depth research on the coupling interaction between excitations. Similarly, further experimental research on collective excitation damping of two-component systems is also important for understanding the essence of quantum multi-body physics. Because of the complexity of the axisymmetric system in the magnetic trap, the homogeneous system in the box trap is a simple and ideal example that is easy to be calculated in the early theoretical study of collective excitation damping in one-component BECs. At present, the damping of collective excitation in one-component BECs in the box trap has been experimentally studied. With the further development of cold atom technology, the experimental study of collective excitation damping in two-component BECs in the box trap will also be conducted. It is hoped that the theoretical research in this paper can play a certain role in facilitating experimental work on two-component condensates.

Key words quantum optics; elementary excitation; dimensionless damping function; error function; Bogoliubov transform; Fourier transform