# 厄米-高斯光束对手征介质球的远场散射

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摘要 利用复源点方法将厄米-高斯光束展开为球矢量波函数的形式。基于广义洛伦兹米氏理论,应用手征介质球 与自由空间分界面处电磁场切向连续的边界条件以及球矢量波函数的正交性,得到手征介质球远区散射场展开系 数。研究了厄米-高斯光束对手征介质球的散射特性。数值计算了厄米-高斯光束对手征介质球的远区散射场分 布,分析了波束模式、手征参数和手征球尺寸等对散射特性的影响。 关键词 激光光学;手征介质;厄米高斯模;散射

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## Far-Field Scattering of a Chiral Sphere Located in a Hermite-Gaussian Beam

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**Abstract** The expression of Hermite–Gaussian beam in terms of spherical vector wave functions is given with the complex source point method. Based on the generalized Lorenz–Mie theory, the scattering coefficients are obtained by applying the continuous boundary conditions of tangential components of electromagnetic field at the interface between chiral medium and free space. The scattering of Hermite–Gaussian beam by a chiral sphere is investigated. The angle distribution of far region scattering field by a chiral sphere located in the Hermite–Gaussian beam is numerically simulated. The influences of beam mode, chiral parameter and sphere radius on the scattering properties sections are analyzed in detail.

**Key words** laser optics; chiral media; Hermite–Gaussian mode; scattering **OCIS codes** 290.5850; 140.3430; 160.1585

## 1 引 言

自洛伦兹米氏理论被提出以来,电磁波/光束与粒子的相互作用已经广泛地应用在粒度分析、环境监测 及激光光镊等领域。从波动理论可知,只有基模的高斯光束并不能完全准确地描述空间中任意旁轴场分 布。早在1966年,Kogelnik等<sup>[1]</sup>就提出了波动方程近似解的高阶模,在直角坐标系下表示为厄米-高斯模。 之后学者们相继利用复源点方法对厄米-高斯光束进行了研究。Kim等<sup>[2]</sup>利用复源点法将厄米-高斯光束展 开为球矢量波函数的形式,推导了未知展开系数的具体表达式,研究了厄米-高斯光束对各向同性均匀球的

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散射及辐射力,并给出了物理解释。1984年,Kojima<sup>[3]</sup>研究了厄米-高斯光束对两个平行导体圆柱的散射问题。Yokota等先后研究了厄米-高斯光束对平行介质圆柱<sup>[4]</sup>、理想导体圆柱<sup>[5]</sup>、铁氧体涂覆导体圆柱<sup>[6]</sup>,平行导体圆柱<sup>[7]</sup>及手征球<sup>[8]</sup>的散射问题。Chang等<sup>[9-10]</sup>研究了圆极化的TEM<sub>01</sub>模厄米-高斯光束对均匀球和分层介质球的辐射力和扭矩问题。1998年,Novotny等<sup>[11]</sup>探讨了通过厄米-高斯光束与尖锐的金属杆相互作用产生的散射增强场,实现了高分辨近场光成像。李海英等研究了厄米-高斯光束对多层球<sup>[12]</sup>及各向异性等离子体球<sup>[13]</sup>的散射。Zhang等<sup>[14]</sup>研究了厄米-高斯光束对高斯谢尔模型离散媒质的散射。罗慧等<sup>[15]</sup>基于互谱密度函数和瑞利散射理论,研究了部分相干复宗量厄米-高斯光束对折射率不同的两类粒子的捕获。然而关于厄米-高斯光束与手征介质的散射问题鲜有报道。Yokota等<sup>[8]</sup>研究了厄米-高斯光束对手征介质球的散射,给出了最低阶模和HG<sub>10</sub>模的数值散射结果。

手征媒质作为一种旋光媒质,可以改变电磁波的传播和散射特性,已广泛地应用于手征微带天线、手征 波导,天线罩等方面,而且自然界中的许多大分子和生物结构也都是手征媒质。由于光在手征媒质中传输 时的特殊性质,许多学者对手征介质的散射问题进行了研究。Holzwarth等<sup>[16]</sup>利用米氏理论,探讨了手征介 质球的散射特性。Bohren<sup>[17]</sup>采用分解法研究了手征球和柱对平面波的散射。利用T矩阵方法,Lakhtakia等 <sup>[18]</sup>研究了任意形状的有耗手征散射体问题。Yung等<sup>[19]</sup>给出了手征涂覆导体球对平面波的散射解。Chiu等<sup>[20]</sup> 研究了等离子体柱涂覆手征介质的散射和隐身问题。Wu等<sup>[21]</sup>通过引入瑞卡提-贝塞尔函数对数导数和迭代 公式,克服了大尺寸介质球散射问题计算时的数值溢出问题。同时对手征球的彩虹现象<sup>[22]</sup>,高斯波束对手征 介质球的辐射力和扭矩<sup>[23]</sup>及手征球对零阶贝塞尔波束的散射<sup>[24]</sup>问题进行探讨。Zhai等<sup>[25]</sup>研究了在轴高斯波 束入射手征介质柱的近表面场。刘锦景等<sup>[26]</sup>研究了双层手征结构在光频段的光学特性。

由于厄米-高斯光束和拉盖尔-高斯光束特殊的强度分布,使其具有更高的捕获效率<sup>[27-28]</sup>,同时,拉盖尔-高斯光束可以表示为厄米-高斯光束的叠加形式<sup>[29-30]</sup>,所以研究厄米-高斯光束对手征粒子的散射是研究厄 米-高斯光束及拉盖尔-高斯光束对手征粒子进行光学操控的基础。本文在厄米-高斯光束的复源点方法展 开基础上,研究了厄米-高斯光束对手征介质球的散射特性,数值计算分析了光束模阶数、手征参数及粒子 尺寸等对高阶厄米-高斯光束散射的影响。所用时谐因子为 exp(-iωt)。

2 理论分析

#### 2.1 厄米-高斯光束的球矢量波函数展开

除了 Davis 波束理论,复源点方法在有形波束的描述方面也是非常重要的一种方法。复源点方法,即对 多极子场中的振荡点源分配复位置坐标,可以将球面波场转化为严格满足波动方程的定向波场。厄米-高 斯光束也可以通过分配复位置坐标给振荡点源来产生。对于复坐标为(-x<sub>0</sub>,-y<sub>0</sub>,-z<sub>0</sub>+ib)的复源点辐射的电 磁场可以用矢量势 A<sub>4</sub>表示为:

$$\boldsymbol{E} = \frac{\mathrm{i}}{\omega \mu_0 \varepsilon_0} \nabla \times \nabla \times \boldsymbol{A}_H \quad , \tag{1}$$

$$H = \frac{1}{\mu_0} \nabla \times A_H \quad , \tag{2}$$

式中 $\varepsilon_0$ , $\mu_0$ 表示自由空间中的介电常数和电导率。

根据Siegman<sup>31</sup>得出的结论,高阶复波束场可以由位于复源点的电磁多极子场得到。uv阶的高阶厄米-高斯光束的矢量势可以表示为:

$$A_{H}^{u,v} = \hat{\mathbf{x}} C \psi_{u,v} = \hat{\mathbf{x}} C \frac{\partial^{u+v}}{\partial x^{u} \partial \gamma^{k}} \frac{\exp(ikR)}{ikR} \quad , \tag{3}$$

式中  $R = [(x + x_0)^2 + (y + y_0)^2 + (z + z_0 - ib)^2]^{1/2}$  为观察点 (x, y, z) 与复源点  $(-x_0, -y_0, -z_0 + ib)$  之间的距离,  $k_0$  是自由空 间波数,  $\hat{x}$  是极化方向的单位矢量。C为归一化常数,  $b = k_0 w_0^2/2$ ,  $w_0$  是光束的束腰半径。

对于高阶光束,在用球矢量波函数表示之前可用球标量函数展开为[32]:

$$\psi_{uv} = \sum_{n=0}^{\infty} \sum_{m=-n}^{n} H_{u,v}^{(1)}(n,m) f_{mn}^{(1)}, \qquad |r| < |r_0 \exp(\pm i\Delta)| \quad , \tag{4}$$

式中  $f_{mn}^{(1)} = j_n(kr)P_n^{|m|}(\cos\theta)\exp(im\phi), r_0 = [x_0^2 + y_0^2 + (z_0 - ib)^2]^{1/2}, \Delta = \arccos\frac{xx_0 + yy_0 + z(z_0 + ib)}{rr_0}, P_n^{|m|}(\cos\theta)$ 是连带勒让德函数,  $\phi = \arctan y/x_0$ 

为了推导电磁场的球矢量波函数展开形式,先设矢量势的球矢量波函数展开为凹:

$$\boldsymbol{A}_{H}^{u,v} = C \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ \boldsymbol{\alpha}^{(1)}(u,v,n,m) \boldsymbol{M}_{mn}^{(1)} + \boldsymbol{\beta}^{(1)}(u,v,n,m) \boldsymbol{N}_{mn}^{(1)} \right],$$
(5)

根据  $M_{mn}^{(1)}$  和  $N_{mn}^{(1)}$  的正交性,通过计算内积,可以得到展开系数  $\alpha^{(1)}(u,v,n,m)$  和  $\beta^{(1)}(u,v,n,m)$  的表达式为<sup>[12]</sup>:

$$\boldsymbol{\alpha}^{(1)}(u,v,n,m) = \frac{1}{C} \frac{\left\langle \boldsymbol{A}_{H}^{u,v} \middle| \boldsymbol{M}_{mn}^{(0)} \right\rangle}{\left\langle \boldsymbol{M}_{mn}^{(0)} \middle| \boldsymbol{M}_{mn}^{(0)} \right\rangle} = \frac{\left\langle \hat{\boldsymbol{x}} \boldsymbol{\psi}_{uv} \middle| \boldsymbol{M}_{mn}^{(0)} \right\rangle}{\left\langle \boldsymbol{M}_{mn}^{(0)} \middle| \boldsymbol{M}_{mn}^{(0)} \right\rangle} , \qquad (6)$$

$$\boldsymbol{\beta}^{(i)}(u,v,n,m) = \frac{1}{C} \frac{\left\langle \boldsymbol{A}_{H}^{u,v} \middle| \boldsymbol{N}_{mn}^{(i)} \right\rangle}{\left\langle \boldsymbol{N}_{mn}^{(i)} \middle| \boldsymbol{N}_{mn}^{(i)} \right\rangle} = \frac{\left\langle \hat{\boldsymbol{x}} \boldsymbol{\psi}_{uv} \middle| \boldsymbol{N}_{mn}^{(i)} \right\rangle}{\left\langle \boldsymbol{N}_{mn}^{(i)} \middle| \boldsymbol{N}_{mn}^{(i)} \right\rangle} ,$$
(7)

内积定义为:

$$\left\langle A_{H}^{u,v} \middle| \boldsymbol{M}_{mn}^{(l)} \right\rangle = \int_{0}^{\infty} \mathrm{d}r \int_{0}^{\pi} \sin \theta \,\mathrm{d}\theta \int_{0}^{2\pi} A_{H}^{u,v} \cdot \boldsymbol{M}_{mn}^{(l)^{*}} \mathrm{d}\phi \quad , \tag{8}$$

因为  $\hat{x} = \sin\theta\cos\phi\hat{r} + \cos\theta\cos\phi\hat{\theta} - \sin\phi\hat{\phi}$ ,将(4)式代入(6)式和(7)式,利用厄米多项式的递推关系<sup>[2]</sup>:

$$H_{u,v+1}(n,m) = \frac{ik}{2} \left\{ \left[ H_{u,v}(n-1,m-1) + (n-m-1)(n-m)H_{u,v}(n-1,m+1) \right] + \frac{1}{2n+3} \left[ H_{u,v}(n+1,m-1) + (n+m+1)(n+m+2)H_{u,v}(n+1,m+1) \right] \right\},$$
(9)

$$H_{u+1,v}(n,m) = \frac{k}{2} \left\{ \frac{1}{2n-1} \left[ -H_{u,v}(n-1,m-1) + (n-m-1)(n-m)H_{u,v}(n-1,m+1) \right] + \frac{1}{2n+3} \left[ -H_{u,v}(n+1,m-1) + (n+m+1)(n+m+2)H_{u,v}(n+1,m+1) \right] \right\}$$
(10)

其初值为:

$$H_{0,0}^{(0)} = \frac{(2n+1)(n-|m|)!}{(n+|m|)!} \mathbf{h}_{n}^{(0)}(kr_{0}) \mathbf{P}_{n}^{|m|}(\cos\theta_{0}) \exp(-\mathrm{i}m\phi_{0}) , \qquad (11)$$

式中 $\phi_0 = \arctan y_0/x_0, \cos \theta_0 = (z_0 - ib)/r_0, h_n^{(l)}(kr_0)$ 表示第一类汉克尔函数。

结合球矢量波函数的正交性,积分计算可得:

$$\alpha^{(l)}(u,v,n,m) = \frac{i}{2n(n+1)} [H_{u,v}^{(l)}(n,m-1) + (n-m)(n+m+1)H_{u,v}^{(l)}(n,m+1)], \qquad (12)$$

$$\beta^{(i)}(u,v,n,m) = \frac{i}{2n(n+1)} \left\{ \frac{(n+1)}{(2n-1)} [H^{(i)}_{u,v}(n-1,m-1) - (n-m)(n-m-1)H^{(i)}_{u,v}(n-1,m+1)] + \frac{n}{(2n+3)} [H^{(i)}_{u,v}(n+1,m-1) - (n+m+2)(n+m+1)H^{(i)}_{u,v}(n+1,m+1)] \right\}$$
(13)

当厄米-高斯光束阶数为u=v=0,即退化为基模高斯波束,若波束束腰中心宽度 $w_0 \rightarrow \infty$ 时,此时高斯波束退化为平面波,波束展开系数只有在 $m=\pm 1$ 时有值,

$$\alpha^{(l)}(0,0,n,1) = \beta^{(l)}(0,0,n,1) = C \frac{\exp(kb)}{kb} \frac{2n+1}{2n(n+1)} \mathbf{i}^{n-1} \quad , \tag{14}$$

$$\alpha^{(0)}(0,0,n,-1) = -\beta^{(0)}(0,0,n,-1) = C \frac{\exp(kb)}{kb} \frac{2n+1}{2} i^{n-1} , \qquad (15)$$

对比平面波的矢量球波函数展开系数[33],可以得到归一化波束因子为:

$$C = \frac{kb}{\exp(kb)} \quad , \tag{16}$$

将(12)式和(13)式的系数代入到(5)式中,根据(1)式和(2)式可以得到入射厄米-高斯光束的球矢量波函数展开 形式为: 光学学报

$$\boldsymbol{E}_{H}^{i} = i\omega C \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ \alpha^{(l)}(u,v,n,m) \boldsymbol{M}_{mn}^{(l)}(r,k_{0}) + \boldsymbol{\beta}^{(l)}(u,v,n,m) \boldsymbol{N}_{mn}^{(l)}(r,k_{0}) \right] \quad , \tag{17}$$

$$\boldsymbol{H}_{H}^{i} = i\omega C \frac{k_{0}}{i\omega\mu_{0}} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ \boldsymbol{\beta}^{(l)}(u,v,n,m) \boldsymbol{M}_{mn}^{(l)}(r,k_{0}) + \boldsymbol{\alpha}^{(l)}(u,v,n,m) \boldsymbol{N}_{mn}^{(l)}(r,k_{0}) \right].$$
(18)

#### 2.2 手征球内场及散射场展开

对任意的手征介质,其本构关系表示有很多种定义,这里采用Condon-Rosenfeld本构关系[18]:

$$\begin{cases} D = \varepsilon_c E + i\kappa \sqrt{\varepsilon_0 \mu_0} H \\ B = -i\kappa \sqrt{\varepsilon_0 \mu_0} E + \mu_c H \end{cases},$$
(19)

式中 $\varepsilon_{e},\mu_{e},\kappa$ 为手征介质的介电常数、磁导率和手征参数,可以为任意复数, $\varepsilon_{0},\mu_{0}$ 是真空中的介电常数和磁导率。

由于在手征媒质中任意电磁波会被分解为波数为 k<sub>R</sub>的右旋圆极化波和波数为 k<sub>L</sub>的左旋圆极化波。根据 Bohren<sup>117</sup>的方法,手征介质球内部的电磁场的球矢量波函数展开表达式为:

$$\boldsymbol{E}^{1} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{mn} \boldsymbol{M}_{mn}^{(0)}(r,k_{R}) + A_{mn} \boldsymbol{N}_{mn}^{(0)}(r,k_{R}) + B_{mn} \boldsymbol{M}_{mn}^{(0)}(r,k_{L}) - B_{mn} \boldsymbol{N}_{mn}^{(0)}(r,k_{L}) \right],$$
(20)

$$\boldsymbol{H}^{1} = Q \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{mn} \boldsymbol{N}_{mn}^{(1)}(r, k_{R}) + A_{mn} \boldsymbol{M}_{mn}^{(1)}(r, k_{R}) + B_{mn} \boldsymbol{N}_{mn}^{(1)}(r, k_{L}) - B_{mn} \boldsymbol{M}_{mn}^{(1)}(r, k_{L}) \right],$$
(21)

式中 Ama 表示右旋圆极化波的展开系数, Bma 表示左旋圆极化波的展开系数,

根据第一类和第二类汉克尔函数的渐近展开和物理意义,可以得到手征介质球的散射场用球矢量波函数展 开的表达式:

$$\boldsymbol{E}^{s} = \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{mn}^{s} \boldsymbol{M}_{mn}^{(3)}(r,k_{0}) + B_{mn}^{s} \boldsymbol{N}_{mn}^{(3)}(r,k_{0}) \right], \qquad (23)$$

$$\boldsymbol{H}^{s} = \frac{k_{0}}{\mathrm{i}\omega\mu_{0}} \sum_{n=1}^{\infty} \sum_{m=-n}^{n} \left[ A_{mn}^{s} \boldsymbol{N}_{mn}^{(3)}(r,k_{0}) + B_{mn}^{s} \boldsymbol{M}_{mn}^{(3)}(r,k_{0}) \right].$$
(24)

#### 2.3 散射系数求解

在手征球形粒子边界 r=a 处,电磁场的切向分量满足连续性条件:

$$\boldsymbol{E}^{1\theta} = \boldsymbol{E}^{i\theta} + \boldsymbol{E}^{s\theta}, \qquad \boldsymbol{H}^{1\phi} = \boldsymbol{H}^{i\phi} + \boldsymbol{H}^{s\phi} , \qquad (25)$$

将电磁场的入射场,内场和散射场表达式代入(25)式,利用球矢量波函数的正交完备性,联立方程可以得到散 射系数。然而,随着大宗量球贝塞尔函数的阶数增加,直接利用贝塞尔函数的递推公式,由截断误差及计算 机的迭代误差和累计误差,将导致计算结果失真,甚至计算发散。这里采用文献中常用的引入瑞卡提-贝塞 尔函数及其对数导数<sup>[21]</sup>的方法,即:

$$D_{n}^{(1)}(z) = \psi_{n}'(z)/\psi_{n}(z), \qquad D_{n}^{(3)}(z) = \xi_{n}'(z)/\xi_{n}(z), \qquad (26)$$

式中 $\psi_n(z) = z j_n(z)$ ,和 $\xi_n(z) = z h_n^{(0)}(z)$ 分别表示第一和第三类瑞卡提-贝塞尔函数,可以避免数值计算误差。可以求得散射系数为:

$$A_{mn}^{s} = A_{n}^{sa} a_{mn}^{ix} + A_{n}^{sb} b_{mn}^{ix} , \qquad (27)$$

$$B_{mn}^{s} = B_{n}^{sa} a_{mn}^{ix} + B_{n}^{sb} b_{mn}^{ix} , \qquad (28)$$

式中:

$$A_{n}^{sa} = \frac{\psi_{n}(x_{0})}{\xi_{n}(x_{0})} \frac{\frac{D_{n}^{(0)}(x_{n}) - \eta_{n} D_{n}^{(0)}(x_{0})}{\eta_{n} D_{n}^{(0)}(x_{n}) - D_{n}^{(3)}(x_{0})} + \frac{D_{n}^{(0)}(x_{L}) - \eta_{n} D_{n}^{(0)}(x_{0})}{\eta_{n} D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{L}) - D_{n}^{(3)}(x_{0})}{\frac{\eta_{n} D_{n}^{(3)}(x_{0}) - D_{n}^{(0)}(x_{n})}{\eta_{n} D_{n}^{(0)}(x_{n}) - D_{n}^{(3)}(x_{0})} + \frac{\eta_{n} D_{n}^{(3)}(x_{0}) - D_{n}^{(0)}(x_{L})}{\eta_{n} D_{n}^{(0)}(x_{n}) - D_{n}^{(3)}(x_{0})} + \frac{\eta_{n} D_{n}^{(3)}(x_{0}) - D_{n}^{(0)}(x_{L})}{\eta_{n} D_{n}^{(0)}(x_{L}) - D_{n}^{(3)}(x_{0})},$$
(29)

$$\frac{\mathcal{H}}{B_{n}^{sa}} = A_{n}^{sb} = \frac{\psi_{n}(x_{0})}{\xi_{n}(x_{0})} \frac{\frac{\eta_{r}D_{n}^{(0)}(x_{R}) - D_{n}^{(0)}(x_{0})}{\eta_{r}D_{n}^{(0)}(x_{R}) - D_{n}^{(0)}(x_{0})} - \frac{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})}{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})} + \frac{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})}{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})} + \frac{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})}{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})} + \frac{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})}{\eta_{r}D_{n}^{(0)}(x_{L}) - D_{n}^{(0)}(x_{0})}$$
(30)

$$B_{n}^{sb} = \frac{\psi_{n}(x_{0})}{\xi_{n}(x_{0})} \frac{\frac{\eta_{r}D_{n}^{(*)}(x_{R}) - D_{n}^{(*)}(x_{0})}{D_{n}^{(0)}(x_{R}) - \eta_{r}D_{n}^{(3)}(x_{0})} + \frac{\eta_{r}D_{n}^{(*)}(x_{L}) - D_{n}^{(*)}(x_{0})}{D_{n}^{(0)}(x_{L}) - \eta_{r}D_{n}^{(3)}(x_{0})} + \frac{\eta_{r}D_{n}^{(*)}(x_{L}) - \eta_{r}D_{n}^{(3)}(x_{0})}{D_{n}^{(0)}(x_{R}) - \eta_{r}D_{n}^{(0)}(x_{R})} + \frac{D_{n}^{(3)}(x_{0}) - \eta_{r}D_{n}^{(0)}(x_{L})}{D_{n}^{(0)}(x_{L}) - \eta_{r}D_{n}^{(3)}(x_{0})} ,$$
(31)

$$a_{mn}^{ix} = i\omega C \alpha^{(l)}(u, v, n, m), b_{mn}^{ix} = i\omega C \beta^{(l)}(u, v, n, m) , \qquad (32)$$

 $\vec{x} \oplus x_0 = ka, x_R = k_R a, x_L = k_L a, \eta_r = \sqrt{\varepsilon_0/\mu_0} / \sqrt{\varepsilon_c/\mu_c}$ 

根据解方程得到的散射系数,散射场可以表示为:

$$\sigma = \lim_{r \to \infty} 4\pi r^2 \left| E^s \right|^{r} \left| E^{i}_{H} \right|^{2} = \frac{4\pi}{k_0^2} \left[ \left| \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (-i)^n \exp(im\phi) \exp(im\phi) (mA^s_{mn}\pi_{mn} + B^s_{mn}\pi_{mn}) \right|^{2} + \left| \sum_{n=1}^{\infty} \sum_{m=-n}^{n} (-i)^{n+1} \exp(im\phi) (A^s_{mn}\pi_{mn} + mB^s_{mn}\pi_{mn}) \right|^{2} \right] , \quad (33)$$

式中 $E_{\mu}^{i}$ 是入射场。

## 3 数值计算结果分析

基于前面的散射理论计算了厄米-高斯光束对手征介质球的散射特性。下面的计算中,入射波长  $\lambda = 0.6328 \mu m$ ,厄米-高斯光束的束腰半径取 $w_0 = \lambda$ ,手征介质球位于光束束腰中心位置 $(x_0, y_0, z_0) = (0.0, 0.0, 0.0)\lambda$ 。HG<sub>w</sub>表示厄米-高斯光束沿x方向和y方向的模阶数分别为u和v。

图 1(a) 计算了厄米-高斯光束的 HG<sub>01</sub>的球矢量波函数展开形式的展开系数的收敛性。从图 1 (a)可以看出,随着阶数 n 的增加,展开系数 α(u,v,n,m),β(u,v,n,m) 收敛较快,证明了文中推导的展开系数的正确性。图 1 (b)给出了厄米-高斯光束退化为平面波时对手征球的散射与文献[34]中图 2 的对比。可以看出结果吻合得较好,证明了理论和程序的正确性。





Fig.1 (a) Convergence of Hermite-Gaussian beam expansion coefficients; (b) comparison between the result of Hermite-Gaussian beam degenerated into plane wave and that of Ref.[34]

图 2 计算了 厄米-高斯光束的模阶数对散射场角分布的影响,手征介质球的介电参数、磁导率和手征参数分别为  $\varepsilon_e = (3.25 + i0.2)\varepsilon_0$ ,  $\mu_e = (1.2 + i0.3)\mu_0$ ,  $\xi_e = (0.6 + i0.1)(\varepsilon_0\mu_0)^{1/2}$ 。由于随着光束模阶数的增加, 厄米-高斯光束不像高斯光束一样轴上强度最大, 当模阶数为奇数时, 沿此方向的轴上强度为零, 模阶数为偶数时, 沿此方向中心轴上存在强度分布的极值, 所以 HG<sub>00</sub>(高斯光束)散射场的前向和后向取最大值, 而高阶的厄米-高斯光束随着光束模阶数增大, 光束宽度增大, 沿 x和 y方向呈瓣状分布, 所以散射场沿各个方向有多个极值, 侧向散射场增大。





Fig.2 Effects of beam mode on the angular distribution of RCS. (a) E-plane; (b) H-plane

图 3 给出了手征参数对厄米-高斯光束入射手征介质球散射场分布特性的影响,其中,A=0.0,B=0.2,C= 0.4,D=0.6。从图 3 中可以看出,在E面,随着手征参数增大,散射截面分布先增大,后减小,且振荡越来越明显。其中,前向和后向散射截面变化不大,侧向影响较大。



图 3 手征参数对散射场角分布的影响。(a) E 面;(b) H 面

Fig.3 Effects of chirality parameters on the angular distribution of RCS. (a) *E*-plane; (b) *H*-plane

图 4 给出了 厄米-高斯光束入射不同尺寸手征介质球的散射场角分布。图 4(a)和图 4(c)分别是 E 面和 H 面的散射场角分布,中间伪色图 4 (b)是 HG<sub>31</sub>模的强度分布。手征球介电常数、磁导率和手征参数分别为:  $\varepsilon_e = (3.25 + i0.2)\varepsilon_0, \mu_e = (1.2 + i0.3)\mu_0, \xi_e = (0.4 + i0.1)(\varepsilon_0\mu_0)^{1/2}$ 。由于对厄米-高斯光束,光斑尺寸不再像高斯光束 一样保持不变  $\left[w_{CB}(z) = w_0 \left(1 + z^2/z_R^2\right)^{1/2}\right]$ ,沿 x 和 y 方向的束腰宽度分别为: $w_{HC}(z)_x = w_{CB}(z)(2u + 1)^{1/2}$ ,  $w_{HC}(z)_y = w_{CB}(z)(2v + 1)^{1/2}$ 。对 HG<sub>31</sub>阶模,沿 x 和 y 方向束腰宽度分别为  $\sqrt{7}\lambda$  和  $\sqrt{3}\lambda$ ,如图 4 所示。当手征球半



图4 介质球尺寸对散射场角分布的影响及HG31强度分布。(a) E面;(b) HG31强度分布;(c) H面 Fig.4 Effects of sphere radius on the angular distribution of RCS and the intensity distribution of HG31. (a) E-plane; (b) intensity distribution of HG31; (c) H-plane

径大于 2λ 时,沿 y方向已经超出光束的照射区域,当手征球半径大于 3λ 时,沿 x方向已经超出光束的照射区域。所以,在厄米-高斯光束的可照射范围内,随着手征介质球半径增加,前向散射逐渐减小,在 E 面的侧向 出现多个散射极值,后向散射变化较小。

### 4 结 论

利用复源点方法,研究了手征介质球对厄米-高斯光束的远场散射特性。数值分析了厄米-高斯光束的 模阶数、手征参数及粒子尺寸对散射场角分布的影响。结果表明:随着厄米-高斯光束模阶数增大,散射截 面分布不会呈现类似高斯光束入射的分布特性,散射场分布依赖于光束模阶数。随着手征参数的增大,散 射截面先增大后减小,且对E面的侧向影响更显著。该结果可以为厄米-高斯光束对手征生物细胞的辐射 力研究提供一些理论参考。

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