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# 相位损耗腔中 Tavis-Cummings 模型的熵特性\*

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摘要: 研究了 Tavis-Cummings 模型中光场存在相位损耗时,原子-光场系统线性熵、原子线性熵和光场线性熵的 时间演化特性,讨论了光场的不同量子统计性质、不同的衰变常数和两原子的初始状态对各线性熵的影响。结果 表明:光场的不同量子统计性质使得系统线性熵、原子线性熵和光场线性熵均发生明显改变。系统线性熵的演化 与原子的初始状态无关,而原子线性熵的演化较强地依赖于原子的初始状态,但与腔场的相位损耗无关。 关键词: 量子光学; Tavis-Cummings 模型; 相位损耗; 线性熵; 压缩真空场

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## The Tavis-Cummings Model Entropy Property in a Phase Damping Cavity

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**Abstract:** The time evolution of the linear entropies for the atom-field coupling system, the atom and the field in Tavis-Cummings model in the presence of phase damping of field is investigated. The effects of the initial field states, different value of decay constant and the initial atomic states on all the entropies are discussed. The results show that the different properties of the initial field prepared in the cavity can change the characters of all the linear entropies significatly. The evolution of the linear entropy of the system is independent of initial atomic state and independent of the cavity phase damping.

**Key words**: quantum optics; Tavis-Cummings model; phase damping; linear entropy; squeezed vacuum field

### 1 引 言

自 Phoenix 和 Knight 等<sup>[1~4]</sup>将熵理论应用于 量子光学领域以来,光场与原子相互作用系统中熵 的演化规律的研究很快引起了人们的广泛关注,得 到了一系列有重要意义的结果,根据该量子熵理论, 在原子-光场相互作用系统中,如果系统初始处于纯

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态,并且在系统中不存在损耗过程,那么光场(原子) 熵的演化行为反映了光场与原子关联程度的演化特 性,熵越高,关联越强,纠缠程度越高。由 Zurek 等<sup>[5]</sup>的研究表明,线性熵也能很好地用来测量量子 态的纯度,对线性熵的讨论有利于各量子态的纯度 分析。然而,以往基于 Jaynes-Cummings(J-C)模型 的有关原子-光场系统中线性熵的研究,没有考虑原 子间的相互作用。Tavis-Cummings(T-C)模型是描 述光场与原子相互作用的理想模型,研究表明,原子 间的偶极相互作用对光场-原子相互作用系统的量

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子特性具有非常重要的影响<sup>[6,7]</sup>。而在许多实际情况中,光场与原子相互作用过程,总伴随着腔场损耗的出现,因此研究光场与原子相互作用系统必须考虑腔场损耗的存在。Peixoto de Faria等<sup>[8]</sup>研究了单个二能级原子与单模腔场相互作用,在大失谐的情形下,腔场的损耗会导致原子从纯态演化到混合态。Zheng等<sup>[9]</sup>提出,在单模腔场中的两个二能级原子,在大失谐情况下,原子之间的相互作用可导致两个原子演化到 Einstein-Podolsky-Rosen (EPR)态,并且该 EPR 态不随腔场的衰减而发生衰变。Yamaguchi等<sup>[10]</sup>指出,双原子与腔场的非共振相互作用所诱导的原子间的相互作用可用来实现Grover 搜索。但耦合双原子与单模光场相互作用

系统,在腔场存在相位损耗时熵的演化特性,至今尚 未见报到。而腔场损耗对光场与原子相互作用系统 的动力学性质<sup>[11,12]</sup>的影响是明显的,因此,深入研 究腔场的相位损耗对耦合双原子与光场相互作用动 力学行为的影响是十分有意义的。

#### 2 密度算符及线性熵

考虑两个全同的二能级原子与单模光场相互作用,在大失谐条件(即光场一原子失谐量 $\Delta$ 比光场与原子耦合常数g大得多)时,在旋波近似下,系统的有效哈密顿<sup>[9,13]</sup>为( $\hbar$ =1)

$$H = \Omega \sum_{i=1,2} \left[ \left( \left| 1 \right\rangle_{jj} \left\langle 1 \right| aa^{+} - \left| 0 \right\rangle_{jj} \left\langle 0 \right| a^{+} a \right) + \left( s_{1}^{+} s_{2}^{-} + s_{1}^{-} s_{2}^{+} \right) \right], \tag{1}$$

式中 $\Omega = g^2 / \Delta, a^+(a)$ 为光场的产生和湮没算符, $S_i^\pm$ 为第i个原子的赝自旋算符。在相位损耗条件下,系统的密度算符满足

$$\frac{\mathrm{d}}{\mathrm{d}t}\boldsymbol{\rho}(t) = -\mathrm{i}[H,\boldsymbol{\rho}(t)] + L_{\mathrm{c}}\boldsymbol{\rho}(t), \qquad (2)$$

式中 $L_{o}\rho(t)$  描述腔中光场的相位损耗,在有限温度下可写为<sup>[12,14]</sup>

$$L_{c}\boldsymbol{\rho}(t) = \nu [2a^{+} a\boldsymbol{\rho}(t)a^{+} a - a^{+} aa^{+} a\boldsymbol{\rho}(t) - \boldsymbol{\rho}(t)a^{+} aa^{+} a], \qquad (3)$$

式中 $\nu = \nu' k T, \nu'$ 为衰变常数。

将密度算符写成矩阵形式[15]:

$$\boldsymbol{\rho}(t) = \begin{bmatrix} \rho_{1111}(t) & \rho_{1110}(t) & \rho_{1101}(t) & \rho_{1100}(t) \\ \rho_{1011}(t) & \rho_{1010}(t) & \rho_{1001}(t) & \rho_{1000}(t) \\ \rho_{0111}(t) & \rho_{0110}(t) & \rho_{0101}(t) & \rho_{0100}(t) \\ \rho_{0011}(t) & \rho_{0010}(t) & \rho_{0001}(t) & \rho_{0000}(t) \end{bmatrix},$$
(4)

则(2)式可写为

$$\begin{cases} \dot{\rho}_{1111}(t) = -2i\Omega[a^{+} a, \rho_{1111}(t)] + L_{c}\rho_{1111}(t), \\ \dot{\rho}_{1010}(t) = -i\Omega[\rho_{0110}(t) - \rho_{1001}(t)] + L_{c}\rho_{1010}(t), \\ \dot{\rho}_{0101}(t) = -i\Omega[\rho_{1011}(t) - \rho_{0110}(t)] + L_{c}\rho_{0101}(t), \\ \dot{\rho}_{0000}(t) = 2i\Omega[a^{+} a, \rho_{0000}(t)] + L_{c}\rho_{0000}(t), \\ \dot{\rho}_{1101}(t) = \dot{\rho}_{1111}^{*}(t) = -i\Omega[2a^{+} a\rho_{1101}(t) + \rho_{1101}(t) - \rho_{1101}(t)] + L_{c}\rho_{1101}(t), \\ \dot{\rho}_{1101}(t) = \dot{\rho}_{0111}^{*}(t) = -i\Omega[2a^{+} a\rho_{1101}(t) + \rho_{1101}(t) - \rho_{1101}(t)] + L_{c}\rho_{1101}(t), \\ \dot{\rho}_{1100}(t) = \dot{\rho}_{0111}^{*}(t) = -2i\Omega[a^{+} a\rho_{1100}(t) + \rho_{1100}(t)a^{+} a + \rho_{1100}(t)] + L_{c}\rho_{1100}(t), \\ \dot{\rho}_{1001}(t) = \dot{\rho}_{0110}^{*}(t) = -i\Omega[\rho_{0101}(t) - \rho_{1010}(t)] + L_{c}\rho_{1001}, \\ \dot{\rho}_{1000}(t) = \dot{\rho}_{0011}^{*}(t) = -i\Omega[2\rho_{1000}(t)a^{+} a + \rho_{1000}(t) + \rho_{1000}(t)] + L_{c}\rho_{1000}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{1000}(t)] + L_{c}\rho_{1000}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{1000}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{1000}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{0100}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{0100}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{0100}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{0100}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{0100}(t) = \dot{\rho}_{0001}^{*}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a + \rho_{0100}(t) + \rho_{0100}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{0100}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a^{+} h_{0100}(t) + \rho_{0100}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{010}(t) = -i\Omega[2\rho_{0100}(t)a^{+} a^{+} h_{0100}(t) + \rho_{0100}(t)] + L_{c}\rho_{0100}(t), \\ \dot{\rho}_{010}(t) = -i\Omega[2\rho_{010}(t)a^{+} a^{+} h_{010}(t) + \rho_{010}(t)] + L_{c}\rho_{010}(t), \\ \dot{\rho}_{010}(t) = -i\Omega[2\rho_{010}(t) + \rho_{010}(t)$$

这里

$$\left|\psi_{a}^{(j)}(0)\right\rangle = \cos\left(\frac{\theta_{j}}{2}\right)\left|1\right\rangle_{j} + \sin\left(\frac{\theta_{j}}{2}\right)\exp(\mathrm{i}\varphi_{j})\left|0\right\rangle_{j}, \ (0 \leqslant \theta_{j} < \pi, -\pi \leqslant \varphi_{j} < \pi, j = 1, 2)$$
(7)

#### 式中 θ<sub>i</sub>/2 的余弦和正弦分别表示两原子处于基态和激发态的概率幅,而初始光场为任意光场

$$\psi_f(0)\rangle = \sum_n F_n |n\rangle, \qquad (8)$$

由(5)~(8)式,可得

$$\begin{split} C_{11} &= \cos^{2}(\theta_{1}/2)\cos^{2}(\theta_{2}/2), \\ C_{12} &= C_{21}^{*} = \cos^{2}(\theta_{1}/2)\cos(\theta_{2}/2)\sin(\theta_{2}/2)\exp(-i\varphi_{2}), \\ C_{13} &= C_{31}^{*} = \cos^{2}(\theta_{2}/2)\cos(\theta_{1}/2)\sin(\theta_{1}/2)\exp(-i\varphi_{1}), \\ C_{14} &= C_{41}^{*} = \cos(\theta_{1}/2)\sin(\theta_{1}/2)\cos(\theta_{2}/2)\sin(\theta_{2}/2)\exp[-i(\varphi_{1}+\varphi_{2})], \\ C_{22} &= \cos^{2}(\theta_{1}/2)\sin^{2}(\theta_{2}/2), \\ C_{23} &= C_{32}^{*} = \cos(\theta_{1}/2)\sin(\theta_{1}/2)\cos(\theta_{2}/2)\sin(\theta_{2}/2)\exp[-i(\varphi_{1}-\varphi_{2})], \\ C_{24} &= C_{42}^{*} = \sin^{2}(\theta_{2}/2)\cos(\theta_{1}/2)\sin(\theta_{1}/2)\exp(-i\varphi_{1}), \\ C_{33} &= \cos^{2}(\theta_{2}/2)\sin^{2}(\theta_{1}/2), \\ C_{34} &= C_{43}^{*} = \sin^{2}(\theta_{1}/2)\cos(\theta_{2}/2)\sin(\theta_{2}/2)\exp(-i\varphi_{2}), \\ C_{44} &= \sin^{2}(\theta_{1}/2)\sin^{2}(\theta_{2}/2), \end{split}$$

利用(4)式可得原子的约化密度算符<sup>[15]</sup>为

$$\boldsymbol{\rho}_{a}(t) = \operatorname{Tr}_{t}\boldsymbol{\rho}(t) = \sum_{n} |F_{n}|^{2} \langle C_{11} | 1,1 \rangle \langle 1,1 | + \frac{1}{2} [(C_{22} + C_{33}) + (C_{22} - C_{33}) \cos 2\Omega t + i(C_{23} - C_{32}) \sin 2\Omega t] | 1,0 \rangle \langle 1,0 | + \frac{1}{2} [(C_{22} + C_{33}) - (C_{22} - C_{33}) \cos 2\Omega t - i(C_{23} - C_{32}) \sin 2\Omega t] | 0,1 \rangle \langle 0,1 | + C_{44} | 0,0 \rangle \langle 0,0 | +$$

$$\left\{ \frac{1}{2} \left[ (C_{12} + C_{13}) + (C_{12} - C_{13}) \exp(-2i\Omega t) \right] \exp(-2i\Omega t) | 1,1 \rangle \langle 1,0 | + \frac{1}{2} \left[ (C_{12} + C_{13}) - (C_{12} - C_{13}) \exp(-2i\Omega t) \right] \exp(-2i\Omega t) | 1,1 \rangle \langle 0,1 | + C_{14} \exp(-2i\Omega t) \exp(-4in\Omega t) | 1,1 \rangle \langle 0,0 | + \frac{1}{2} \left[ (C_{23} + C_{32}) + (C_{23} - C_{32}) \cos 2\Omega t + i(C_{22} - C_{33}) \sin 2\Omega t \right] | 1,0 \rangle \langle 0,1 | + \frac{1}{2} \left[ (C_{24} - C_{34}) + (C_{24} + C_{34}) \exp(-2i\Omega t) \right] \exp(-2in\Omega t) | 1,0 \rangle \langle 0,0 | + \frac{1}{2} \left[ (C_{34} - C_{24}) + (C_{34} + C_{24}) \exp(-2i\Omega t) \right] \exp(-2in\Omega t) | 0,1 \rangle \langle 0,0 | + h.c. \rangle \right\},$$

利用(4)式可得光场的约化密度算符[15]为

$$\boldsymbol{\rho}_{\mathrm{f}}(t) = \mathrm{Tr}_{\mathrm{a}}\boldsymbol{\rho}(t) = \sum_{m,n} F_{n}F_{m}^{*} \exp\left[-\nu(n-m)^{2}t\right] \times \left\{C_{\mathrm{u}} \exp\left[-2\mathrm{i}(n-m)Ot\right] + C_{\mathrm{u}} + C_{\mathrm{u}} + C_{\mathrm{u}} \exp\left[2\mathrm{i}(n-m)Ot\right]\right\} |n\rangle\langle m|$$
(11)

Zurek 等人利用线性熵<sup>[5]</sup>对谐振子系统在环境影响下,从初始纯态演化到混合态进行的研究,表明线性熵也能很好地用来测量量子态的纯度。Peixoto de Faria 和 Nemes 应用线性熵清晰地说明了原子和系统的 纯度演化。

为了说明相位损耗对系统任意时刻量子态纯度的影响,我们讨论系统线性熵的时间演化。由(4)式和(9)式可得系统线性熵<sup>[5,12]</sup>为

$$S(t) = 1 - \operatorname{Tr} \boldsymbol{\rho}^{2}(t) = 1 - \sum_{m,n} |F_{n}|^{2} |F_{m}|^{2} \exp[-2\nu(n-m)^{2}t], \qquad (12)$$

为了说明原子任意时刻量子态纯度变化情形,我们讨论原子线性熵的时间演化。由(10)式可得原子线 性熵为

$$S_{a}(t) = 1 - \operatorname{Tr} \boldsymbol{\rho}_{a}^{2}(t) = \frac{1}{2} (\sin^{2}\theta_{1} + \sin^{2}\theta_{2}) - \frac{3}{8} \sin^{2}\theta_{1} \sin^{2}\theta_{2} - \sum_{m,n} |F_{n}|^{2} |F_{m}|^{2} \times \left[ \frac{1}{2} (\sin^{2}\theta_{1} + \sin^{2}\theta_{2} - \sin^{2}\theta_{1} \sin^{2}\theta_{2}) \cos 2(n-m)\Omega t + \frac{1}{8} \sin^{2}\theta_{1} \sin^{2}\theta_{2} \cos 4(n-m)\Omega t \right], \quad (13)$$

为了说明光场任意时刻量子态纯度变化情况,我们讨论光场线性熵的时间演化。由(11)式可得光场线 性熵为

$$S_{\rm f}(t) = 1 - \operatorname{Tr} \boldsymbol{\rho}_{\rm f}^{2}(t) = 1 - \sum_{m,n} |F_{n}|^{2} |F_{m}|^{2} \exp[-2\nu(n-m)^{2}t] \Big[ 1 - \frac{1}{2}(\sin^{2}\theta_{1} + \sin^{2}\theta_{2}) + \frac{3}{8}\sin^{2}\theta_{1}\sin^{2}\theta_{2} + \frac{1}{2}(\sin^{2}\theta_{1} + \sin^{2}\theta_{2} - \sin^{2}\theta_{1}\sin^{2}\theta_{2})\cos 2(n-m)\Omega t + \frac{1}{8}\sin^{2}\theta_{1}\sin^{2}\theta_{2}\cos 4(n-m)\Omega t \Big].$$
(14)

#### 3 数值分析及讨论

由(12)式可知,系统线性熵由腔场的相位损耗 和初始光场的统计性质所决定,而与原子-光场耦合 程度以及原子的初态无关,这是因为在考虑大失谐 情形下,原子与光场相互耦合的有效哈密顿由(1)式 给出,原子与光场之间无光子交换,系统从纯态演化 为混合态的唯一原因是腔场损耗的影响。(13)式表 明,原子线性熵与初始光场的统计性质、耦合常数和 原子的初态有关,但与腔场相位损耗无关,这是因为 光腔的相位损耗使得初始光场的相干性随着时间的 演化逐渐消失, 而光场的能量则不发生衰减, 因此, (10)式表明原子约化密度算符不依赖于光腔相位损 耗衰变常量ν,使得原子线性熵与腔场相位损耗无 关<sup>[12]</sup>。但从(14)式可发现, 光场的线性熵不仅与腔 场相位损耗、初始光场的统计性质有关, 而且与其耦 合常数和原子初态也有关, 其原因在于原子与光场 相互作用时, 由(1)式可知, 原子不同能级具有不同 的动态斯塔克移位, 使得光场的性质与原子-光场的 耦合常数和原子初态有关。

下面讨论不同量子统计性质、不同衰变常数和 原子初态对系统、光场和原子各线性熵的影响。 如果初始时,光场为压缩真空态场,即

$$\begin{cases} |\psi_{\rm f}(0)\rangle = S[\gamma \exp(i\eta)]|0\rangle = \sum^{n} F_{n} |n\rangle, \\ F_{n} = (n!\mu)^{-1/2} \left(\frac{\lambda}{2\mu}\right)^{n/2} H_{n}(0), \end{cases}$$
(15)

式中 $\mu = ch\gamma, \lambda = exp(i2\eta)sh\gamma, H_n(0)$ 为厄米多项式,  $S[\gamma exp(i\eta)] =$ 

$$\exp\left\{\frac{1}{2}\gamma\left[\exp(-\mathrm{i}\eta)a^{2}-\exp(\mathrm{i}\eta)a^{+2}\right]\right\}$$

为单模压缩算符<sup>[15]</sup>, $\gamma$ 为压缩参量, $\eta$ 为压缩角,取  $\eta = 0$ 。实际上,若 $\nu = 0$ , $\theta_1 = \theta_2 = \pi/2$ ,由(9)式容



易得到原子-光场在任意时刻的态矢为

$$\begin{aligned} |\psi(t)\rangle &= \frac{1}{2} \{ |1,1\rangle \otimes S[\gamma \exp(-2i\Omega t)] |0\rangle + \\ |1,0\rangle \otimes S(\gamma) |0\rangle + |0,1\rangle \otimes S(\gamma) |0\rangle + \\ |0,0\rangle \otimes S[\gamma \exp(2i\Omega t)] |0\rangle \}, \end{aligned}$$

上式表明,体系在 $t \neq n\pi/\Omega(n=1,2,\dots)$ 时,体系处 于原子-光场的纠缠态,这是因为原子不同能级的动 态斯塔克移位导致光场相位发生改变。由(12) ~ (15)式作数值计算可得系统、光场和原子的线性熵 随时间演化规律如图1所示。



Fig. 1 Time evolution of  $S, S_f, S_a$ . (a)  $\theta_1 = \theta_2 = \pi/2, \nu = 0. \ 1\Omega, \gamma = 0. \ 3$ , (b)  $\theta_1 = \theta_2 = \pi/2, \nu = 0. \ 1\Omega, \text{sh}^2 \gamma = 1$ , (c)  $\theta_1 = 0, \theta_2 = \pi/2, \nu = 0. \ 1\Omega, \gamma = 0. \ 3$ , (d)  $\theta_1 = 0, \theta_2 = \pi/2, \nu = 0. \ 2\Omega, \gamma = 0. \ 3$ 

当初始时刻,两原子均处于激发态和基态的等 概率叠加态(即 $\theta_1 = \theta_2 = \pi/2$ )时,从图1(a)、图1(b) 可看出,系统的线性熵随时间的增加而增大,初始时 其值为零,由于腔场的相位损耗,系统的线性熵经一 段时间后趋于一大于零的稳定值,该稳定值随压缩 光场的压缩程度增加而增大。光场的线性熵随时间 作振幅减小的衰减振荡,其衰减振荡周期为 $\pi/\Omega$ ,该 振荡经一段时间后停止,而使光场线性熵趋于与系 统线性熵相同的稳定值,其振荡时间与压缩参量无 关。这是由于经足够长时间后,因相位损耗使密度 算符中非对角元均为零,而只存在对角元,由(12)和 (14)式可知:

 $S_{t}(t \to \infty) = 1 - \sum_{n} |F_{n}|^{4} = S(t \to \infty).$ 原子线性熵随时间作等幅周期振荡,其振荡周期也 为  $\pi/\Omega$ , 在  $t_d = n\pi/\Omega(n = 1, 2, \cdots)$  处,  $\rho(t_d) = \rho_a(t_d) \otimes \rho_i(t_d)$ ,  $\operatorname{Tr} \rho_a^2(t_d) = 1$ , 即  $S_a(t_d) = 0$ ,  $S_i(t_d) = S(t_d) > 0$ , 且该周期不随  $\gamma$  的改变而变化。由此表明, 系统和光场除初始时刻呈现纯态外, 其它任何时刻均处于混合态, 其混合程度经一段时间后保持不变, 该时间与压缩光场的压缩程度无关, 但混合程度的高低与压缩光场的压缩程度有关, 且随压缩程度的增加而升高, 在 $t_d$  时刻, 原子与光场完全退耦合, 原子退回到初始纯态, 而光场仍处于混合态。

当初始时两原子分别处于激发态  $|1\rangle$ (或基态  $|0\rangle$ )和等概率叠加态(如 $\theta_1 = 0, \theta_2 = \pi/2$ )时,从图 1(c)可看出,系统线性熵随时间演化没有改变,光场 和原子线性熵随时间的演化只是周期振荡更规则, 随着衰减常数加大,系统线性熵和光场线性熵趋渐 于稳定值的时间快速缩短,而原子线性熵并不随之 改变[见图 1(c)、图 1(d)]。

如果初始光场为相干态场,即

$$F_n = \exp\left(-\frac{1}{2} |\alpha|^2\right) \frac{\alpha^n}{\sqrt{n!}}, \qquad (16)$$



Fig. 2 Time evolution of  $S, S_r, S_a$  for  $\nu = 0, 1\Omega, \overline{n} = 1$ . (a)  $\theta_1 = \theta_2 = \pi/2$ , (b)  $\theta_1 = 0, \theta_2 = 0$ 

当初始时两原子均处相干叠加态(即 $\theta_1 = \theta_2 = \pi/2$ )时,在初始光场强度相同的条件下,比较图1(b)和 图2(a)可知,系统线性熵和光场线性熵的稳定值在 初始光场为压缩真空态较为相干态时小;光场线性 熵趋于稳定值的时间在初始光场为压缩真空态比为 相干态时要短;原子线性熵随时间的演化均呈现出 较完美的周期振荡,其振荡周期在初始光场为压缩 真空态较为相干态要短,其振荡幅度要小。由此可 见,初始光场的统计性质不同对系统的动力学特征 会产生较大影响。

当初始时刻两原子均处于基态( $\theta_1 = \theta_2 = \pi$ )或 激发态( $\theta_1 = \theta_2 = 0$ )或其中之一处于基态而另一个 处于激发态时,从图 2(b)可看出,光场线性熵随时 间的演化规律与系统线性熵完全相同[见(12)式、 (14)式],原子线性熵不再随时间变化,且  $S_a = 0$ ,可 从(13)式得到,这表明原子始终处于初始纯态。与 文献[6]相比较,在不考虑腔场损耗情形下,耦合双 原子与相干态场相互作用时,场熵随时间做不规则 振荡,这说明腔场的损耗对 T-C 模型的动力学性质 有明显的影响。

**结论** 本文研究了 Tavis-Cummings 模型中光场存 在相位损耗时,原子一光场系统线性熵、光场线性熵 和原子线性熵的时间演化特性,讨论了光场的不同 量子统计性质、不同的衰变常数和两原子的初始状 态对各线性熵的影响。结果表明:系统线性熵随时 间的增加而增大,经一段时间后趋于一大于零的稳 定值,该稳定值在初始光场为压缩真空态较为相干 态时小,对初始光场为压缩真空态而言,该值随压缩 系数的增加而增大。若初始两原子均处于激发态和 由(16)式和(12)~(14)式,利用数值计算可得原子--光场系统、原子和光场的线性熵随时间的演化规律, 结果如图 2 所示。



的处相干叠加态(即 $\theta_1 = \theta_2 = \pi/$ 基态的等概率叠加态,光场线性熵随时间作振幅减

小的振荡,经一段时间后趋于与系统稳定值相等的 稳定值,该时间随初始光场的统计性质不同而改变, 且随衰减常数的增大而迅速减小,但不随压缩真空 的压缩程度而改变;原子线性熵呈现较完美的周期 振荡,其振荡周期与初始光场的统计性质有关,如初 始光场为真空压缩态,其振荡周期为 π/Ω,在 t<sub>d</sub> 时 刻,原子与光场完全退耦合,原子退回到初始纯态, 但不随压缩真空的压缩程度改变。如初始光场为相 干态,原子的振荡周期延长。若初始两原子处于基 态或激发态中任意态时,原子始终处于初始纯态。

#### 参考文献

- 1 Phoenix S J D, Knight P L. Periodicity, phase and entropy in models of two-photon resonance. J. Opt. Soc. Am. (B), 1990, 7(1):116~124
- 2 Fang Maofa, Chen Jumei. Entropic uncertainty relation and entropic squeezing of the field. Acta Optica Sinica (光 学学报), 2001, 21(1):8~12 (in Chinese)
- 3 Fang M F, Liu X. Influence of the Stark shift on the evolution of field entropy and entanglement in two-photon processes. *Phys. Lett.*, 1996, A210(1):11~20
- 4 Wu Y, Yang X X. J-C model for a trapped ion in any position of a standing wave. *Phys. Rev. Lett.*, 1997, 78 (16):3086~3088
- 5 Zurek W H, Habib S, Paz J P. Coherent states via decoherence. *Phys. Rev. Lett.*, 1993, **70**(9):1187~1190
- 6 Huang C J, Zhou M, Li J F *et al*.. The entropy squeezing properties in a system of single-mode field interacting with coupled atoms. *Acta Physica Sinica* (物理学报), 2002, **51**(5): 1049~1053 (in Chinese)
- 7 Liu Sanqui, Wan Lin, Liu Sumei. Squeezing effect of two coupling atoms interacting with a single mode light field in Kerr medium. *Acta Optica Sinica* (光学学报), 2002, **22** (8):902~906 (in Chinese)

- 8 Peixoto de Faria J G, Nemes M C. Dissipative dynamics of the Jaynes-Cummings model in the dispersive approximation: Analytical results. *Phys. Rev.* (A), 1999, 59(5):3918~3925
- 9 Zheng S B, Guo G C. Efficient scheme for two-atom entanglement and quantum information processing in cavity QED. Phy. Rev. Lett., 2000, 85(11):2392~2395
- 10 Yamaguchi F, Milman P, Brune M et al.. Quantum search with two-atom collisions in cavity QED. Phys. Rev. (A), 2002, 66(1): 010302-1~010302-4
- 11 Li G X, Peng J S, Wu S P et al.. Population inversion of a V-type three-level atom due to the effect of cavityinduced quantum interference. Chin. Phys. Lett. (中国物 理快报), 2001, 18(1):48~50 (in Chinese)

- 12 Zhang Lihui, Li Gaoxiang, Peng Jinsheng. Evolution of the entropy in the Jaynes-Cummings model with large detuning inside a phase daping cavity. *Acta Optica Sinica* (光学学报), 2002, **22**(8):907~911 (in Chinese)
- 13 Guo G P, Li C F, Li J et al.. Scheme for the preparation of multiparticle entanglement in cavity QED. Phys. Rev. (A), 2002, 65(4):042102-1~042102-4
- 14 Walls D F, Milburn G J. Effect of dissipation on quantum coherence. Phys. Rev. (A), 1985, 31 (4): 2403~2408
- 15 Peng Jinsheng, Li Gaoxiang. An Introduction to Modern Quantum Optics. Singapore: World Scientific, 1998. 15~24,163~171