第 23 卷 第 8 期
2003 年 8 月

光 学 学 报
ACTA OPTICA SINICA

Vol. 23, No. 8
August, 2003

文章编号：0253-2239(2003)08-0964-6

仅有相邻波导耦合的定向耦合器的一般解法*

孟义朝[1,3] 谭维翰[2] 黄肇明[3]

1 上海大学光纤研究所，上海 201800
2 上海大学物理系，上海 200436
3 上海大学通信学院光波技术实验室，上海 200072

摘要： 应用切比雪夫多项式，求得了仅有相邻波导耦合的 $(N+1)\times(N+1)$ 定向耦合器在弱耦合、无损耗和忽略正交偏振模场分量间耦合的情形下波导耦合方程的通解。列出线形排列 2×2，3×3，4×4 定向耦合器的解的表达式，并对环形排列 3×3 定向耦合器的解作了较详细的分析。

关键词： 导波光学；定向耦合器；耦合模理论；切比雪夫多项式；通解

中图分类号：TN256 文献标识码：A

1 引 言

波导型光纤定向耦合器是一类重要的光子器件，在光通信和光传感领域得到了广泛应用，功率分束器是它的主要应用之一[1]。该器件可直接用在光纤网络中，也可用来组成结构复杂的光子器件，例如多干涉臂密集型波分复用器[2]。

在光波导理论中，波导耦合方程的求解是一个常见的问题，在多波导耦合器的文献中，已有很多作者研究过这一问题[3~16]。本文主要应用了切比雪夫(Chebyshev)多项式关系，求在弱耦合、无损耗情况下，包括线形排列与环形排列的 $(N+1)\times(N+1)$ 定向耦合器的通解，列出了 2×2，3×3，4×4 线形排列定向耦合器的解的表达式。并以 3×3 环形排列定向耦合器为例，作了较详细的分析。

2 $(N+1)\times(N+1)$ 定向耦合器的通解

2.1 线形排列定向耦合器的通解

如图 1 所示，$N+1$ 个线形排列的波导，在弱耦合及无损耗的情况下，忽略非相邻波导间的耦合和正交偏振模场分量间的耦合，耦合波方程写作

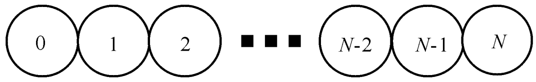

Fig. 1 Scheme of directional coupler in a line

$$\left.\begin{aligned}\frac{\partial\mu_i}{\partial z}&=\mathrm{j}\,\frac{K}{2}\mu_{i-1}+\mathrm{j}\,\frac{K}{2}\mu_{i+1},\ 0<i<N\\ \frac{\partial\mu_0}{\partial z}&=\mathrm{j}\,\frac{K}{2}\mu_1,\\ \frac{\partial\mu_N}{\partial z}&=\mathrm{j}\,\frac{K}{2}\mu_{N-1},\end{aligned}\right\}\tag{1}$$

上式中 $K/2$ 表示相邻波导间的耦合系数，μ_i 表示第 i 通道的耦合模。光波导间的耦合系数已在多篇文献中给出[3,4]。参照文献[3]，给出弱耦合时

$$\frac{K}{2}=\frac{n_\mathrm{A}U^2K_0(Wd/a)}{n_\mathrm{CO}aV^3[K_1(W)]^2},\tag{2}$$

式中 $n_\mathrm{A}=(n_\mathrm{CO}^2-n_\mathrm{Cl}^2)^{1/2}$，归一化频率 $V=\dfrac{2\pi a n_\mathrm{A}}{\lambda}$，$U=a(k^2n_\mathrm{CO}^2-\beta^2)^{1/2}$ 和 $W=a(\beta^2-k^2n_\mathrm{Cl}^2)^{1/2}$ 分别表示波导的芯层和包层中基模的本征值，式中 d 代表相邻光纤的间距，a 表示光纤的芯径，k 表示自由空间的波数，n_CO 和 n_Cl 分别表示光纤芯和包层的折射率。耦合系数的大小取决于波导之间的距离和介质的折射率等参量，弱耦合时耦合系数的下限为 0，上限由弱耦合和强耦合的界限确定。(2) 式是在弱导近似和包层无限大的假定下得到的，应根据具体结构由耦合系数的通式求解[4]，耦合系数的讨论不再作为本文的重点。

设 $\mu_i=\phi_i(\beta/K)\exp[\mathrm{j}\beta(z-\omega t/n_\mathrm{CO})]$，$\beta$ 表示模的传播常量，可由本征方程求出[3]。弱导近似下，β

* 上海市教委青年基金(02AQ77)和上海市重点学科建设资金资助课题。

E-mail：mengyichao@sh163.net

收稿日期：2002-06-14；收到修改稿日期：2002-09-02

$\approx kn_{\rm CO}=(\omega/c)n_{\rm CO}$，相当于芯层中的波数。令 $x=\beta/K$，即以 K 为单位的波数，代入(1) 式得

$$\left.\begin{aligned}&2x\phi_i(x)=\phi_{i-1}(x)+\phi_{i+1}(x),\\&i=1,\cdots,N-1\\&2x\phi_0(x)=\phi_1(x),\\&2x\phi_N(x)=\phi_{N-1}(x),\end{aligned}\right\}\tag{3}$$

式中 $\phi_i(x)$ 可用如下第二类切比雪夫多项式表示[19]：

$$\left.\begin{aligned}&\phi_n(x_m)=\sqrt{\frac{2}{N+2}}\sin[(n+1)\varphi_m],\\&\varphi_m=\frac{m\pi}{N+2},\\&x_m=\beta_m/k=\cos\varphi_m,\\&n=0,\cdots,N,\quad m=1,\cdots N+1,\end{aligned}\right\}\tag{4}$$

容易证明(4) 式是(3) 式的通解。当 φ 取本征值 φ_m 时，$\sin(N+2)\varphi_m=0$，故

$$2x_m\phi_N(x_m)=\sqrt{\frac{2}{N+2}}\sin(N\varphi_m)=\phi_{N-1}(x_m),$$

则 $\phi_n(x_m)$ 为正交完备系，满足以下关系：

$$\left.\begin{aligned}&\sum_{m=1}^{N+1}\phi_k(x_m)\phi_l(x_m)=\delta_{k,l},\\&\sum_{n=0}^{N}\phi_n(x_m)\phi_n(x_l)=\delta_{m,l},\end{aligned}\right\}\tag{5}$$

当本征值 x_m 为各异时，可用特征解 $\mu_n(x_m)=\phi_n(x_m)\exp(\mathrm{j}x_m\tau)$ 解矩阵

$$\boldsymbol{X}(\tau)=\begin{bmatrix}\phi_0(x_1)\exp(\mathrm{j}x_1\tau)&\cdots&\phi_0(x_{N+1})\exp(\mathrm{j}x_{N+1}\tau)\\\vdots&\ddots&\vdots\\\phi_N(x_1)\exp(\mathrm{j}x_1\tau)&\cdots&\phi_N(x_{N+1})\exp(\mathrm{j}x_{N+1}\tau)\end{bmatrix},\tag{6}$$

式中 $\tau=K(z-ct/n_{\rm CO})$，则通解 $\mu(\tau)$ 可写作

$$\boldsymbol{\mu}(\tau)=\boldsymbol{X}(\tau)\begin{bmatrix}\sum_{l=0}^{N}c_l\phi_l(x_1)\\\sum_{l=0}^{N}c_l\phi_l(x_2)\\\vdots\\\sum_{l=0}^{N}c_l\phi_l(x_{N+1})\end{bmatrix}=\begin{bmatrix}\sum_{m=1}^{N+1}\sum_{l=0}^{N}c_l\phi_l(x_m)\phi_0(x_m)\exp(\mathrm{j}x_m\tau)\\\sum_{m=1}^{N+1}\sum_{l=0}^{N}c_l\phi_l(x_m)\phi_1(x_m)\exp(\mathrm{j}x_m\tau)\\\vdots\\\sum_{m=1}^{N+1}\sum_{l=0}^{N}c_l\phi_l(x_m)\phi_N(x_m)\exp(\mathrm{j}x_m\tau)\end{bmatrix},\tag{7}$$

当 $\tau\to0$，亦即在输入端，应用(5) 中第二式，得

$$\boldsymbol{\mu}(0)=\begin{bmatrix}c_0\\c_1\\\vdots\\c_N\end{bmatrix},\tag{8}$$

$c_0,c_1,\cdots,c_N$ 即 N 个输入端口的输入，是解 $\boldsymbol{\mu}(\tau)$ 的初值，(7) 式的物理意义：当输入端取初值 $\boldsymbol{\mu}(0)$，经相位延迟 $x_m\tau$ 后，解 $\boldsymbol{\mu}(\tau)$ 将按(7) 式演化，事实上(7) 式还可以写作

$$\boldsymbol{\mu}(\tau)=\boldsymbol{X}(\tau)\boldsymbol{X}^{-1}(0)\boldsymbol{\mu}(0),$$

$$\boldsymbol{X}^{-1}(0)\boldsymbol{\mu}(0)=\begin{bmatrix}\phi_0(x_1)&\phi_1(x_1)&\cdots&\phi_N(x_1)\\\phi_0(x_2)&\phi_1(x_2)&\cdots&\phi_N(x_2)\\\vdots&\vdots&\cdots&\vdots\\\phi_0(x_{N+1})&\phi_1(x_{N+1})&\cdots&\phi_N(x_{N+1})\end{bmatrix}\begin{bmatrix}c_0\\c_1\\\vdots\\c_N\end{bmatrix}=\begin{bmatrix}\sum_{l=0}^{N}c_l\phi_l(x_1)\\\sum_{l=0}^{N}c_l\phi_l(x_2)\\\vdots\\\sum_{l=0}^{N}c_l\phi_l(x_{N+1})\end{bmatrix},\tag{9}$$

或写作

$$\boldsymbol{\mu}(\tau)=\boldsymbol{X}(\tau)\boldsymbol{X}^{-1}(0)\boldsymbol{\mu}(0)=\boldsymbol{R}(\tau)\boldsymbol{\mu}(0),$$

$$\boldsymbol{R}(\tau)=\begin{bmatrix}\sum_m\phi_0^2(x_m)\exp(\mathrm{j}x_m\tau) & \sum_m\phi_0(x_m)\phi_1(x_m)\exp(\mathrm{j}x_m\tau) & \cdots & \sum_m\phi_0(x_m)\phi_N(x_m)\exp(\mathrm{j}x_m\tau)\\ \sum_m\phi_1(x_m)\phi_0(x_m)\exp(\mathrm{j}x_m\tau) & \sum_m\phi_1^2(x_m)\exp(\mathrm{j}x_m\tau) & \cdots & \sum_m\phi_1(x_m)\phi_N(x_m)\exp(\mathrm{j}x_m\tau)\\ \vdots & \vdots & \ddots & \vdots\\ \sum_m\phi_N(x_m)\phi_0(x_m)\exp(\mathrm{j}x_m\tau) & \sum_m\phi_N(x_m)\phi_1(x_m)\exp(\mathrm{j}x_m\tau) & \cdots & \sum_m\phi_N^2(x_m)\exp(\mathrm{j}x_m\tau)\end{bmatrix},$$

矩阵元

$$R_{n,l}=\sum_m\phi_{n-1}(x_m)\phi_{l-1}(x_m)\exp(\mathrm{i}x_m\tau).\quad(10)$$

2.2 环形分布定向耦合器的通解

如图 2 所示，由 $N+1$ 个波导构成的环形定向耦合器的结构，弱耦合波导方程

$$\left.\begin{aligned}&\frac{\partial\mu_i}{\partial z}=\mathrm{j}\,\frac{K}{2}\mu_{i-1}+\mathrm{j}\,\frac{K}{2}\mu_{i+1},\\&0<i<N,\\&\frac{\partial\mu_0}{\partial z}=\mathrm{j}\,\frac{K}{2}\mu_1+\mathrm{j}\,\frac{K}{2}\mu_N,\\&\frac{\partial\mu_N}{\partial z}=\mathrm{j}\,\frac{K}{2}\mu_0+\mathrm{j}\,\frac{K}{2}\mu_{N-1},\end{aligned}\right\}\quad(11)$$

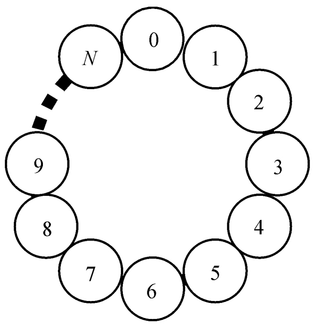

Fig. 2 Scheme of directional coupler in a ring

设 $\mu_i=\phi_i(\beta/K)\exp[\mathrm{j}\beta(z-ct/n_{\mathrm{CO}})]$，并令 $x=\beta/K$，代入(11) 式得

$$\left.\begin{aligned}&2x\phi_i(x)=\phi_{i-1}(x)+\phi_{i+1}(x),\\&0<i<N,\\&2x\phi_0(x)=\phi_1(x)+\phi_N(x),\\&2x\phi_N(x)=\phi_0(x)+\phi_{N-1}(x),\end{aligned}\right\}\quad(12)$$

ϕ_i 的解可用切比雪夫第一类多项式表示[19]。第一类切比雪夫多项式定义为

$$\left.\begin{aligned}&\phi_n(x_m)=\sqrt{\frac{2}{N+1}}\cos\varphi_m\cos(n\varphi_m),\\&m\neq0,\quad 0\leqslant n\leqslant N,\\&\phi_n(x_0)=\sqrt{\frac{1}{N+1}},\\&x_m=\beta_m/K=\cos\varphi_m,\\&\varphi_m=2\pi\frac{m}{N+1},\quad m=0,\cdots,N\end{aligned}\right\}\quad(13)$$

容易证明(13)式是(12)式的解

$$\begin{aligned}&2x_m\phi_n(x_m)=2\sqrt{\frac{2}{N+1}}\cos\varphi_m\cos(n\varphi_m)=\\&\qquad\phi_{n+1}(x_m)+\phi_{n-1}(x_m),\\&\qquad 0<n<N,\\&2x_m\phi_0(x_m)=2\sqrt{\frac{2}{N+1}}\cos\varphi_m=\\&\qquad\sqrt{\frac{2}{N+1}}[\cos\varphi_m+\cos(N\varphi_m)]=\\&\qquad\phi_1(x_m)+\phi_N(x_m),\\&2x_m\phi_N(x_m)=2\sqrt{\frac{2}{N+1}}\cos\varphi_m\cos(N\varphi_m)=\\&\qquad\phi_{N-1}(x_m)+\phi_0(x_m)\end{aligned}$$

以上两式成立的条件是考虑到 $\cos\varphi_m=\cos(N\varphi_m)$，以及

$$\sqrt{\frac{2}{N+1}}\cos[(N+1)\varphi_m]=\sqrt{\frac{2}{N+1}}\cos[0\cdot\varphi_m]=\phi_0(x_m)$$

由(13)式定义的 $\phi_n(x_m)$ 和 $\phi_0(x_m)$ 为完备正交的。如果本征值 $x_m=\cos\varphi_m$ 为各异的，则按(6)式可以求出解矩阵 $\boldsymbol{X}(\tau)$。但当 $x=\cos\varphi_m$ 为简并时，需要另外求解，这种情况在下面第 4 节中讨论。

3 线形排列 2×2，3×3，4×4 定向耦合器的表达式

3.1 线形排列 2×2 定向耦合器(N=1)的计算

$$\left.\begin{aligned}&x_m=\cos\varphi_m=\cos\left(\frac{m\pi}{1+2}\right),m=1,2.\\&x_1=\cos\frac{\pi}{3}=\frac{1}{2},\quad x_2=\cos\frac{2\pi}{3}=-\frac{1}{2},\\&\phi_0(x_1)=\frac{1}{\sqrt{2}},\quad\phi_0(x_2)=\frac{1}{\sqrt{2}},\\&\phi_1(x_1)=\frac{1}{\sqrt{2}},\quad\phi_1(x_2)=-\frac{1}{\sqrt{2}},\end{aligned}\right\}$$

(14)

$$\boldsymbol{\mu}(\tau)=\begin{bmatrix}\frac{1}{\sqrt{2}}\exp(\mathrm{j}\tau/2) & \frac{1}{\sqrt{2}}\exp(-\mathrm{j}\tau/2)\\ \frac{1}{\sqrt{2}}\exp(\mathrm{j}\tau/2) & -\frac{1}{\sqrt{2}}\exp(-\mathrm{j}\tau/2)\end{bmatrix}\begin{bmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\end{bmatrix}=\begin{bmatrix}\cos(\tau/2) & \mathrm{j}\sin(\tau/2)\\ \mathrm{j}\sin(\tau/2) & \cos(\tau/2)\end{bmatrix}\begin{bmatrix}c_0\\ c_1\end{bmatrix},\quad(15)$$

3.2　线形排列 3×3 定向耦合器($N=2$)的计算

$$\left.\begin{aligned}&\varphi_m=\frac{m\pi}{4},\quad m=1,2,3,\quad x_1=\frac{1}{\sqrt{2}},\quad x_2=0,\quad x_3=-\frac{1}{\sqrt{2}}\\ &\boldsymbol{\mu}(\tau)=\begin{bmatrix}\frac{1}{2}\exp(\mathrm{j}\tau/\sqrt{2}) & \frac{1}{\sqrt{2}} & \frac{1}{2}\exp(-\mathrm{j}\tau/\sqrt{2})\\ \frac{1}{\sqrt{2}}\exp(\mathrm{j}\tau/\sqrt{2}) & 0 & -\frac{1}{\sqrt{2}}\exp(-\mathrm{j}\tau/\sqrt{2})\\ \frac{1}{2}\exp(\mathrm{j}\tau/\sqrt{2}) & -\frac{1}{\sqrt{2}} & \frac{1}{2}\exp(-\mathrm{j}\tau/\sqrt{2})\end{bmatrix}\begin{bmatrix}\frac{1}{2} & \frac{1}{\sqrt{2}} & \frac{1}{2}\\ \frac{1}{\sqrt{2}} & 0 & -\frac{1}{\sqrt{2}}\\ \frac{1}{2} & -\frac{1}{\sqrt{2}} & \frac{1}{2}\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\end{bmatrix},\\ &\boldsymbol{\mu}(\tau)=\begin{bmatrix}\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\tau}{\sqrt{2}}\right) & \frac{\mathrm{j}}{\sqrt{2}}\sin\left(\frac{\tau}{\sqrt{2}}\right) & -\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\tau}{\sqrt{2}}\right)\\ \frac{\mathrm{j}}{\sqrt{2}}\sin\left(\frac{\tau}{\sqrt{2}}\right) & \cos\left(\frac{\tau}{\sqrt{2}}\right) & \frac{\mathrm{j}}{\sqrt{2}}\sin\left(\frac{\tau}{\sqrt{2}}\right)\\ -\frac{1}{2}+\frac{1}{2}\cos\left(\frac{\tau}{\sqrt{2}}\right) & \frac{\mathrm{j}}{\sqrt{2}}\sin\left(\frac{\tau}{\sqrt{2}}\right) & \frac{1}{2}+\frac{1}{2}\cos\left(\frac{\tau}{\sqrt{2}}\right)\end{bmatrix}\begin{bmatrix}c_0\\ c_1\\ c_2\end{bmatrix},\end{aligned}\right\}\quad(16)$$

(16)式的解与文献[17]求得的结果相同。

3.3　线形排列 4×4 定向耦合器($N=3$)的矩阵表示

$$\left.\begin{aligned}&\varphi_m=m\pi/5,\quad m=1,2,3,4\\ &x_1=\cos(\pi/5),\quad x_2=\cos(2\pi/5),\quad x_3=-x_2,\quad x_4=-x_1,\quad R_{n,l}=4r_{n,l}/5\\ &r_{1,1}=r_{4,4}=(1-x_1^2)\cos(x_1\tau)+(1-x_2^2)\cos(x_2\tau),\\ &r_{2,2}=r_{3,3}=(1-x_2^2)\cos(x_1\tau)+(1-x_1^2)\cos(x_2\tau),\\ &r_{1,2}=r_{2,1}=r_{3,4}=r_{4,3}=\mathrm{j}\sqrt{(1-x_1^2)(1-x_2^2)}[\sin(x_1\tau)+\sin(x_2\tau)],\\ &r_{1,3}=r_{2,4}=r_{3,1}=r_{4,2}=\sqrt{(1-x_1^2)(1-x_2^2)}[\cos(x_1\tau)-\cos(x_2\tau)],\\ &r_{1,4}=r_{4,1}=\mathrm{j}(1-x_1^2)\sin(x_1\tau)-\mathrm{j}(1-x_2^2)\sin(x_2\tau),\\ &r_{2,3}=r_{3,2}=\mathrm{j}(1-x_2^2)\sin(x_1\tau)-\mathrm{j}(1-x_1^2)\sin(x_2\tau),\end{aligned}\right\}\quad(17)$$

4　有简并情形环形定向耦合器的求解

按上面方法，对 3×3 环形分布定向耦合器($N=2$)求解

$$\varphi_m=2\pi\frac{m}{1+2},\qquad m=0,1,2,\qquad(18)$$

对应的本征值

$$x_m=\cos\varphi_m=1,-1/2,-1/2,\qquad(19)$$

为二重简并的，这时解 μ_1,μ_2,μ_3 可写作

$$\left.\begin{aligned}&\mu_1=\alpha_{11}\exp(\mathrm{j}\beta z)+(\alpha_{12}+\alpha_{13}z)\exp(-\mathrm{j}\beta z/2),\\ &\mu_2=\alpha_{21}\exp(\mathrm{j}\beta z)+(\alpha_{22}+\alpha_{23}z)\exp(-\mathrm{j}\beta z/2),\\ &\mu_3=\alpha_{31}\exp(\mathrm{j}\beta z)+(\alpha_{32}+\alpha_{33}z)\exp(-\mathrm{j}\beta z/2),\end{aligned}\right\}\quad(20)$$

将(20)式代入(11)式，并比较系数得

$$\left.\begin{aligned}&\alpha_{11}=\alpha_{21}=\alpha_{31}=A/3,\\ &\alpha_{12}=2B,\\ &\alpha_{22}=C-B,\\ &\alpha_{32}=-C-B,\\ &\alpha_{13}=\alpha_{23}=\alpha_{33}=0,\end{aligned}\right\}\quad(21)$$

故有

$$\left.\begin{aligned}&\mu_1=\frac{A}{3}\exp(\mathrm{j}\beta z)+2B\exp(-\mathrm{j}\beta z/2),\\ &\mu_2=\frac{A}{3}\exp(\mathrm{j}\beta z)+(C-B)\exp(-\mathrm{j}\beta z/2),\\ &\mu_3=\frac{A}{3}\exp(\mathrm{j}\beta z)-(C+B)\exp(-\mathrm{j}\beta z/2),\end{aligned}\right\}\quad(22)$$

设处 $z=0$，μ_1,μ_2,μ_3 的初值为 a_1,a_2,a_3，则有

$$a_1 = A/3+2B, a_2 = A/3+C-B, a_3 = A/3-(C+B)$$

$$\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix} = \begin{bmatrix}\frac{a_1+a_2+a_3}{3}\exp(\mathrm{j}\beta z)+\frac{2a_1-a_2-a_3}{3}\exp(-\mathrm{j}\beta z/2)\\ \frac{a_1+a_2+a_3}{3}\exp(\mathrm{j}\beta z)+\frac{-a_1+2a_2-a_3}{3}\exp(-\mathrm{j}\beta z/2)\\ \frac{a_1+a_2+a_3}{3}\exp(\mathrm{j}\beta z)+\frac{-a_1-a_2+2a_3}{3}\exp(-\mathrm{j}\beta z/2)\end{bmatrix} = \begin{bmatrix}\gamma_1 & \gamma_2 & \gamma_2\\ \gamma_2 & \gamma_1 & \gamma_2\\ \gamma_2 & \gamma_2 & \gamma_1\end{bmatrix}\cdot\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix}, \tag{23}$$

式中 $\gamma_1=\frac{1}{3}\exp(\mathrm{j}\beta z)+\frac{2}{3}\exp(-\mathrm{j}\beta z/2)$，$\gamma_2=\frac{1}{3}\exp(\mathrm{j}\beta z)-\frac{1}{3}\exp(-\mathrm{j}\beta z/2)$，调节耦合区的长度 z，使(23)式中的 γ_1、γ_2 分别为

$$\gamma_1=\frac{1}{\sqrt{3}}\exp(\mathrm{j}\varphi_2),\qquad \gamma_2=\frac{1}{\sqrt{3}}\exp(\mathrm{j}\varphi_1), \tag{24}$$

(φ_1,φ_2) 有两组解：$\left(\frac{11\pi}{18},-\frac{\pi}{18}\right)$；$\left(-\frac{\pi}{18},\frac{5\pi}{18}\right)$。相应地 $\Delta\varphi=\varphi_2-\varphi_1=\left(-\frac{2\pi}{3},\frac{\pi}{3}\right)$，若将第一组解 $\Delta\varphi=-\frac{2\pi}{3}$ 代入(23)式得

$$\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix}=\frac{1}{\sqrt{3}}\begin{bmatrix}\exp\left(-\mathrm{j}\frac{2\pi}{3}\right) & 1 & 1\\ 1 & \exp\left(-\mathrm{j}\frac{2\pi}{3}\right) & 1\\ 1 & 1 & \exp\left(-\mathrm{j}\frac{2\pi}{3}\right)\end{bmatrix}\begin{bmatrix}a_1\\ a_2\\ a_3\end{bmatrix}\exp\left(-\mathrm{j}\frac{\pi}{18}\right), \tag{25}$$

若取初值 $(a_1,a_2,a_3)=\left\{0,\frac{-1}{\sqrt{2}\mathrm{j}}\exp\left[\mathrm{j}\left(-\frac{\delta}{2}+\frac{\pi}{18}\right)\right],\frac{1}{\sqrt{2}\mathrm{j}}\exp\left[\mathrm{j}\left(\frac{\delta}{2}+\frac{\pi}{18}\right)\right]\right\}$，则得

$$\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix}=\sqrt{\frac{2}{3}}\begin{bmatrix}\sin(\delta/2)\\ \sin[(\delta+2\pi/3)/2]\exp(-\mathrm{j}\pi/3)\\ \sin[(\delta-2\pi/3)/2]\exp(-\mathrm{j}\pi/3)\end{bmatrix}, \tag{26}$$

$$\begin{cases}P_1=|\mu_1|^2=\frac{2}{3}\sin^2\frac{\delta}{2}=\frac{1}{3}(1-\cos\delta),\\ P_2=|\mu_2|^2=\frac{2}{3}\sin^2\frac{1}{2}\left(\delta+\frac{2\pi}{3}\right)=\frac{1}{3}\left[1-\cos\left(\delta+\frac{2\pi}{3}\right)\right],\\ P_3=|\mu_3|^2=\frac{2}{3}\sin^2\frac{1}{2}\left(\delta-\frac{2\pi}{3}\right)=\frac{1}{3}\left[1-\cos\left(\delta-\frac{2\pi}{3}\right)\right],\end{cases} \tag{27}$$

(27)式即 Vance 得到的结果[18]。若取另一种初值

$$(a_1,a_2,a_3)=\left\{\exp\left[\mathrm{j}\left(\frac{2\pi}{3}+\delta+\frac{\pi}{18}\right)\right],\exp\left(\mathrm{j}\frac{\pi}{18}\right),\exp\left[\mathrm{j}\left(-\delta+\frac{\pi}{18}\right)\right]\right\}, \tag{28}$$

代入(25)式，经简单计算，得

$$\begin{bmatrix}\mu_1\\ \mu_2\\ \mu_3\end{bmatrix}=\frac{1}{\sqrt{3}}\begin{bmatrix}A\\ B\\ C\end{bmatrix}, \tag{29}$$

式中

$$A=\frac{\sin(3\delta/2)}{\sin(\delta/2)},$$

$$B=\frac{\sin[3(\delta+4\pi/3)/2]}{\sin[(\delta+4\pi/3)/2]}\exp\left(-\mathrm{j}\frac{2\pi}{3}\right),$$

$$C=\frac{\sin[3(\delta+2\pi/3)/2]}{\sin[(\delta+2\pi/3)/2]},$$

不论(27)式还是(29)式，均实现了相位差为 $2\pi/3$ 的三波分复用。

结论 本文采用切比雪夫多项式，给出弱耦合、无损耗 $(N+1)\times(N+1)$ 定向耦合器的一般解法，包括线形和环形分布两种情形。具体给出线形分布 2×2，3×3，4×4 三种定向耦合器的解析解，详细讨论

了环形分布定向耦合器的解。该方法同样适合耦合系数各异的定向耦合系统。

参　考　文　献

1 Huang Yong, Zeng Qingji. A novel structure single-mode optical fiber splitter. *Acta Optica Sinica* (光学学报), 1995, **15**(2):248～251 (in Chinese)
2 Yao Shouquan, Wang Zihua. A dense-wavelength-division-multiplexer by using a three arm Mach-Zehnder interferometer. *Acta Optica Sinica* (光学学报), 2000, **20**(7):952～956 (in Chinese)
3 Falcial R, Scheggi A M, Schena A. Approximate calculation method for predicting selective properties of fused monomode biconical couplers. *Interna. J. Optoelectron.*, 1990, **5**(1):41～46
4 Snyder A W. Coupled mode theory for optical fiber. *J. Opt. Soc. Am.*, 1972, **62**(6):1267～1277
5 Mortimore D B. Theory and fabrication of 4×4 single-mode fused optical fiber couplers. *Appl. Opt.*, 1990, **29**(3):371～374
6 Haus H A, Molter-Ore L. Coupled multiple waveguide systems. *J. Quant. Electron.*, 1983, **QE-19**(5):840～844
7 Chuang Shun-lien. A coupled mode theory for multiwaveguide systems satisfying the reciprocity theorem and power conservation. *J. Lightwave Technol.*, 1987, **5**(1):174～183
8 She Shouxian, Qiao Li. Analysis of three channel waveguide directional couplers by a variational method and weighted residual method. *Opt. Commun.*, 1988, **87**(4):271～276
9 Kubo H, Yasumoto K. Numerical analysis of three-parallel embedded optical waveguides. *J. Lightwave Technol.*, 1989, **7**(12):1924～1931
10 Paney F P, Hussey C D, Yataki M S. Modeling fused single mode fiber couplers. *Electron. Lett.*, 1985, **21**(4):461～462
11 Chen Y. Asymmetric triple-core couplers. *Opt. and Quant. Electron.*, 1991, **24**(4):539～553
12 Naoto Kishi, Eikichi Yamashita, Hirokazu Kawabta. Modal and coupling field analysis of optical fibers with linearly distribution multiple cores. *J. Lightwave Technol.*, 1989, **7**(6):902～907
13 Ankiewicz A, Synder A W, Zheng X H. Coupling between parallel optical fiber cores-critical examination. *J. Lightwave Technol.*, 1986, **4**:1317～1323
14 Hardy A, Streifer W. Coupled mode solutions of multi-waveguide systems. *IEEE J. Quant. Electron.*, 1986, **QE-22**(4):528～534
15 Hardy A, Streifer W, Osinski M. Weak coupling of parallel waveguides. *Opt. Lett.*, 1988, **13**(2):161～163
16 Sun Liping, Ye Peida. General analysis of 3×3 optical fiber directional couplers. *Microwave and Optical Technol. Lett.*, 1989, **2**(2):52～54
17 Chew Y H, *et al.*. Performance of single- and Double-ring resonators using 3×3 optical fiber coupler. *J. Lightwave Technol.*, 1993, **11**(12):1998～2008
18 Vance R W C, Love J D. Design procedures for passive planar coupled waveguide devices. *IEE Proc. Optoelectron.*, 1994, **141**(4):231～241
19 Wang Zhuxi, Guo Dunren. *Introduction to Special Function* (特殊函数概论). Beijing: Peking University Press, 2000. 168, 645 (in Chinese)
20 Gradsheyn I S, Ryshik I M. *Table of Integrals*. Series and Products, New York: Academic Press, 1990. 29

General Solutions to Directional Couplers with Coupling Between Neighbored Waveguides Only

Meng Yichao[1,3]　Tan Weihan[2]　Huang Zhaoming[3]

1 *Institute of Fiber Optics, Shanghai University, Shanghai* 201800
2 *Physics Department, Shanghai University, Shanghai* 200436
3 *School of Communication and Information Engineering, Shanghai University, Shanghai* 200072

(Received 14 June 2002; revised 02 September 2002)

Abstract: General analytical solutions are given by means of Chebyshev polynomial for directional couplers, which are made up by lossless multi-waveguides in a line or in a ring, weak coupling existing only between neighbored waveguides and polarization coupling being not considered also. As examples, expressions for the 2×2, 3×3 and 4×4 directional couplers in the form of line are deduced. Besides, solution to the 3×3 directional coupler arranged in a ring is given and analyzed in detail.

Key words: guided wave optics; directional coupler; mode coupling theory; Chebyshev polynomial; general solution