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用频域法实现圆环形目标物的计数*

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摘要： 研究了用频域法实现圆环形目标物计数的方法。对空域中含有圆环形目标物的二值图像，经傅里叶变换后在频域内进行滤波，再经傅里叶反变换后取一阈值进行处理，则对应于圆环形目标物即变为一个实心体，然后对实心体进行计数即可实现对圆环形目标物的计数。对该方法进行了数学描述，并分别用模拟图像和实际图像进行了实验。结果表明，该方法消除噪声的效果好，对部分封闭圆的检测同样有效，且可用于目标物有粘连的情况。该方法对圆环形目标物的尺寸相差不大的情况更为有效。

关键词： 光学信息处理；图像处理；频域法；圆环形；计数；傅里叶变换

中图分类号：O438.2　　文献标识码：A

1 引　　言

自然界中很多物体都与圆形或圆环形有关，对图像中圆形或圆环形目标物的有效检测和计数是图像分析中重要的内容。目前检测圆的最常用的方法是 Hough 变换法。将 Hough 变换用于圆的检测首先是由 Duda 和 Hart 报导的[1]，Kimme 等[2]首先将其用于实际图像中检测圆。目前，用 Hough 变换检测圆的方法可归纳为三大类：第一类是基于对参量的分解，减少累加阵列的维数[3~8]；第二类是采用几个处理器在多处理器系统中实现变换[9~14]；第三类是从算法着手，设计有效算法[15~24]。本文对空域中含有圆环形孢子的二值图像，经傅里叶变换后在频域内进行滤波，再经傅里叶反变换后取一阈值进行处理，则对应于圆环形的孢子变为一个实心体，对实心体进行计数即可实现对圆环形目标物的计数。该方法具有抗噪声能力强、且不影响对部分封闭圆检测等特点。

2 原　　理

设一圆环图像的内径为 $2b$，外径为 $2(b+a)$（如图 1）。通过圆心的任一直线和圆环相交，则可得到如图 2 的一维波形（设坐标的起始点为该直线与圆环相交的起始位置）。其中，横坐标为该直线对应的像素位置，纵坐标为其对应的灰度值（实际图像为二值化图像。圆环上点的灰度为 255，背景为 0，为简化起见，亮值设为 1，暗值为 0）。

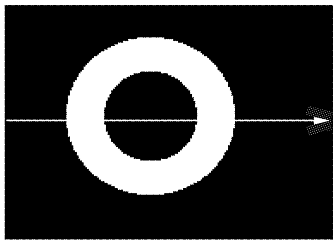

Fig. 1　Model image of spore with circular ring shape

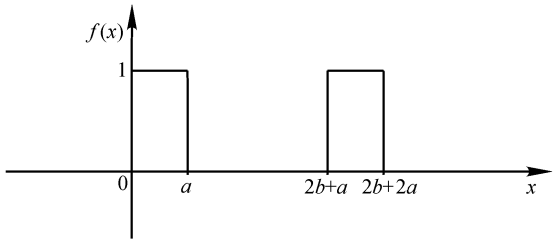

Fig. 2　Function waveform corresponding to circular ring

其函数表达式为

$$f(x)=\begin{cases}1, & 0\leqslant x<a\\ 0, & a\leqslant x<2b+a\\ 1, & 2b+a\leqslant x<2b+2a\\ 0, & \text{Other}\end{cases}\tag{1}$$

对 $f(x)$ 进行傅里叶变换，得

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$$F(u)=\int_{-\infty}^{+\infty} f(x)\cdot\exp(-\mathrm{j}2\pi ux)\mathrm{d}x=\int_{0}^{a}\exp(-\mathrm{j}2\pi ux)\mathrm{d}x+\int_{2b+a}^{2b+2a}\exp(-\mathrm{j}2\pi ux)\mathrm{d}x=$$
$$2a\cdot\cos\pi u(a+2b)\cdot\frac{\sin\pi ua}{\pi ua}\cdot\exp[-\mathrm{j}2\pi u(b+a)], \tag{2}$$

其频谱的模为

$$|F(u)|=\left|2a\cdot\cos\pi u(a+2b)\cdot\frac{\sin\pi ua}{\pi ua}\right|, \tag{3}$$

对图 2 的函数进行离散化采样，N 为采样点数，采样间隔为 1 个像素单位，则采样序列为 $f(0)$、$f(1)$、$f(2)$、…、$f(N-1)$。作连续傅里叶变换，通过矩形近似将连续积分过程变为数值积分，则其离散频谱为

$$\left|F\left(u_{\max}\frac{n}{N}\right)\right|=\left|2a\cdot\cos\left[\frac{\pi u_{\max}n(a+2b)}{N}\right]\cdot\frac{\sin(\pi u_{\max}na/N)}{\pi u_{\max}na/N}\right|\qquad n=0,1,\cdots,N-1, \tag{4}$$

频谱曲线图如图 3 所示。

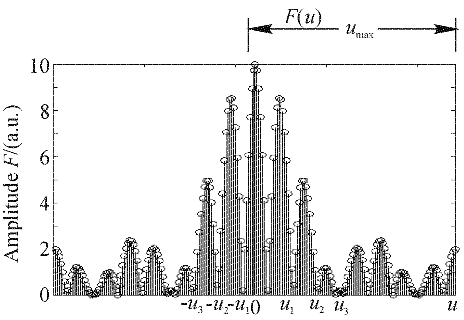

Fig. 3 Amplitude of Fourier spectrum

图 3 中，包络线形状是由辛克(sinc)函数决定的，包络线下面的波动是由 cos 函数决定。由(3)式不难得出

$$u_1=\frac{1}{2(a+2b)},u_2=\frac{3}{2(a+2b)},u_3=\frac{5}{2(a+2b)},$$

其中 u_1、u_2、u_3 分别为傅里叶频谱中 0 级谱、1 级谱和 2 级谱右端的过零点频率值。为了分析各频段的作用，在频域内进行滤波处理，本文采用理想的带通滤波器进行滤波，设其滤波函数为 $H(u)$，则

$$H(u)=\begin{cases}1, & u_{\mathrm{D}}<u<u_{\mathrm{G}}\\ 0, & \text{other}\end{cases}$$

其中，u_{D}、u_{G} 分别为带通滤波器的低截止频率和高截止频率。

则 $\mathrm{G(u)=H(u)F(u)}$，

经带通滤波器滤波后的输出函数 $f'(x)$ 为对 $G(u)$ 的反傅里叶变换，采用矩形数值积分公式，可表为

$$f'(x)=F^{-1}\{G(u)\}=\int_{-\infty}^{+\infty}H(u)F(u)\exp(\mathrm{j}2\pi ux)\mathrm{d}u=\int_{u_{\mathrm{D}}}^{u_{\mathrm{G}}}F(u)\exp(\mathrm{j}2\pi ux)\mathrm{d}u\doteq$$
$$\frac{u_{\max}}{N}\sum_{n=[u_{\mathrm{D}}N/u_{\min}]}^{[u_{\mathrm{G}}N/u_{\max}]}F\left(u_{\max}\frac{n}{N}\right)\exp\left(\mathrm{j}2\pi xu_{\max}\frac{n}{N}\right), \tag{5}$$

式中$[X]$表示不超过 X 的最大整数。

分别对$(-u_1,u_1)$，(u_1,u_2) 和$(-u_2,-u_1)$，(u_2,u_3) 和$(-u_3,-u_2)$ 等频带取出进行分析，反变换后的函数波形如图 4 所示(其中取 $a=5$，$b=10$)。然后，确定一个阈值，函数值大于该阈值时置1，小于该阈值时置零。这样，图 2 的函数波形变为在$b+a$ 处某一范围内的值为 1，在其它地方的值为零(如图 5 所示)。为了阈值的选取和了解在$b+a$ 处值为 1 的宽度，下面进一步分析图 4 的波形随 a,b 变化的情况。对图 4 分析可知，对频段(u_1,u_2) 和$(-u_2,-u_1)$ 反变换的效果最好。所以，以下分析选择(u_1,u_2) 和$(-u_2,-u_1)$ 频段反变换的结果。

设在 $b+a$ 处，也即圆环中心处的峰值为 $f'_{0\max}$，设该波峰与 x 轴的两交点为 x_1、x_2，其宽度为 $\Delta x=x_2-x_1$。这里 Δx 的解析式无法明确地表达出来，但可对各个离散点进行分析求解，可由中心点 $x=(b+a)$ 分别向左、向右的第一个最低点之差所得到。设次波峰值为 $f'_{1\max}$。针对我们的应用要求，反变换后波形的 $f'_{0\max}$ 越高、Δx 越小、$f'_{0\max}/f'_{1\max}$ 越大，检测的效果就越好。

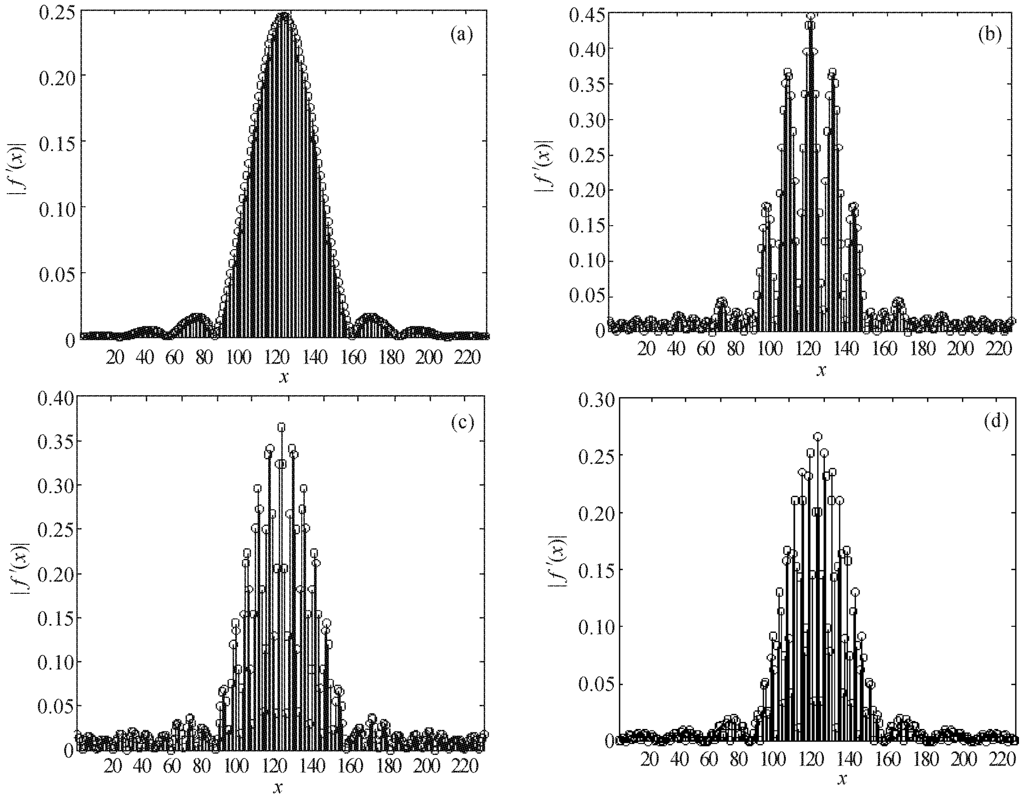

Fig. 4　Waveform obtained from filtered spectrum by using inverse Fourier transformation. (a) Inverse transformation on spectrum band $(-u_1, u_1)$; (b) Inverse transformation on spectrum band $(-u_2, -u_1)$ and (u_1, u_2); (c) Inverse transformation on spectrum band $(-u_3, -u_2)$ and (u_2, u_3); (d) Inverse transformation on spectrum band $(-u_4, -u_3)$ and (u_3, u_4)

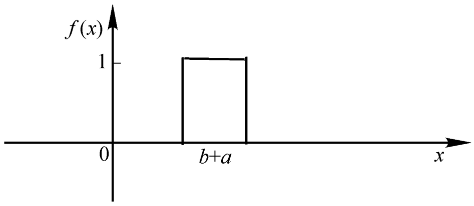

Fig. 5　Function waveform using thresholding

为了分析对不同尺寸孢子的检测效果，根据(5)式分别画出 $f'_{0\max}$、Δx、$f'_{0\max}/f'_{1\max}$ 随 a、b 的变化图如图 6、图 7、图 8 所示。

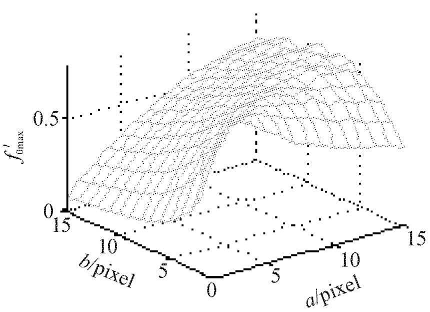

Fig. 6　Variation of $f'_{0\max}$ with respect to a, b

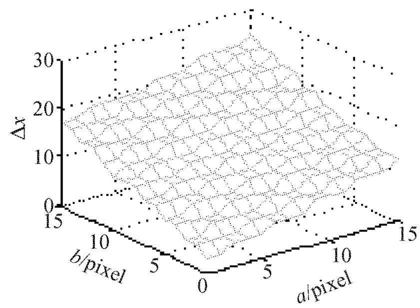

Fig. 7　Variation of Δx with respect to a, b

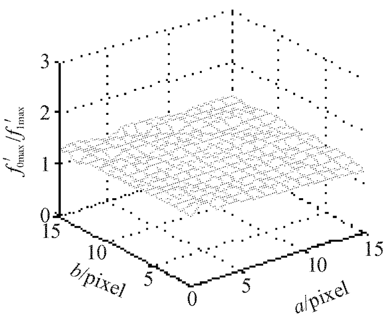

Fig. 8　Variation of $f'_{0\max}/f'_{1\max}$ with respect to a, b

从图 6、图 7 可看出：对固定的 b，随着 a 的增大，$f'_{0\max}$ 增大，Δx 也增大。即当圆环的内径不变，环的宽度增加时，中心波峰峰值增大，宽度也增加。对固定的 a，随着 b 的增大，$f'_{0\max}$ 减小，Δx 却增大。即当圆环的环宽度不变，内径增加时，中心波峰峰值减小，宽度却增加。从图 8 可看出，对不同的 a，b，$f'_{0\max}/f'_{1\max}$ 的值都大于 1.3，说明中心波峰高出其它波峰，所以只要所有圆环的 a，b 值相差在一定范围内，$f'_{0\max}$ 相差不是很大，这样可以选取某个阈值将所有圆环的中心波峰分离出来。因此，用频域法对圆环状孢子进行计数，一般它们的 a，b 值都相差不大，所以此法可适用于对实际圆环孢子的检测（如图 18，19 所示）。

上述分析是基于直角坐标下的一维情形，要推广到圆形从理论上推导较复杂，以下从定性方面进行分析。根据傅里叶变换的性质，若 $f(x)$ 在空间内旋转一个角度，则它的傅里叶变换 $F(u)$ 在频域内也旋转同一角度。所以对圆环状目标物经傅里叶变换，在频域内频谱的形状相当于图 3 沿 $u=0$ 轴旋转 360°得到的环状谱。环状谱的中心为 0 级，由内到外的最大峰依次对应于 1 级谱，2 级谱，3 级谱等。将 1 级谱取出后对其进行傅里叶反变换，再选取一阈值，对幅值高于阈值的部分置为 1（对二值图像，该值为 255），低于阈值的部分置 0。这样，在每一个圆环的圆心周围将得到亮的一个连续区域。然后对图像中亮区域进行计数，即可得圆环孢子的数量。

3 实验结果及讨论

本文分别用模拟图像和实际图像对上述方法进行了实验。图 9～图 17 是对各种情况下的模拟图像实验所得到的结果。可看出，该方法消除噪声的效果很好，且对部分封闭圆的检测不受影响。将本

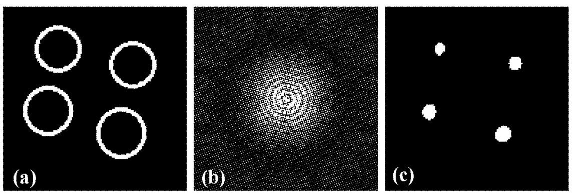

Fig. 9 Detection of occluded circular rings with multi-pixels boundaries. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

文的方法用于实际图像的检测，也取得了很好的效果，其结果如图 18、19 所示。

从图 17 可看出，对于目标物有粘连情况，其傅里叶频谱图从环状上仍然容易分辨，得到的结果也较理想，所以该方法可适用目标物有粘连的情况。

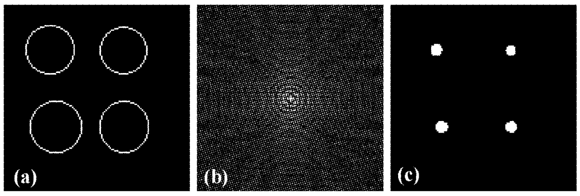

Fig. 10 Detection of occluded circular rings with single-pixel boundaries. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

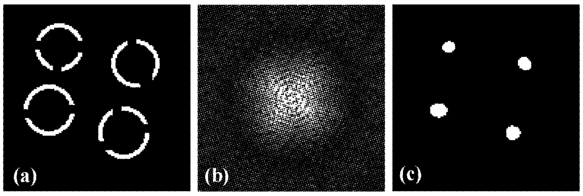

Fig. 11 Detection of partially occluded circular rings with multi-pixels boundaries. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

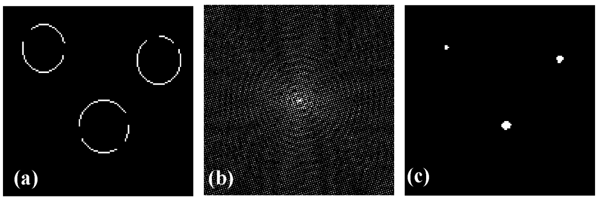

Fig. 12 Detection of partially occluded circular rings with single-pixel boundaries. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

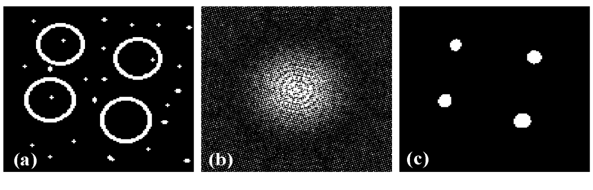

Fig. 13 Detection of occluded circular rings with multi-pixels boundaries under noise background. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

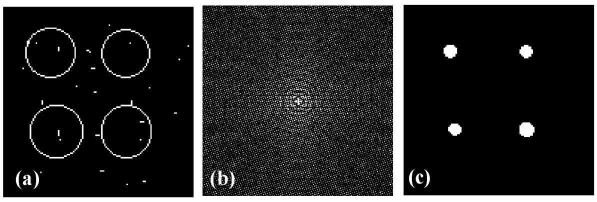

Fig. 14 Detection of occluded circular rings with single-pixel boundaries under noise background. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

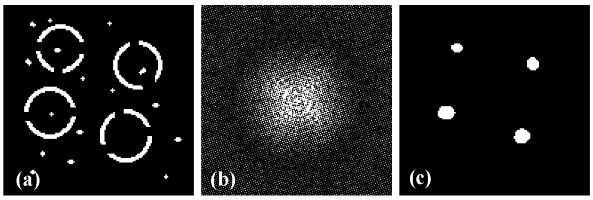

Fig. 15 Detection of partially occluded circular rings with multi-pixels boundaries under noise background. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

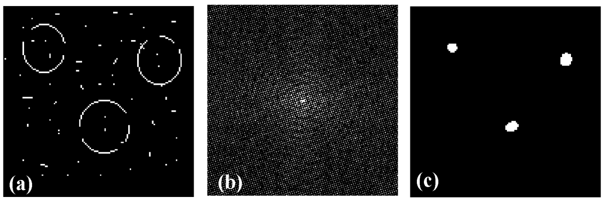

Fig. 16 Detection of partially occluded circular rings with single-pixel boundaries under noise background. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

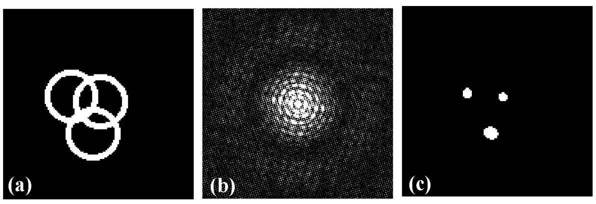

Fig. 17 Detection of overlapping rings with multi-pixels boundaries. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

当圆环目标物大小差别较大时，其傅里叶频谱较模糊，滤波器带宽不易选择，结果不理想。

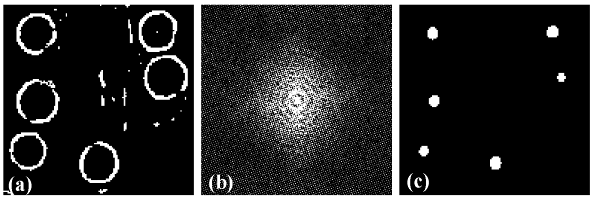

Fig. 18 Detection of real spores image. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

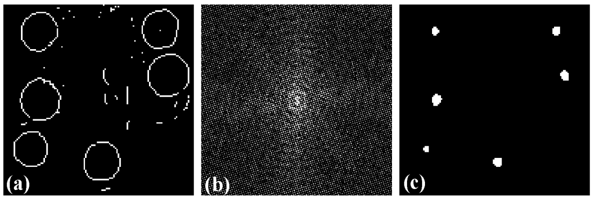

Fig. 19 Detection of thinning spores image. (a) Original image; (b) Fourier spectrum; (c) Image obtained by inverse Fourier transform on filtered spectrum

结束语　本文提出用频域法实现圆环形目标物的计数，建立了该方法的理论，给出了相应的实验结果。因在实际应用中，预处理后的图像往往伴有很多噪声，且一些圆形目标物常常出现不封闭的情况，而本文提出的方法具有抗干扰能力强、对部分封闭圆环的检测同样有效等特点，所以，该文的方法具有很好的应用前景。

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The Realization of Circular Ring Counting Using Frequency Domain Method

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Abstract: The method of counting for circular ring objects in frequency domain is proposed. A binarized image with circular rings is processed by Fourier transformation and filtered in frequency domain, then inverse Fourier transformation is carried on. After selecting a suitable threshold, the circular rings become solid particles individually. The number of circular rings is obtained through counting the solid particles. The mathematical model is established and the experimental results for both simulated and real images are also given. The method is robust to noise and the detection for partially occluded circular rings is also available. The method can also be applied to the situation that some objects are overlapped. The method is more effective when the size of the objects in the image is similar to each other.

Key words: optical information processing; image processing; frequency domain; circular ring; counting; Fourier transformation