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在亚泊松泵浦统计和压缩真空条件下多单光子关联辐射光子双稳态*

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摘 要 证明了亚泊松泵浦及原子的相干性可使多单光子双稳态的光场噪声降低, 但不能产生光场的压缩效应。若向腔中注入压缩真空, 可在双稳态的上支产生几乎完全的光子数压缩效应。同时还证明, 双稳态临界合作系数与原子的相干性及集居数都有关。

关键词 多单光子关联辐射双稳态, 泵浦统计, 原子相干性, 压缩真空, 光子数压缩效应。

1 引 言

激光和光学双稳态中的量子噪声猝灭或压缩是近年来量子光学中的一个活跃的课题。有许多方法可使这些装置中的量子噪声降低, 如压缩泵浦噪声[1, 2]、关联自发辐射[2~5]、向腔中注入压缩真空[6]、原子记忆效应[7]、原子关联和多光子跃迁[8]等。但通常的光子双稳态仅能在低通态产生近乎零场强的压缩光[8~10], 或通过平移腔场而在高速态产生压缩光[11]。

本文考虑一种新的单光子双稳态模型, 即所谓关联辐射光子双稳态(CEPB)。将具有 $m+1$ 个近简并上能级和 $n-1$ 个近简并下能级且能级间处于相干叠加态的原子通过亚泊松泵浦注入谐振腔并在腔中经历多单光子跃迁。这时发现, 泵浦噪声的完全抑制并不能使多单光子双稳态在热库条件下产生压缩光。与此成对比的是, 在亚泊松泵浦统计下, 多单光子关联辐射不但可在非线性区[2]而且可在线性区[5]产生压缩光。计算表明, 在注入压缩真空的条件下, 仅可在多单光子关联辐射光子双稳态的高通态产生近乎完全的光子数压缩效应。

2 多单光子关联辐射光子双稳态的密度算符方程

图 1 为本文考虑的多单光子关联辐射光子双稳态的原子能级构型示意图[5]。具有 $n-1$ 个近简并下能级($|1\rangle$, $|2\rangle\cdots$, $|n-1\rangle$) 和 $m+1$ 个近简并上能级($|n\rangle$, $|n+1\rangle$, $\cdots$, $|n+m\rangle$) 的原子与频率为 Ω 的腔模相互作用, 在旋转波近似下, 体系的哈密顿量为[5]

$$H = (\Omega - \omega)a^{+}a + (\omega_0 - \omega)S_z + g\sqrt{(m+1)(n-1)}\,(a^{+}S_{-} + aS_{+}) \quad (1)$$

式中各符号的含义见文献[5]。假定注入速率为 r_a 的原子寿命 $1/\Gamma$ 远远小于原子通过腔的时

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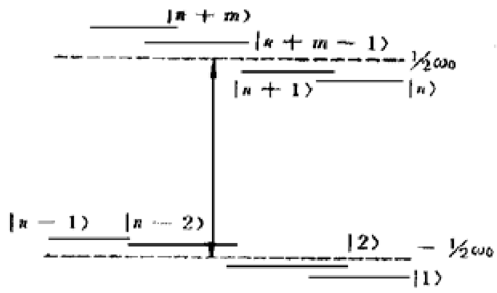

Fig. 1 Atomic levels for multiple one-photon CEOB. The atoms are injected in a coherent superposition of levels |1⟩, |2⟩, ⋯, |n + m⟩

间，并有一宽带压缩真空及一频率为 $\omega = \Omega = \omega_0$ 的经典光场注入谐振腔，可得如下的光场密度算符方程

$$\rho(t) = \frac{r_a}{\eta}\ln\left[1+\eta(\bar{M}-1)\right]\rho(t) - i[V_d,\ \rho] + \rho_l \tag{2}$$

其中 η、r_a、$\bar{M}$ 及 $\rho(t)$ 的含义见文献[5]，哈密顿

$$V_d = i(\epsilon_d a^+ - a\epsilon_d^*) \tag{3}$$

描写腔的驱动。而反映注入压缩真空影响的密度算符方程为[6]

$$\begin{aligned}\rho_l = &-(1/2)[(\gamma+\gamma_t N)(\rho a^+ a - a\rho a^+) + \\ &\gamma_t N(\rho a a^+ - a^+\rho a) + \gamma_t M(2a^+\rho a^+ - \\ &a^+ a^+ \rho - \rho a^+ a^+)] + H.c.\end{aligned} \tag{4}$$

其中损耗速率 $\gamma = \gamma_t + \gamma_a$，$\gamma_a(\gamma_t)$ 表示腔场的吸收(传输)损耗，N、M 满足 $N(N+1) \geqslant |M|^2$。若 $N = M = 0$，则变为通常的真空(热库)。

3 福克-普朗克方程

利用关系

$$\rho(t) = \int P(1,\ \varphi,\ t)\,|\sqrt{I}\exp(\mathrm{i}\varphi)\rangle\langle\sqrt{I}\exp(\mathrm{i}\varphi)|\,\mathrm{d}I\mathrm{d}\varphi \tag{5}$$

可将算符方程(2)转变为关于正规函数 $P(I,\ \varphi,\ t)$ 的福克-普朗克方程

$$\frac{\partial P(I,\ \varphi,\ t)}{\partial t} = \left(-\frac{\partial}{\partial I}d_I - \frac{\partial}{\partial\varphi}d_\varphi + \frac{\partial^2}{\partial I^2}D_{II} + \frac{\partial^2}{\partial\varphi^2}D_{\varphi\varphi} + 2\frac{\partial^2}{\partial I\partial\varphi}D_{I\varphi}\right)P(I,\ \varphi,\ t) \tag{6}$$

其中

$$\begin{aligned}d_I = &\left\{\frac{A}{\Pi}\left[(n-1)\sum_{k,\,k'=n}^{n+m}\rho^a_{kk'} - (m+1)\sum_{j,\,j'=1}^{n-1}\rho^a_{jj'}\right] - \gamma\right\}I + \\ &2SD\sqrt{I}\sin(\theta-\varphi)/\Pi + 2\sqrt{I}|\epsilon_d|\cos(\varphi_d-\varphi)\end{aligned} \tag{7a}$$

$$d_\varphi = -DS\cos(\theta-\varphi)/\sqrt{I} + |\epsilon_d|\sin(\varphi_d-\varphi)/\sqrt{I} \tag{7b}$$

$$\begin{aligned}D_{II} = &\frac{AI}{\Pi^2}\Bigg\{\Bigg[(n-1)\sum_{k,\,k'=n}^{n+m}\rho^a_{kk'} + (n-1)(m+1)^2\frac{B}{A}I\sum_{j,\,j'=1}^{n-1}\rho^a_{jj'} - \\ &2(n-1)(m+1)(BI/A)^{1/2}D\sin(\theta-\varphi)\Bigg] - \\ &\eta\Bigg(D^2\sin^2(\theta-\varphi) + (n-1)(m+1)(BI/A)^{1/2}D\sin(\theta-\varphi)\times \\ &\left[(n-1)\sum_{k,\,k'=n}^{n+m}\rho^a_{kk'} - (m+1)\sum_{j,\,j'=1}^{n-1}\rho^a_{jj'}\right] + \\ &\frac{1}{4}(n-1)(m+1)\left(\frac{BI}{A}\right)\left[(n-1)\sum_{k,\,k'=n}^{n+m}\rho^a_{kk'} - (m+1)\sum_{j,\,j'=1}^{n-1}\rho^a_{jj'}\right]^2\Bigg)\Bigg\} + \\ &\gamma_t I[N + |M|\cos(2\varphi-\psi)]\end{aligned} \tag{7c}$$

$$\begin{aligned}D_{\varphi\varphi} = &\frac{A}{4\Pi I}\Bigg\{(n-1)\sum_{k,\,k'=n}^{n+m}\rho^a_{kk'} + \frac{1}{2}(n-1)(m+1)\left(\frac{BI}{A}\right)\times \\ &\left[(n-1)\sum_{k,\,k'=n}^{n+m}\rho^a_{kk'} + (m+1)\sum_{j,\,j'=1}^{n-1}\rho^a_{jj'}\right] - \\ &(n-1)(m+1)(BI/A)^{1/2}D\sin(\theta-\varphi)\Bigg\} + (\gamma_t/4I)[N - |M|\cos(2\varphi-\psi)]\end{aligned} \tag{7d}$$

$$D_{I\varphi} = (1/4\Pi)(n-1)(m+1)(BI/A)^{1/2} D\cos(\theta-\varphi) - (1/2)\gamma_t|M|\sin(2\varphi-\psi) \quad (7e)$$

其中 $\sum_{j=1}^{n-1}\sum_{k=n}^{n+m}\rho_{kj}^a = D\exp(\mathrm{i}\theta)$，$M = |M|\exp(\mathrm{i}\psi)$，$\epsilon_d = |\epsilon_d|\exp(\mathrm{i}\varphi_d)$，$A = 2\gamma_a g^2/\Gamma^2$，$B = 4Ag^2/\Gamma^2$，$S = \gamma_a g/\Gamma$，$\Pi = 1 + (m+1)(n-1)(BI/A)$。由方程(6)可得关联辐射光子双稳态的稳态运转条件

$$\partial d_I/\partial I < 0, \qquad \partial d_\varphi/\partial\varphi < 0 \quad (8)$$

由(8)式及 $\langle d_\varphi\rangle = 0$ 可得锁相条件 φ_0

$$\varphi_d - \varphi_0 = 0, \qquad \theta - \varphi_0 = \pi/2 \quad (9)$$

由(9)式及 $\langle d_I\rangle = 0$ 可得关联辐射光子双稳态的运转方程

$$y = x + \frac{2C}{\Pi}\{[(m+1)\sum_{j,j'=1}^{n-1}\rho_{jj'}^a - (n-1)\sum_{k,k'=n}^{n+m}\rho_{kk'}^a]x - 2D\} \quad (10)$$

其中 $y = 2|\epsilon_d|/(\gamma\sqrt{n_s})$，$x = (I/n_s)^{1/2}$，$C = A/(2\gamma) = S/(2\gamma\sqrt{n_s})$，$n_s = A/B$，$\Pi = 1 + (n-1)(m+1)x^2$。

若 $m+1 = n-1 = 1$，$\rho_{kk'} = 0$，$D = 0$，则(10)式变为单光子光学双稳态[8~11]。利用

$$\partial d_I/\partial I = -(\gamma/2)\,\mathrm{d}y/\mathrm{d}x, \qquad \partial d_\varphi/\partial\varphi = -(\gamma/2)[(4CD/x) + (y/x)] \quad (11)$$

由(7)式及(8)式可得光子数起伏 $\langle(\Delta n)^2\rangle$ 及光场位相起伏 $\langle(\Delta\varphi)^2\rangle$

$$\langle(\Delta n)^2\rangle = n_0\Big\{1 + \frac{4C}{\Pi^2}\Big((n-1)\sum_{k,k'=n}^{n+m}\rho_{kk'}^a + (n-1)(m+1)^2\sum_{j,j'=1}^{n-1}\rho_{jj'}^a x^2 - 2(n-1)(m+1)Dx - \eta\{D^2 + (n-1)(m+1)D\times[(n-1)\sum_{k,k'=n}^{n+m}\rho_{kk'}^a - (m+1)\sum_{j,j'=1}^{n-1}\rho_{jj'}^a]x + \frac{1}{4}(n-1)(m+1)\times[(n-1)\sum_{k,k'=n}^{n+m}\rho_{kk'}^a - (m+1)\sum_{j,j'=1}^{n-1}\rho_{jj'}^a]^2x^2\}\Big)\Big/\Big|\frac{\mathrm{d}y}{\mathrm{d}x}\Big| - (\gamma_t/\gamma)[1-\exp(-2\beta)]\Big/|\mathrm{d}y/\mathrm{d}x|\Big\} \quad (12)$$

$$\langle(\Delta\varphi)^2\rangle = \frac{1}{4n_0}\Big\{1 + \frac{4C}{\Pi}\Big([n-1]\sum_{k,k'=n}^{n+m}\rho_{kk'}^a + \frac{1}{2}(n-1)(m+1)\times[(n-1)\sum_{k,k'=n}^{n+m}\rho_{kk'}^a + (m+1)\sum_{j,j'=1}^{n-1}\rho_{jj'}^a]x^2 - (n-1)(m+1)Dx\Big)\Big/(4CD/x + y/x) + (\gamma_t/\gamma)[\exp(2\beta) - 1]\Big/(4CD/x + y/x)\Big\} \quad (13)$$

其中

$$\frac{\mathrm{d}y}{\mathrm{d}x} = 1 + \frac{2C}{\Pi^2}\{[(m+1)\sum_{j,j'=1}^{n-1}\rho_{jj'}^a - (n-1)\sum_{k,k'=n}^{n+m}\rho_{kk'}^a]\times[1-(n-1)(m+1)x^2] + 4(n-1)(m+1)Dx\} \quad (14)$$

在方程(12)及(13)中，已取 $N = \sinh^2\beta$，$|N| = \sinh\beta\cosh\beta$ 及 $\varphi_0 = (1/2)(\Psi \pm \pi)$。由(13)式及(14)式知，亚泊松泵浦统计可使光子噪声降低，但对光场位相起伏无影响。而原子的相干性及原子能级的简并度可同时影响光场噪声。详细讨论见下节。

4 关联辐射光子双稳态的运转及光子数压缩

4.1 关联辐射光子双稳态的运转条件

关联辐射光子双稳态的运转条件为

$$dy/dx < 0 \tag{15}$$

若 $d^2y/dx^2 = 0$, 则 dy/dx 取极值。由 $d^2y/dx^2 = 0$ 给出

$$\left[(m+1)\sum_{j,j'=1}^{n-1}\rho^a_{jj'} - (n-1)\sum_{k,k'=n}^{n+m}\rho^a_{kk'}\right][(n-1)(m+1)x^2 - 3]x + 2D[1 - 3(n-1)(m+1)x^2] = 0 \tag{16}$$

方程(16)有三个实根，容易证明仅有根

$$x_e = 2[4D^2/F^2 + 1/(n-1)(m+1)]^{1/2}\cos\Theta + 2D/F \tag{17}$$

能给出关联辐射光子双稳态的运转条件

$$C > C_0 = \Pi_e^2/\{2(n-1)(m+1)F[4D^2/F^2 + 1/(n-1)(m+1)](4\cos^2\Theta - 1)\} \tag{18}$$

且须

$$F(4\cos^2\Theta - 1) > 0 \tag{19}$$

其中

$$\Pi_e = 1 + (n-1)(m+1)x_e^2, \qquad \Theta = (1/3)\arctan\{F/[2D\sqrt{(n-1)(m+1)}]\},$$

$$F = (m+1)\sum_{j,j'=1}^{n-1}\rho_{jj'} - (n-1)\sum_{k,k'=n}^{n+m}\rho^a_{kk'}$$

若 $D = 0$, $F = 1$, (18)式变为通常单光子光学双稳态的结果[12]

$$C > C_0 = 4, \tag{20}$$

取[5] $\rho_{jj'} = \rho_{11}$, $\rho_{kk'} = \rho_{nn}$, $|\rho_{kj}|^2 = |\rho_{n1}|^2 = \rho_{nn}\rho_{11} = \rho_{11}[1-(n-1)\rho_{11}]/(m+1)$ $(j, j' = 1, 2, \cdots, n-1;\ k, k' = n, n+1, \cdots, n+m)$, $(n-1)\rho_{11} + (m+1)\rho_{nn} = 1$, 及

$$\sum_{k,k'=n}^{n+m}\rho^a_{kk'} = (m+1)^q\rho^a_{nn}, \qquad \sum_{j,j'=1}^{n-1}\rho^a_{jj'} = (n-1)^p\rho^a_{11},$$

$$D = \left|\sum_{j=1}^{n-1}\sum_{k=n}^{n+m}\rho^a_{kj}\right| = (n-1)(m+1)|\rho^a_{n1}| \tag{21}$$

此处 q、p 为 2, 分别表示原子上、下近简并能级间有完全的相干性, q、p 为 1, 表示无相干性存在。对 q、p 为 1, 及 $n-1 = m+1 = 1$ 和 $n-1 = m+1 = 2$ 两种情况, 图 2 及图 3, 分别给出了临界合作数 C_0 与 ρ_{11} 的关系。其它情形类似。可以看出, 随 $|1\rangle$ 能级粒子数的增加, 双稳态的临界合作数 C_0 减小。

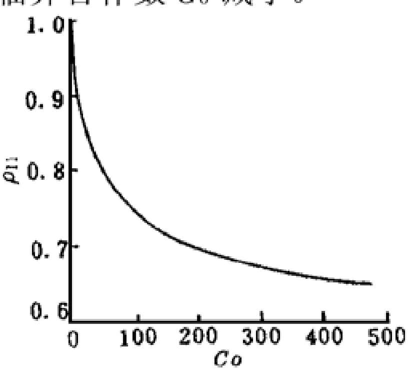

Fig. 2 Plot of critical value C_0 for the operation of CEOB vs atomic population ρ^a_{11} for $n-1 = m+1 = 1$

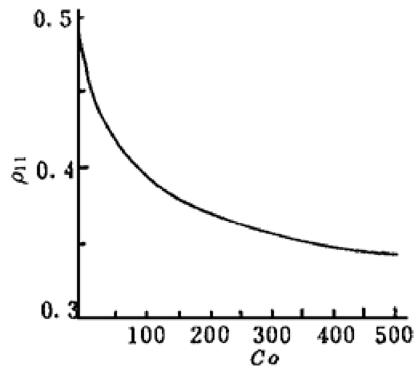

Fig. 3 Same as Fig. 2, but for $n-1 = 2$, $m+1 = 2$, and $q = p = 1$

4.2　关联辐射光子双稳态的运转

在通常的单光子光学双稳态中，若注入经典光场 $y = 0$，则腔场 $x = 0$。但在关联辐射光子双稳态中，因原子上下能级间相干性的存在，使得 $y = 0$ 时 x 并不等于零。对 q、p 为 1，$C = 100$，$n - 1 = m + 1 = 2$ 及不同的 ρ_{11}，图 4 给出了注入光场 y 与腔内光场 x 的双稳态曲线。其它情形类似。因关联辐射光子双稳态本质上是吸收型的光子双稳态，易证明对一定的 x、C 及 $(n-1)\rho_{11}$，y 随 $(n-1)^{p-2}$ 的增加或 $(m+1)^{q-2}$ 的减少而增加。

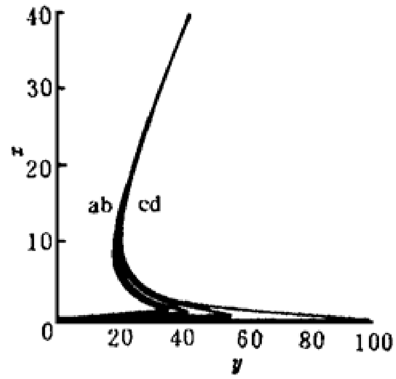

Fig. 4 Plot of normalized cavity field x given by eq. (12) vs normalized input field y for $n - 1 = m + 1 = 2$, $q = p = 1$, and $C = 100$, (a) 0.47; (b) 0.48; (c) 0.49; (d) 0.50

4.3　光子数压缩效应

与单光子关联辐射激光器一样，多单光子关联辐射光子双稳态也无光场位相压缩。尽管在单光子关联辐射激光器中有光子数压缩效应，但计算表明，多单光子关联辐射光子双稳态中并不存在光子数压缩效应。这一点与通常的单光子双稳态不同。当注入压缩真空后，多单光子关联辐射光子双稳态可在高通态产生几乎完全的光子数压缩效应，在低通态仍无光子数压缩效应。图 5 及图 6 给出了在 $\gamma_t \approx \gamma$, $\exp(-2\beta) \ll 1$ 的条件下，q、$p = 1$，$n - 1 = 1 = m + 1$，和 $n - 1 = m + 1 = 2$ 两种情形下，不同的 C 及 ρ_{11}，$\langle(\Delta n)^2\rangle n_0^{-1} - 1$ 与 x 的关系。其它情形类似。从图 5 及图 6 可以看出光子数压缩效应随 ρ_{11} 的增加或 C 的减少而增加。此时测不准关系为 $\langle(\Delta n)^2\rangle\langle(\Delta\varphi)^2\rangle \approx 1/4$，近似为 Glauber 相干态。由(12) 式知，在双稳态的上支，当 x 较大时(饱和，$x \gg 1$)，$dy/dx \to 1$，而(12) 式中大括号内第二项的分子变为 $O(1/x^2)$，此时 $\langle(\Delta n)^2\rangle \approx n_0\{1 + O(1/x^2) - (\gamma_t/\gamma)[1 - \exp(-2\beta)]\} \approx 0$。故光子数压缩效应来源于压缩真空和腔内光场的饱和效应。原子的相干性 D 和亚泊松泵浦可降低光场噪声，但不能产生光子数压缩效应。

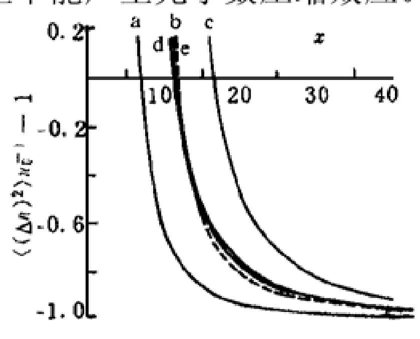

Fig. 5 Plot of $\langle(\Delta n)^2\rangle n_0^{-1} - 1$ as a function of the normalized cavity field for $n - 1 = m + 1 = 1$, and different values of C and $\rho_{11}^a C$, ρ_{11}^a: (a) 17.8, 0.90; (b) 50, 0.90; (c) 100, 0.90; (d) 50, 0.80; (e) 50, 0.90

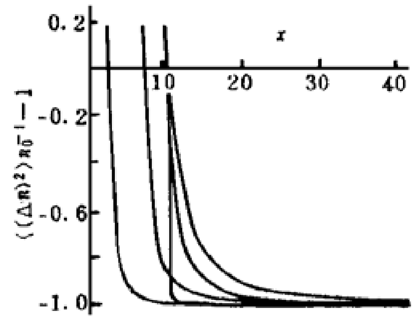

Fig. 6 Same as Fig. 8, but for $n - 1 = m + 1 = 2$, $q = p = 1$, and C, ρ_{11}^a: (a) 5.7, 0.49; (b) 50, 0.49; (c) 100, 0.50; (d) 100, 0.49; (e) 100, 0.47

结　论　多单光子关联辐射光子双稳态运转的稳定条件是其光场位相锁定在由原子相干性及注入场位相决定的值 φ_0。且临界合作数 C_0 与原子能级构型，初始原子相干性及原子能级集居数有关。亚泊松泵浦可降低光子数噪声，但对位相噪声无影响。原子相干性可降低光子数噪声或位相噪声。但原子相干性及亚泊松泵浦统计在任何条件下都不能产生多单光子关联辐射

光子双稳态的光场压缩。只在注入压缩真空的情形下可在双稳态的高通态产生几乎完全的光子数压缩，但低通态无压缩效应。高通态的压缩效应随原子吸收的增加而增加。这种压缩效应来源于压缩真空及光场饱和。原子的相干性、能级构型及亚泊松泵浦统计对压缩效应无明显影响，因这些因素被光场的饱和性掩盖。

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Multiple One-Photon Correlated-Emission Optical Bistavility with Regular Pumping Statistics and Injected Squeezed Vacuum

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Abstract We show that the refular pumping statistics and atomic coherences can reduce the field noise of multiple one-photon optical bistability, but can not result in field squeezing from an ordinary vacuum. If a squeezed vacuum is injected into a cavity, almost perfect photon-number squeezing can be obtained in the upper branch. It is verified that the critical values of cooperativity C are related with the atomic coherences and atomic population.

Key words multiple one-photon correlated emission optical bistability, pump statistics, atomic coherences, squeezed vacuum, photo-numbber suqeezing.