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粗糙表面相关结构对远场散斑相位差统计性质的影响

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摘　要　假设弱散射体粗糙表面高度起伏服从高斯统计，在表面高度起伏分别服从高斯相关、指数相关、圆型相关的情况下，分析了由弱散射体产生的远场高斯斑场相位差的条件统计分布。

关键词　粗糙表面相关结构，　散斑，　相位差条件统计性质。

散斑相位差的统计性质在散斑干涉计测中有重要应用，一直受到重视。Donati[1] 等人最早研究了由强散射体产生的高斯散斑场相位差的统计性质。Kadono 等人[2]首先实验研究了由弱散射体产生的两衍射场高斯散斑场干涉相位差的所谓自由统计性质，但未能给出理论解释。作者[3, 4]研究了弱散射体产生的远场高斯散斑相位差的条件统计分布，给出了相位差条件概率密度函数，以及相位差条件标准偏差的近似解析表达式，并分析了相位差条件统计分布与有关参数的关系。本文在文献[3，4]的基础上，讨论相位差的条件统计分布与散射体粗糙表面相关结构的关系。

1　理论推导

如图1所示，D为透明的相位式弱散射体，ξ-η 平面为粗糙表面，散射体由高斯激光束照明，束腰位于散射体的粗糙表面，散射体在远场产生高斯散斑，x-y 平面为远场观察平面。假设透过散射体的激光束的相位为零平均的平稳高斯随机变量，则散斑场 P_2 和 P_1 两点相位差 $\theta=\theta_2-\theta_1$ 的条件概率密度函数为[4]

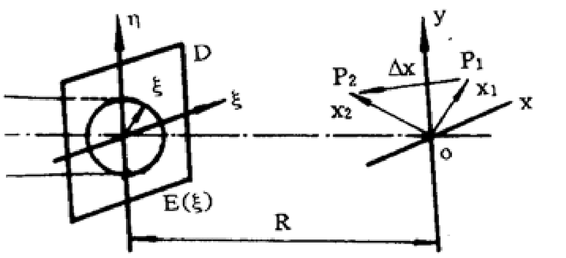

Fig. 1 Optical configuration for producing far-field Gaussian speckle fields by a weak diffuser

$$P(\theta|I_1,\theta_1)=\frac{\tau}{2\pi\alpha}\{1+\sqrt{\pi}\ \varepsilon\exp(\varepsilon^2)\ [1+\mathrm{erf}\ (\varepsilon)]\}\exp(-\gamma/2\tilde{\sigma}_{r_2}^2) \tag{1}$$

相位差条件标准偏差定义为[4]

$$\sigma_\theta|_{I_1,\theta_1}=[\int_{-\pi}^{\pi}(\theta-\langle\tilde{\theta}\rangle)^2P(\theta|_{I_1,\theta_1})\ \mathrm{d}\theta]^{1/2}. \tag{2}$$

为了考察粗糙表面相关结构对相位差条件统计分布的影响，下面就几种典型的相关结构进行

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计算。若透过散射体粗糙表面的激光束相位起伏相关函数由 $\rho_\phi(\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2)$ 表示，则根据文献[3]可推导出 P_1 和 P_2 两点散斑复振幅实部和虚部相关矩表示式为：

$$\begin{aligned}\langle \Delta A_{r_1}\Delta A_{r_2}\rangle &= \langle [A_r(\boldsymbol{x}_1)-\langle A_r(\boldsymbol{x}_1)\rangle][A_r(\boldsymbol{x}_2)-\langle Ar(\boldsymbol{x}_2)\rangle]\rangle \\ &= \frac{1}{2}\iiiint_{-\infty}^{+\infty} E(\boldsymbol{\zeta}_1)E(\boldsymbol{\zeta}_2)\{W_+(0)\cos[H(\boldsymbol{\zeta}_1,\boldsymbol{x}_1)-H(\boldsymbol{\zeta}_2,\boldsymbol{x}_2)] \\ &\quad + W_-(0)\cos[H(\boldsymbol{\zeta}_1,\boldsymbol{x}_1)+H(\boldsymbol{\zeta}_2,\boldsymbol{x}_2)\}\,\delta(\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2)\,d^2\boldsymbol{\zeta}_1 d^2\boldsymbol{\zeta}_2 \end{aligned} \tag{3}$$

$$\begin{aligned}\langle \Delta A_{i_1}\Delta A_{i_2}\rangle &= \langle [A_i(\boldsymbol{x}_1)-\langle A_i(\boldsymbol{x}_1)\rangle][A_i(\boldsymbol{x}_2)-\langle A_i(\boldsymbol{x}_2)\rangle]\rangle \\ &= \frac{1}{2}\iiiint_{-\infty}^{+\infty} E(\boldsymbol{\zeta}_1)E(\boldsymbol{\zeta}_2)\{W_+(0)\cos[H(\boldsymbol{\zeta}_1,\boldsymbol{x}_1)-H(\boldsymbol{\zeta}_2,\boldsymbol{x}_2)] \\ &\quad - W_-(0)\cos[H(\boldsymbol{\zeta}_1,\boldsymbol{x}_1)+H(\boldsymbol{\zeta}_2,\boldsymbol{x}_2)]\}\,\delta(\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2)\,d^2\boldsymbol{\zeta}_1 d^2\boldsymbol{\zeta}_2 \end{aligned} \tag{4}$$

式中 W_+ 和 W_- 分别为 R_+ 和 R_- 的傅里叶变换，而 $R_+=\exp(-\sigma_\phi^2)\{\exp[\sigma_\phi^2\rho_\phi(\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2)-1]\}$，$R_-=\exp(-\sigma_\phi^2)\{\exp[-\sigma_\phi^2\rho_\phi(\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2)-1]\}$。

2.1 表面服从高斯相关

表面相关函数等各量为：

$$\rho_\phi(\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2)=\exp(-|\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2|^2/\alpha^2) \tag{5}$$

$$W_+(0)=\pi\alpha^2\exp(-\sigma_\phi^2)h_+,\qquad W_-(0)=\pi\alpha^2\exp(-\sigma_\phi^2)h_-, \tag{6}$$

考虑一维坐标情形，经推导得[3]：

$$\left.\begin{aligned}
\langle A_{r_1}\rangle &= \pi\omega_0^2\exp(-\sigma_\phi^2/2)\exp(-\hat{x}_1^2), \\
\langle A_{r_2}\rangle &= \pi\omega_0^2\exp(-\sigma_\phi^2/2)\exp[-(\hat{x}_1+\Delta\hat{x})^2] \\
\sigma_{r_1} &= (\pi/2)\,\omega_0\alpha\,[h_+-h_-\exp(-2\hat{x}_1^2)]^{1/2}\exp(-\sigma_\phi^2/2) \\
\sigma_{i_1} &= (\pi/2)\,\omega_0\alpha\,[h_++h_-\exp(-2\hat{x}_1^2)]^{1/2}\exp(-\sigma_\phi^2/2) \\
\sigma_{r_2} &= (\pi/2)\,\omega_0\alpha\exp(-\sigma_\phi^2/2)\{h_+-h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\}^{1/2} \\
\sigma_{i_2} &= (\pi/2)\,\omega_0\alpha\exp(-\sigma_\phi^2/2)\{h_++h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\}^{1/2} \\
\gamma_r &= \frac{\exp(-\Delta\hat{x}^2/2)[h_+-h_-\exp(-2\hat{x}_1^2)\exp(-2\hat{x}_1\Delta\hat{x})]}{([h_+-h_-\exp(-2\hat{x}_1^2)]\{h_+-h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\})^{1/2}} \\
\gamma_i &= \frac{\exp(-\Delta\hat{x}^2/2)[h_++h_-\exp(-2\hat{x}_1^2)\exp(-2\hat{x}_1\Delta\hat{x})]}{([h_++h_-\exp(-2\hat{x}_1^2)]\{h_++h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\})^{1/2}}
\end{aligned}\right\} \tag{7}$$

式中 $\Delta\hat{x}=\hat{x}_2-\hat{x}_1$，$\hat{x}_1$ 和 $\hat{x}_2$ 分别为由 $(\lambda R/\pi\omega_0)$ 归一化的坐标 x_1 和 x_2。

2.2 表面服从指数相关

相关函数等各量为：

$$\rho_\phi(\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2)=\exp(-|\boldsymbol{\zeta}_1-\boldsymbol{\zeta}_2|/\alpha) \tag{8}$$

$$W_+(0)=2\pi\alpha^2\exp(-\sigma_\phi^2)h_+,\qquad W_-(0)=2\pi\alpha^2\exp(-\sigma_\phi^2)h_-, \tag{9}$$

经过推导得：

$$
\left.\begin{aligned}
&\langle A_{r_1}\rangle = \pi\omega_0^2 \exp(-\sigma_\phi^2/2)\exp(-\hat{x}_1^2),\\
&\langle A_{r_2}\rangle = \pi\omega_0^2 \exp(-\sigma_\phi^2/2)\exp[-(\hat{x}_1+\Delta\hat{x})^2],\\
&\sigma_{r_1} = \pi\,\omega_0 a\,[h_+ - h_-\exp(-2\hat{x}_1^2)]^{1/2}\exp(-\sigma_\phi^2/2)\\
&\sigma_{i_1} = \pi\,\omega_0 a\,[h_+ + h_-\exp(-2\hat{x}_1^2)]^{1/2}\exp(-\sigma_\phi^2/2)\\
&\sigma_{r_2} = \pi\,\omega_0 a\exp(-\sigma_\phi^2/2)\{h_+ - h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\}^{1/2}\\
&\sigma_{i_2} = \pi\,\omega_0 a\exp(-\sigma_\phi^2/2)\{h_+ + h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\}^{1/2}\\
&\gamma_r = \frac{\exp(-\Delta\hat{x}^2/2)[h_+ - h_-\exp(-2\hat{x}_1^2)\exp(-2\hat{x}_1\Delta\hat{x})]}{([h_+ - h_-\exp(-2\hat{x}_1^2)]\{h_+ - h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\})^{1/2}}\\
&\gamma_i = \frac{\exp(-\Delta\hat{x}^2/2)[h_+ + h_-\exp(-2\hat{x}_1^2)\exp(-2\hat{x}_1\Delta\hat{x})]}{([h_+ + h_-\exp(-2\hat{x}_1^2)]\{h_+ + h_-\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\})^{1/2}}
\end{aligned}\right\}\tag{10}
$$

2.3　表面服从圆型相关

表面相关函数等各量为：　$$\rho_\phi(\boldsymbol{\xi}_1-\boldsymbol{\xi}_2) = \mathrm{circ}(|\boldsymbol{\xi}_1-\boldsymbol{\xi}_2|/a)\tag{11}$$

$$W_+(0) = \pi a^2\exp(-\sigma_\phi^2)[\exp(\sigma_\phi^2)-1],\qquad W_-(0) = \pi a^2\exp(-\sigma_\phi^2)[\exp(-\sigma_\phi^2)-1]\tag{12}$$

经过推导得：

$$
\left.\begin{aligned}
&\langle A_{r_1}\rangle = \pi\omega_0^2\exp(-\sigma_\phi^2/2)\exp(-\hat{x}_1^2),\\
&\langle A_{r_2}\rangle = \pi\omega_0^2\exp(-\sigma_\phi^2/2)\exp[-(\hat{x}_1+\Delta\hat{x})^2],\\
&\sigma_{r_1} = \pi/2\ \omega_0 a\{\exp(\sigma_\phi^2)-1-[\exp(-\sigma_\phi^2)-1]\exp(-2\hat{x}_1^2)\}^{1/2}\exp(-\sigma_\phi^2/2)\\
&\sigma_{i_1} = \pi/2\ \omega_0 a\{\exp(\sigma_\phi^2)-1+[\exp(-\sigma_\phi^2)-1]\exp(-2\hat{x}_1^2)\}^{1/2}\exp(-\sigma_\phi^2/2)\\
&\sigma_{r_2} = \pi/2\ \omega_0 a\exp(-\sigma_\phi^2/2)\{\exp(\sigma_\phi^2)-1-[\exp(-\sigma_\phi^2)-1]\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\}^{1/2}\\
&\sigma_{i_2} = \pi/2\ \omega_0 a\exp(-\sigma_\phi^2/2)\{\exp(\sigma_\phi^2)-1+[\exp(-\sigma_\phi^2)-1]\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\}^{1/2}\\
&\gamma_r = \exp(-\Delta\hat{x}^2/2)\{\exp(\sigma_\phi^2)-1-[\exp(-\sigma_\phi^2)-1]\exp(-2\hat{x}_1^2)\exp(-2\hat{x}_1\Delta\hat{x})\}\\
&\qquad /(\{\exp(\sigma_\phi^2)-1-[\exp(-\sigma_\phi^2)-1]\exp(-2\hat{x}_1^2)\}\\
&\qquad \times\{\exp(\sigma_\phi^2)-1-[\exp(-\sigma_\phi^2)-1]\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\})^{1/2}\\
&\gamma_i = \exp(-\Delta\hat{x}^2/2)\{\exp(\sigma_\phi^2)-1+[\exp(-\sigma_\phi^2)-1]\exp(-2\hat{x}_1^2)\exp(-2\hat{x}_1\Delta\hat{x})\}\\
&\qquad /(\{\exp(\sigma_\phi^2)-1+[\exp(-\sigma_\phi^2)-1]\exp(-2\hat{x}_1^2)\}\\
&\qquad \times\{\exp(\sigma_\phi^2)-1+[\exp(-\sigma_\phi^2)-1]\exp[-2(\hat{x}_1+\Delta\hat{x})^2]\})^{1/2}
\end{aligned}\right\}\tag{13}
$$

2　结果及讨论

根据(1)式、(7)式、(10)式和(13)式可计算不同相关情形下的相位差条件概率密度函数曲线，结果如图 2 所示。根据方程(2)式、(7)式、(10)式和(13)式，通过数值计算可得到相位差条件标准偏差的分布曲线。图 3 所示的是不同相关情形下的 $\sigma_\theta|_{I_1,\theta_1}\sim\Delta\hat{x}$ 分布曲线。

在上述计算中，对于不同的相关情形，采用的表面粗糙度和表面相关长度值是一样的。从图 2、图 3 可看出，粗糙表面高度起伏的相关函数形式对散斑场两点相位差的条件统计分布是有一定影响的。但从图 3 可看出，$\Delta\hat{x}$ 值减小，即散斑场两点相关性越好，表面相关函数的形式对散斑场两点相位差的条件标准偏差影响越小。对散斑场两点相位差条件标准偏差分

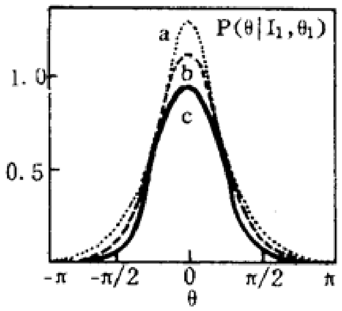

Fig. 2 Conditional probability density function of the phase difference under the condition of $I_1 = 25\langle I_1\rangle$ and $\theta_1 = 0$ with $N = 10$, $\sigma_\phi = 1$, $\hat{x}_1 = 0$ and $\Delta\hat{x} = 2$ for the cases that the rough surface obeys (a) Gaussian correlation, (b) circle correlation and (c) exponential correlation

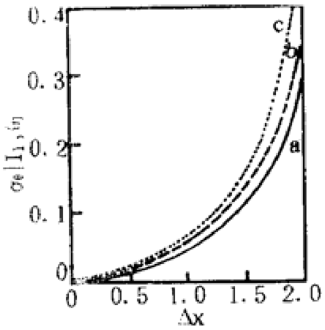

Fig. 3 Conditional standard deviation of the phase difference $\sigma_\theta|_{I_1,\theta_1}$ as a function of the normalized interval $\Delta\hat{x}$ under the condition of $I_1 = 25\langle I_1\rangle$ and $\theta_1 = 0$ with $N = 10$, $\sigma_\phi = 1$, $\hat{x}_1 = 0$ and $\Delta\hat{x} = 2$ for the cases that the rough surface obeys (a) Gaussian correlation, (b) circle correlation and (c) exponential

析的直接应用是估计激光粗糙表面干涉测量的精度。上述结果表明，在 $\Delta\hat{x}$ 值较大的情形下，即在散斑场两点相关性不好的情形下，粗糙表面相关结构对其影响是不可忽略的。因此在理论计算散斑相位差条件标准偏差时，应针对具体的粗糙表面采用与实际表面相符的相关模型。经验表明，对于各向同性的精密表面，采用高斯相关函数是符合实际的。

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Effects of Correlative Structure of Rough Surface on the Conditional statistical Distributions of Phase Difference for Gaussian Speckle in Far-Field

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Abstract The Gaussian random variable is assumed for the height variation of rough surface of weak diffuser. The conditional statistical distributions of phase difference for Gáussian speckle produced by a weak diffuser in the far-field is discussed in the case that the height variation of rough surface obeyes Gaussian correlation, exponential correlation and circle correlation respectively.

Key words correlative structure of rough surface, speckle, conditional statistical distribution of phase difference.