第16卷 第5期 光 学 学 报 Vol. 16, No. 5
1996年5月 ACTA OPTICA SINICA May, 1996

Circular Disc Linear Refractometer for Liquids

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Abstract A circular disc refractometer with linear output response of photo-electrical signal to liquid refractive index has been developed. The multiple reflection within the circular disc provides a design feature to reduce the influence of non-total internal reflection and significantly improves the measurement linearity. In addition, the circular disc refractometer can achieve quite good linearity even for non-parallel light beams.

Key words circular disc, refractometer.

1 Introduction

There are several types of well known refractometers, most of them are based on measurement of the deflection angle due to the refraction of light[1]. These refractomers have a non-linear scale with changes in refractive index and use optical methods to read the deflection angle. This type of refractometer is not suitable for on-line test, especially in chemical or food processing industries. Geake developed a linear refractometer based on critical reflection of light in the part of a circular cylinder[2]. Geake's refractometer can give a good direct reading of refractive index on a linear scale, but can not produce linear output by means of electrical signal with simple measures.

In this paper, an on-line linear refractometer is presented which uses a circular disc as the principle component. This circular disc refractometer provides quite good linearity with refractive index to be measured even for non-parallel light beams. This significant feature is provides by the multiple reflection within the circular disc which very effectively reduces the non-total internal reflection influence and improves the lineatity. The principle of the circular disc refractometer is discussed. A circular section U rod was chosen for the prototype due to low cost and ease of manufacture. The experiment results from this prototypt give good agreement with the computed results from the theoretical model. It is believed that the circular disc refractometer could be a

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Received 30 December 1994; revised 25 May 1995

promising piece of test equipment being used in industry production line, and also could be set in a pocket model for laboratory uses.

2 Principle of Circular Disc Refractometer

Fig. 1 shows the principle of circular disc refractometer which is constructed mainly by a half part of disc and with two light windows on the diameter section plane, and immersed in a liquid with refractive index n_l, to be measured. Direct a collimated light beam perpendicular to the diameter section plane into the disc. Generally, the light beam will be refracted out from the disc at the cylinder boundary according to the Snell' s refractive law. Suppose PA is an incident ray perpendicular to the diameter section plane, x is the distance between the ray PA and the cylinder boundary, O is the centre of curvature of the circular disc with the curvature radius r. AO is the normal at the incident point A to the outside surface. So the angle θ_i subtended by PA and AO is the incident angle, θ_r is the corespondent refractive angle (Fig. 1). If the refractive index of circular disc glass substrate is n_s and the liquid to be measured is n_l, for some pair of indexes n_s and n_l, there will be no refraction and total internal reflection will take place, that is

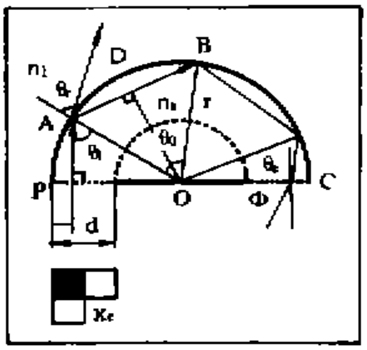

Fig. 1 Principle diagram of circular disc refractometer

$$\sin\theta_c = n_l/n_s \tag{1}$$

θ_c is called the critical angle. On the other hand, from the geometry in Fig. 1, it follows

$$\sin\theta_i = (r - x)/r \tag{2}$$

From equations (1) and (2), for the total internal reflection

$$n_l = n_s(1 - x_c/r) = k_c x_c + b_c \tag{3}$$

where $k_c = -n_s/r$, $b_c = n_s$, and x_c is the cirtical distance for the particular n_l. The parallel rays left to the x_c have the incident angle larger than the critical angle and will be totally internal reflected, while the rays right to the x_c will be partially reflected and refracted. Knowing the position of x_c, the associated liquid refractive index can be obtained.

If the intensity distribution of incident lgiht is homogeneous, then P_c, the reflected input light intensity with respect to the critical condition, will be as follows

$$P_c = P_0 x_c d_h \tag{4}$$

P_0 is the light intensity of the source per unit area, and d_h is the vertical hight of the circular disc. From equations (3) and (4), finally we have

$$P_c = k n_l + b \tag{5}$$

were $k = -P_0 d_h r/n_s$, and $b = P_0 d_h r$. Equation (5) shows us that the reflected light intensity corespondent to the critical condition is linear against n_l. If a photo-receiver is arranged at the exit of the circular disc to collect the output light, the output photo-current would be linear with n_l.

3 Light Transmission in Circular Disc

In order to know the real output intensity after the light rays have passed through the circular disc and out at the exit, there is a need to understand in more detail how the light is transmitted and finally how much the light intensity can get to the receiver. In the pressent paper, only the discussion on the optical transmission of the collimated incident light rays is carried out, the situation with the non-parallel rays can be found in another paper[3].

3.1 Light Transmission only at Outside Boundary

Parallel incident rays can not reach the inside boundary of the diameter plane of the circular disc before out the exit. In Fig. 1, drawing a perpendicular line OD to the AB from the curvature O, it is clear that the ΔPAO is congruent to the ΔADO, so the distance DO is equal to the distance PO. This means that the reflected ray of a perpendicular input ray can not touch the diameter section boundary and only reflect at the outside boundary of the circular disc.

3.2 Reflection Numbers

One of the main features of this design is the multiple reflection of a light ray in the circular disc. Because of the circular symmetry of the disc, an incident ray will be reflected several times with the same reflection angle until it gets out from the exit. In Fig. 1, θ_0 (i. e. $\angle AOB$) is the central angle subtended by the two sub-reflections, POC is the exit plane, so the reflection number N_c in the circular disc is

$$N_c = \mathrm{INT}\left(\frac{\angle AOC}{\angle AOB}\right) + 1 = \mathrm{INT}\left(\frac{\pi/2 + \theta_c}{\pi - 2\theta_c}\right) + 1 \quad (6)$$

where INT is a function which returns the largest integer less than or equal to the value of the numeric expression. The reflection number N_c from equation (6) is shown in Fig. 2, the curvature of the circular disc is $r = 50$ mm. It can be seen from Fig. 2 that for the range of liquid refractive index (from 1.33 to 1.52), the reflection numbers are larger than 3, the large reflection number is very welcome for reducing the non-total reflection effects, as will be shown later. As a reference the critical distance x_c corresponding to the liquid refractive index are also shown.

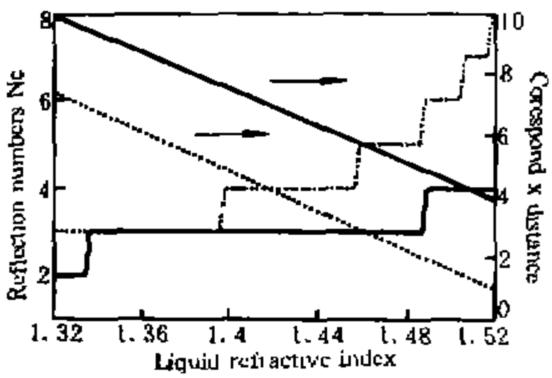

Fig. 2 Reflection numbers in the circular disc refractometer for different substrate refractive index $n_s = 1.55$ and $n_s = 1.65$ respectively

······ n_s(1.55) ------ x(1.55)

—— n_s(1.65) —— x(1.65)

3.3 Exit Rays from the Circular Disc

The transmitted rays which reach the exit could not be totally internal reflected back into the circular disc. In Fig. 1, θ_e is the input angle of output ray at the exit. The minimum incident critical angle is $\theta_{cm} = \sin^{-1}(n_{1m}/n_s)$, n_{1m} is the minimum n_1 to be measured. From the geometry in Fig. 1, $\theta_e \leqslant \pi/2 - \theta_{cm}$, generally, θ_{cm} is larger than 45°, so this incident angle θ_e is less than the minimum critical angle. This means that every ray which was transmitted throught the circular disc will be refracted out from the exit.

3.4 Open Angle of the Output Beams

At the exit, $\sin\phi = n_s \sin\theta_e = n_s \cos\theta_c$, ϕ is the open angle which will be maximum for lowest liquid refractive index n_{1m} to be measured, i. e.

$$\phi_M = \sin^{-1}(n_s \cos\theta_{cm}) = \sin^{-1}\sqrt{n_s^2 - n_{1m}^2} \quad (7)$$

the acceptance angle of photo-detector or focusing lens at the exit should be matched with the maximum open angle ϕ_M.

4 Transmission of Optical Energy in Circular Disc

The circular disc refractometer will be used as an on-line equipment by providing electrical signals proportional to the liquid refractive index. It is, therefore, important to develop a relationship between refractive index and the associated output light intensity.

4.1 Reflectivity of the Light In Circular Disc

All of the light in the circular disc is transmitted by means of reflection (total or non-total internal reflection). If input light intensity is P, reflected part is R, from Fresnel's equation[4],

$$R_{\parallel} = P_{\parallel} \frac{\tan^2(\theta_i - \theta_r)}{\tan^2(\theta_i + \theta_r)}, \qquad R_{\perp} = P_{\perp} \frac{\sin^2(\theta_i - \theta_r)}{\sin^2(\theta_i + \theta_r)} \tag{8}$$

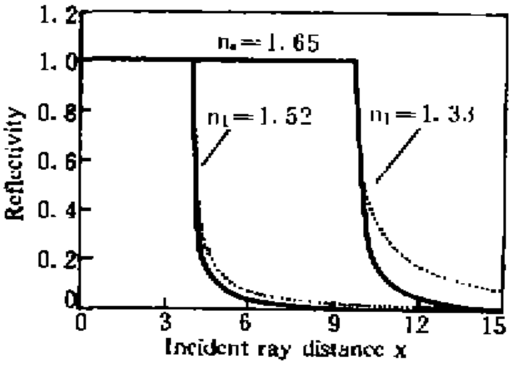

Fig. 3 Reflectivity via distance x in circular disc, substrate refractive index $n_s = 1.65$

where suffixes $\parallel$ and $\perp$ refer to the light polarised parallel and perpendicular to the incident plane respectively, the angles θ_i and θ_r are the incident and refractive angles respectively. Fig. 3 shows reflectivity against x in the circular disc for two typical extreme liquid refractive indexes, the curvature of the circular disc is $r = 50$ mm, and the window width is $d = 15$ mm. Refractive indexes 1.333 and 1.514 are corespondent to 0° and 90° Brix of sugar solution, respectively (degree Brix is a percentage weight measure of the sucrose sugar solution). Fig. 3 shows that although the reflectivity is rapidly reduced right to the critical position, there is small neighbourhood range where the reflectivity has still some value and can not be simply ignored. The lower the liquid refractive index, the higher the contribution from the non-total reflection. Hence the measurement of the lower liquid index is more affected by the non-total reflection. It can also be shown that parallel polarised light has less non-total reflectivity than the perpendicular polarised light. Numerical calculation shows that for $n_l = 1.52$, if the incident angle is 1° below the critical angle, only about 15% is reflected, if 2° below the critical angle, then 10% is reflected. However, for $n_l = 1.33$, the equivalent numbers are 1° below, 30% reflected, while 2° below still 20% reflected.

4.2 Total Internal Reflection

From equation (8), it can be shown that for total internal reflection $\theta_r = \pi/2$, $R_{\parallel} = P_{\parallel}$, $R_{\perp} = P_{\perp}$, $R_{\parallel} + R_{\perp} = P$. The optical intensity carried by the rays with the incident angle larger than or equal to the critical angle corresponding to the particular n_l, is transmitted completely in the circular disc with no theoretical loss. This part of the optical intensity would give expected linear response in the circular disc refractometer, using a photoelectric detector instead of measuring the separation line between the dark and bright area[2].

4.3 Nearly Total Internal Reflection

In addition to this total internal reflection, there are non-total reflections especially near-total reflections, which will make a contribution to the intensity output. The near-total internal reflection has quite a big influence on the output signal. Because this influence is not the same for different n_l, it will have the affect of introducing non-linearity. Generally the reflectivity is much less than unit, so if this non-total reflection takes place more times, the final reflected intensity associated with the non-total reflection will be reduced according to the power series due to the multiple reflections. Hence the non-linearity due to the non-total reflction could be improved significantly.

4.4 Effect of Multiple Reflection

Fig. 4 shows the normalised reflected output optical intensity variation with liquid refractive index. The curvature of the circular disc is $r = 50$ mm, and the window width $d = 15$ mm. The doted line is the total output intensity by single reflection, while the centre-lined curve is the output optical intensity from the exit after the ray was reflected multiple time in the circular disc. The non-total reflected intensity is averaged from the parallel and perpendicular polarised light. Fig. 4 shows that the optical intensity by means of the single reflection (total plus non-total reflected intensity) is not linear. However, after multiple reflection the output intensity is very linear and nearly coincident with the one corresponding to the total internal reflected intensity which is known to be linear.

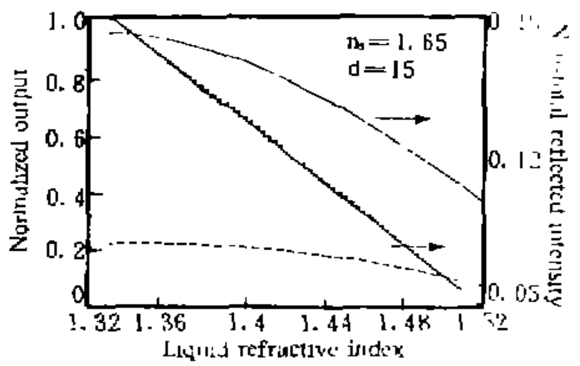

Fig. 4 Single and multiple total internal reflection results in the circular disc, substrate refractive index n_s = 1.65. Non-total reflected intensities are also shown

Fig. 4 shows that the refelcted part of parallel polarised light is nearly uniform over the whole measurement range and lower than the perpendicular part. Therefore, if high accuracy is need, the use of a polarised source in the parallel plane would be preferred.

Fig. 4 also shows that for some range (e.g. liquid refractive index from 1.33 to 1.44 for n_s = 1.55, or from 1.33 to 1.46 for n_s = 1.65) the non-total reflected part is nearly homogensous. In this kind of range high accuracy of measurement could be achieved. It is possible to optimise the circular disc design parameters (n_s, r and d) so that the no-linear influence of non-total reflection contribution can be reduced significantly, improving linearity over the whole measurement range. From Fig. 3 and Fig. 4, if the window width d is large enough the non-total reflection contribution can be nearly uniform for the whole measurement range. Based on the circular disc parameters and from the theoretical calculations and experiment data, it is possible to find a formula to calibrate the measurement data and give a true measure of refractive index.

The optical intensity associated with total internal reflection is linear, but combining with the non-total internal reflection (i.e. single reflection) it becomes non-linear. From the section 3.2 the reflection times in a circular disc is generally larger than 3. The biggest single reflected non-total intensity will be around 0.1 of the total output. After another two reflections this non-total reflection intensity will be reduced to 0.001. Then the final output becomes quite good linear. This is the most important feature of the circular disc refractometer. The linear feature of the circular disc refractometer is preserved even for non-parallel incident rays[5].

5 Circular Section U Rod

Using a circular glass or transparent plastic rod and bending to a U shape a simpe simulating prototypt for circular disc refractometer can be obtained. The important feature of this prototype is that there are two curvatures at any point of the curved part in a circular section U rod.

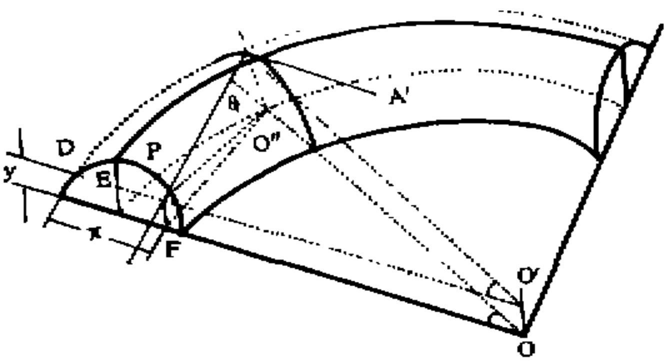

Fig. 5 Light rays in the circular section U rod

Suppose PA is an incident ray parallel to the axis of the straight part, x is the distance of the ray from the vertical plane at the left most original point, y is the vertical distance to the axial horizontal plane (see Fig. 5). The two curvatures at the incident point A are as follows. The first is in the plane parallel to the horizontal axial plane and with the curvature radius r_p (the first curvature, or global curvature) from the curvature centre O'. The second curvature is in the plane $AO'O$ vertical to the axial horizontal plane and the curvature radius is $d/2$ (d is the diameter of the circular section of the U rod), the curvature centre O'' is on the axis of the curved rod (local curvature). The tangential plane at the point A, however, is perpendicular to AO'', so AO'' is the unique normal to the surface at the point A. Then, the correspondent incident angle θ_i is $\angle PAO''$ (Fig. 5). From the geometry in the Fig. 5

$$\cos\theta_i = \frac{(2/r_p)(r - d/2 - r_p)(r - x)^2 + r_p^2 - r^2 + rd - y^2}{r_p d \cos\theta_p} \tag{9}$$

where $\theta_p = \angle PAO'$. So, if $\theta_i = \theta_c = \sin^{-1}(n_1/n_s)$, then the corresponding rays will be reflected totally. Generally $\theta_i < \theta_p$, for instance, when $x = d/4$, $y = d/4$, then $\theta_i = 71.8°$, $\theta_p = 74.34°$.

Let θ_i equal to the critical angle, then rewriting the equation (9),

$$f_1(r - x)^2 - n_{is} d\sqrt{r_p^2 - (r - x)^2} + f_2 = 0 \tag{16}$$

where $f_1 = (2/r_p)(r - r_p - d/2)$, $f_2 = r_p^2 - r^2 - rd - y^2$, $n_{is} = \sqrt{1 - n_l^2/n_s^2}$

Equation (10) is the critical condition for the incident rays and could be solved to find the solution of $(r - x)$ in terms of y and other parameters. In Fig. 6, the curve $22''$ is obtained from equation (10), the area to the left of $22''$ corresponds to te critical input.

Fig. 7 shows the normalised caculated output result for a circular section U rod. The solid and dashed lines represent the output intensity for single and multiple reflection respectively. The caculated refractive indexes ranged from 1.349 to 1.442 and were chosen for easy comparison with experiment data.

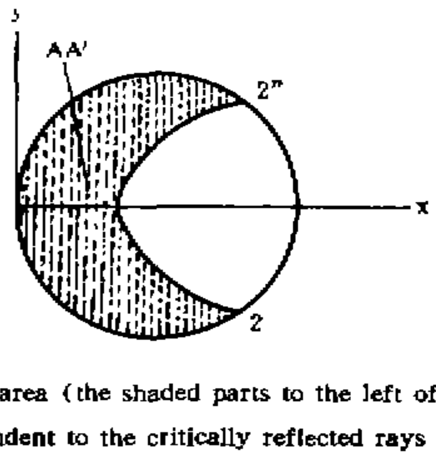

Fig. 6 Section area (the shaded parts to the left of 22') corespondent to the critically reflected rays in the circular section U rod

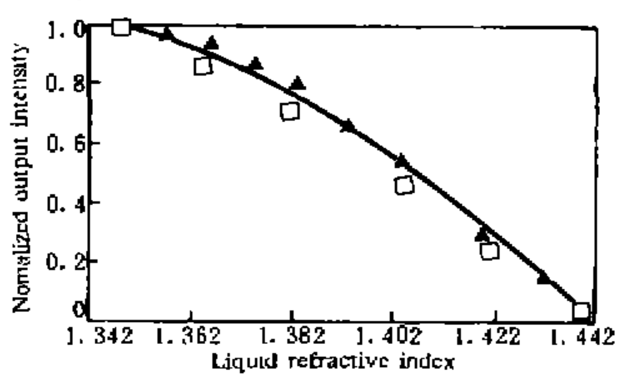

Fig. 7 Normalised theoretical output and experimental data from the circular section U rod refractometer prototype

6 Experiment Prototype

In order to check the mathematical model for the circular disc refractometer, a prototype has been developed. A glass rod was bent into U rod with the outside curvature approximately $r = 60$ mm. The refractive index of the substrate is $n_s = 1.475$ (Pyrex glass). The diameter of the rod cross-section is $d = 10$ mm. A light proof housing was designed and fabricated to hold the U rod and the liquids to be measured.

A LED was chosen to provide good light purity and good constancy. A photodiode, manufactured by Hamamatsu Photonics, was chosen as the photoreceiver, an op-amp detector circuit is used to magnify and convert the photocurrent to voltage output.

Sugar water was used as test liquids. Different concentrations of sugar produced the refractive index range from 1.349 to 1.442, calibrated before testing with an Abbe's refractometer. The experiments were carried out with silvered and non-silvered circular section U rod. Only the critical surface was exposed in the silvered rod and this was used to measure the effect of one reflection only. Multiple reflection were observed with the non-silvered type.

The test results, which have been normalised, compare well with the theoretical calculations, are shown in Fig. 7. This means that the theoretical analysis on the ray transmission in circular section U rod is reasonable correct. Of course some differences between the theoretical calculation and the experiment data do exist and could be due to the following reasons.

(1) The global curvature of the U rod is not homogeneous, so during transmission of the light ray in the U rod, some rays would be totally reflected or some non-totally reflected.

(2) The silver coating of the U rod had an uncoated window size larger than the critical surface, so partial rays were reflected once while others were reflected twice, encountering different non-total reflection losses.

7 Improving the Accuracy of Circular Disc Refractometer

From equation (3), the possible parameters for design consideration are n_s, r, and x. Differentiating equation (3),

$$\frac{\delta|\Delta n_l|}{\Delta n_l} = \frac{|\delta n_s|}{n_s} + \frac{|\delta r|}{r} + \frac{|\delta x|}{x} \tag{11}$$

where $\Delta n_l = n_s - n_l$. Generally the first and second terms are far less than the third term in equation (11), so the main error would come from the third term, i.e. errors in the determination of the critical position x_c. Equation (11) shows that in order to achieve the higher accuracy, the larger the parameters n_s, r, and x, the better the circular disc refractometer will be. The larger n_s not only reduces the materials non-homogeneous problem, but also gains the advantage of evening out the contribution of non-total internal reflection. Larger x_c will benefit the reduction of the third term in equation (11), but this will reduce the reflection numbers, so this would need optimising. Using a parallel polarised light source is another way to reduce the non-total reflection influence. Parallel polarised light not only has a lower but also nearly uniform value of reflectivity. A parallel beam is preferred, however, non-parallel rays can also achieve quite good linearity[3]. Non-critical incident rays will come out from the circular disc, these rays can re-enter the circular disc and appear to be an error source. However, entering the circular disc means that the incident angle is less than the critical angle inside the outside boundary. The effect will be that the most of the power will refract out and just a small non-total reflected part is transmitted. This part will encounter multiple reflection and will reduce significantly at the exit so that ray re-entry is not a problem.

Conclusion This paper shows that a circular disc structure can be used to provide an on-line linear refractometer with electric output. Its construction is simple, its linearity is quite good, and the prototype is easy to realise. The total internal reflection gives linear response, but single reflection produces non-linear effects, fortunately the multiple reflection in the circular disc recovers the linearity again. The theoretical analysis and experimental results with a circular section U rod can achieve good linearity. It is believed that a circular disc refractometer could be a promising test equipment for industry use such as food processing and chemical manufacturing. A circular disc refractometer is in preparation and experimental result will be available shortly.

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圆盘型液体线性光电折射率计

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(收稿日期:1994年12月30日;收到修改稿日期:1995年5月25日)

摘 要 提出一种具有线性响应的圆盘型液体光电折射率计。光线在圆盘内的多次反射显著地消除了非全内反射的影响，有效地改善了测量线性度。这种圆盘型光电折射率计甚至可对非平行入射光束仍保持相当好的线性度。

关键词 圆盘型， 折射率计。

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