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泵浦统计、原子相干性及多单光子激光场光子数压缩效应*

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摘 要 利用朗之万方程理论证明，在多单光子激光振荡的线性区域，泵浦统计、原子相干性及原子位相起伏可导致光子数压缩效应。

关键词 多单光子关联辐射激光器， 泵浦统计， 原子相干性， 光子数压缩效应。

1 引 言

如何猝灭或压缩激光场的量子噪声是现代量子光学的主要课题之一。有许多方法可达到这一目的，如利用原子记忆效应[1,2]、抑制泵浦噪声[2~10]及关联辐射激光器(CEL)[11,12]等。对非相干泵浦单光子激光器，泵浦统计只在高光强时才显得重要，而对关联辐射激光器，泵浦统计在低光强时就已显得重要了[9]。但在激光运转的线性区域，无论相干还是非相干泵浦，也无论是双能级构型体系[2,9]还是 Λ 能级构型体系[10]，泵浦统计都不能导致单光子激光场的光子数压缩效应。

本文利用朗之万方程理论证明，若考虑原子的相干性及原子的位相起伏，在激光运转的线性区域，对一般 V 能级构型体系，泵浦统计可导致多单光子激光场的光子数压缩效应。

2 多单光子关联辐射激光器的朗之万方程

图 1 为本文考虑的多单光子关联辐射激光器体系的示意图。近简并的 $n-1$ 个原子能级 $|1\rangle$、$|2\rangle$、…，$|n-1\rangle$ 为激光上能级，而能级 $|n\rangle$ 为激光下能级，若 $n-1=2$，即为 V 能级构型体系[13]。将一束初始时刻各能级处在相干叠加态的原子注入激光腔，并与单模光场(频率为 ω) 作用，在旋转波近似下，体系的哈密顿量为($\hbar=1$)：

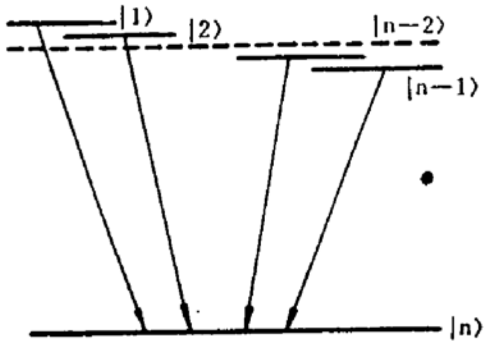

Fig. 1 Schematic diagram of the level structure for the correlated spontaneous-emission lasers

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$$H=\omega a^{+}a+\sum_{j}\sum_{l=1}^{n-1}\omega_{l}|l\rangle\langle l|^{j}+g\sum_{j}\theta(t-t_{j})\sum_{l=1}^{n-1}\left[a^{+}\sigma_{nl}^{j}+a(\delta_{nl}^{j})^{+}\right]. \tag{1}$$

其中 a、a^{+} 分别为光场湮灭、产生算符，$\delta_{nl}^{j}=|n\rangle\langle l|^{j}$ 为原子极化算符，g 为耦合常数，$\theta(t)$ 为阶跃函数。由(1)可得如下的海森堡方程：

$$\left.\begin{aligned}
\dot{a}&=-i\omega a-\frac{\gamma}{2}a-ig\sum_{j}\theta(t-t_{j})\sum_{l=1}^{n-1}\sigma_{nl}^{j}+f_{a},\\
\dot{\sigma}_{nn}^{j}&=-\Gamma\sigma_{nn}^{j}-jg\theta(t-t_{j})\sum_{l=1}^{n-1}\left[a^{+}\sigma_{nl}^{j}-a(\sigma_{nl}^{j})^{+}\right]+f_{nn}^{j},\\
\dot{\sigma}_{ll}^{j}&=-\Gamma\sigma_{ll}^{j}-ig\theta(t-t_{j})\left[a(\sigma_{nl}^{j})^{+}-a^{+}\sigma_{nl}^{j}\right]+f_{ll}^{j},\\
\dot{\sigma}_{ll'}^{j}&=(-i\omega_{l'l}-\Gamma)\sigma_{ll'}^{j}-ig\theta(t-t_{j})\left[a(\sigma_{nl}^{j})^{+}-a^{+}\sigma_{nl'}^{j}\right]+f_{ll'}^{j},\\
\dot{\sigma}_{nl}^{j}&=(-i\omega_{ln}-\Gamma)\sigma_{nl}^{j}+ig\theta(t-t_{j})a\Big(\sum_{l'=1}^{n-1}\sigma_{l'l}^{j}-\sigma_{nn}^{j}\Big)+f_{nl}^{j}.
\end{aligned}\right\} \tag{2}$$

其中 $l=1,2,\cdots,n-1$，$\sigma_{mm}^{j}=|m\rangle\langle m|^{j}$，$\omega_{mn}=\omega_{m}-\omega_{n}$，$\gamma$ 为腔场衰减常数，Γ 为各原子能级寿命，f 为朗之万噪声算符，满足[14, 15]：

$$\begin{gathered}
\langle f_{a}\rangle=0,\qquad \langle f_{a^{+}}(t)f_{a}(t')\rangle=\gamma\bar{n}\delta(t-t'),\\
\langle f_{a}(t)f_{a}(t')\rangle=0,\qquad \langle f_{m}^{j}(t)f_{m}^{j}(t')\rangle=\Gamma\langle\sigma_{mm}^{j}(t)\rangle\delta(t-t'),\\
\langle(f_{nl}^{j}(t))^{+}f_{nl'}^{j}(t')\rangle=\Gamma\langle\sigma_{ll'}^{j}(t)\rangle\delta(t-t'),\\
\langle f_{nl}^{j}(t)f_{ll'}^{j}(t')\rangle=\Gamma\langle\sigma_{nl'}^{j}(t)\rangle\,\delta(t-t'),\\
\langle(f_{nl}^{j}(t)^{+}f_{nn}^{j}(t')\rangle=\Gamma\langle(\sigma_{nl}^{j}(t))^{+}\rangle\,\delta(t-t').
\end{gathered}$$

为了方便起见，定义慢变算符

$$\tilde{a}(t)=a(t)\exp(i\nu t),\qquad \tilde{\sigma}_{nl}^{j}(t)=\sigma_{nl}^{j}(t)\exp(i\nu t), \tag{3}$$

ν 为激光振荡频率。由于个别原子算符对绝热近似特别敏感，而平均的宏观原子算符则不然，因而定义下列宏观量：

$$\left.\begin{aligned}
M_{nl}(t)&=-i\sum_{j}\theta(t-t_{j})\,\tilde{\sigma}_{nl}^{j}(t),\\
M_{ll'}(t)&=-i\sum_{j}\theta(t-t_{j})\,\tilde{\sigma}_{ll'}^{j}(t),\qquad (l>l')\\
M_{ll'}(t)&=i\sum_{j}\theta(t-t_{j})\,\tilde{\sigma}_{ll'}^{j}(t),\qquad (l<l')\\
N_{m}(t)&=\sum_{j}\theta(t-t_{j})\,\tilde{\sigma}_{mm}^{j}(t).\qquad (m=n,\ l)
\end{aligned}\right\} \tag{4}$$

由(2)、(3)及(4)可得：

$$\left.\begin{aligned}
\dot{a}&=-i(w-\nu)a-\frac{\gamma}{2}a+g\sum_{l=1}^{n-1}M_{nl}+\tilde{f}_{a},\\
\dot{N}_{n}&=R\rho_{nn}-\Gamma N_{n}+g\sum_{l=1}^{n-1}(a^{+}M_{nl}+aM_{nl}^{+})+F_{nn},\\
\dot{N}_{l}&=R\rho_{ll}-\Gamma N_{l}-g(a^{+}M_{nl}+aM_{nl}^{+})+F_{ll},\\
\dot{M}_{nl}&=-iR\rho_{ln}+[-i(\omega_{ln}-\nu)-\Gamma]M_{ln}+ga\Big(i\sum_{\substack{l'=1\\(l'>l)}}^{n-1}M_{l'l}-i\sum_{\substack{l'=1\\(l'<l)}}^{n-1}M_{l'l}\\
&\qquad+N_{l}-N_{n}\Big)+F_{nl},\\
\underset{(l>l')}{\dot{M}_{ll'}}&=-iR\rho_{l'l}+(-iW_{l'l}-\Gamma)M_{ll'}+ig(a^{+}M_{nl'}+aM_{nl}^{+})+F_{ll'},\\
\underset{(l<l')}{\dot{M}_{ll'}}&=iR\rho_{l'l}+(-iW_{l'l}-\Gamma)M_{ll'}-ig(a^{+}M_{nl'}+aM_{nl}^{+})+F_{ll'}.
\end{aligned}\right\} \tag{5}$$

其中 R 为原子的注入速率，ρ_{mn} 为初始时刻原子的密度矩阵元，而噪声算符 F 为：

$$
\left.\begin{aligned}
F_{nn} &= \sum_j \theta(t-t_j)\,\tilde{f}^j_{nn}(t) + \sum_j \delta(t-t_j)\,\tilde{\sigma}^j_{nn}(t_j) - R\rho_{nn},\\
F_{ll} &= \sum_j \theta(t-t_j)\,\tilde{f}^j_{ll}(t) + \sum_j \delta(t-t_j)\tilde{\sigma}^j_{ll}(t_j) - R\rho_{nn},\\
\underset{(l>l')}{F_{ll'}} &= -i\sum_j \theta(t-t_j)\,\tilde{f}^j_{ll'}(t) - i\sum_j \delta(t-t_j)\,\tilde{\sigma}^j_{ll'}(t_j) + iR\rho_{l'l},\\
\underset{(l<l')}{F_{ll'}} &= i\sum_j \theta(t-t_j)\,f^j_{ll'}(t) + i\sum_j \theta(t-t_j)\,\sigma^j_{ll'}(t_j) - iR\rho_{l'l},\\
F_{nl} &= -i\sum_j \theta(t-t_j)\,f^j_{nl}(t) - i\sum_j \delta(t-t_j)\,\sigma^j_{nl}(t_j) + iR\rho_{ln}.
\end{aligned}\right\} \tag{6}
$$

在导出(5)式时，利用了[8, 9]

$$
\left.\begin{aligned}
\langle -i\sum_j \delta(t-t_j)\,\tilde{\sigma}^j_{mn}\rangle_s &= -iR\int_{-\infty}^{\infty} dt_j\,\delta(t-t_j)\,\langle\tilde{\sigma}^j_{mn}(t_j)\rangle = -iR\rho_{nm},\\
\langle \sum_j \delta(t-t_j)\,\tilde{\sigma}^j_{mm}(t_j)\rangle_s &= R\int_{-\infty}^{\infty} dt_j\,\delta(t-t_j)\langle\tilde{\sigma}^j_{mm}(t_j)\rangle \doteq R\rho_{mm},
\end{aligned}\right\} \tag{7}
$$

下标 s 表示泵浦统计平均。

利用[8, 9]

$$
\left.\begin{aligned}
\langle \sum_j \delta(t-t_j)\delta(t'-t_j)\rangle_s &= \langle\sum_j \delta(t-t_j)\rangle_s\delta(t-t') = R\delta(t-t'),\\
\langle \sum_{\substack{j-k\\ j\neq k}} \delta(t-t_j)\,\delta(t'-t_k)\rangle_s - R^2 &= -\eta R\delta(t-t'),
\end{aligned}\right\} \tag{8}
$$

可得噪声算符 $F(t)$ 的二阶关联函数

$$
\left.\begin{aligned}
\langle F_{nl}(t)F_{nl'}(t')\rangle &= \eta R\rho_{l'n}\rho_{ln}\,\delta(t-t'),\\
\langle F^{+}_{nl}(t)F_{nl'}(t')\rangle &= \big[\Gamma\langle N_l\rangle + i\Gamma(\langle \underset{(l>l')}{M_{ll'}}\rangle - \langle \underset{(l<l')}{M_{ll'}}\rangle)\\
&\quad + R\rho_{l'l} - \eta R\rho^{*}_{ln}\rho_{l'n}\big]\,\delta(t-t'),\\
\langle F^{+}_{nl}(t)F^{+}_{nl'}(t')\rangle &= \eta R\rho^{*}_{l'n}\rho_{ln}\,\delta(t-t').
\end{aligned}\right\} \tag{9}
$$

其中 η 是表示泵浦统计的参数，$\eta=0$ 表示泊松泵浦，$\eta=1$ 表示亚泊松泵浦(注入原子的粒子数起伏为零)。假定原子的衰减速率 Γ 远大于光子的衰减速率 γ，即原子变量 N_m、M_{mn} 为快变量(与光场相比)，可绝热消去 N_m 及 M_{mn}。下面利用线性理论处理问题。忽略 F_{nn}、F_{ll}、$F_{ll'}$，由 $\dot{N}_n=\dot{N}_l=\dot{M}_{ll'}=0$ 且只保留 g^0 项可得：

$$
\left.\begin{aligned}
&N_n = \frac{R}{\Gamma}\,\rho_{nn}, \qquad N_l = \frac{R}{\Gamma}\,\rho_{ll},\\
&M_{ll'} = -i\frac{R}{\Gamma}\,L_{ll'}\,\rho_{l'l}, \qquad (l>l'), \qquad M_{ll'} = i\frac{R}{\Gamma}\,L_{ll'}\rho_{l'l}, \qquad (l<l')\\
&L_{l'l} = \frac{\Gamma}{\Gamma + i\omega_{l'l}}
\end{aligned}\right\} \tag{10}
$$

由 $\dot{M}_{nl}=0$ 并利用(10)式可得

$$
M_{nl} = L_{ln}\Big[-i\frac{R}{\Gamma}\,\rho_{ln} + \frac{Rg}{\Gamma^2}\,(\sum_{l'=1}^{n-1} L_{ll'}\,\rho_{ll'} - \rho_{nn}) + \frac{F_{nl}}{\Gamma}\Big], \tag{11}
$$

其中
$$
L_{ln} = \frac{\Gamma}{\Gamma + i(\omega_{ln}-\nu)}.
$$

将(11)式代入(5)式中关于 $\dot{a}$ 的表达式，有

$$
\dot{a} = -i(\omega-\nu)a - \frac{\gamma}{2}a - is\sum_{l=1}^{n-1} L_{ln}\rho_{ln} + \frac{1}{2}\,\beta a\Big[\sum_{l,l'=1}^{n-1} L_{ll'}\rho_{ll'} - (n-1)\rho_{nn}\Big] + F_a, \tag{12}
$$

其中，
$$
S = \frac{Rg}{\Gamma}, \qquad \beta = \frac{2Rg^2}{\Gamma^2},
$$

噪声算符 F_a 为：

$$F_a=\tilde{f}_a+\frac{g}{\Gamma}\sum_{l=1}^{n-1}L_{ln}F_{nl}.$$

为了简化下面的分析，将算符方程(5)及(12)转为经典 c 数方程。为此选择正规次序：a^+，M_{n1}^+，M_{n2}^+，…，$M_{n,n-1}^+$，$\underset{l>l'}{M_{ll'}^+}$，$N_1$，…，$N_n$，$M_{ll'}$，$M_{n1}$，…，$M_{n,n-1}$，$a$。这样(5)式及(12)式即为 c 数随机微分方程。(12)式为：

$$\dot{\alpha}=-i(\omega-\nu)\sigma-\frac{\gamma}{2}\alpha-is\sum_{l=1}^{n-1}L_{ln}\rho_{ln}+\frac{1}{2}\beta\alpha\Big[\sum_{l,l'=1}^{n-1}L_{ll'}\rho_{ll'}-(n-1)\rho_{nn}\Big]+\mathscr{F}_a \tag{13}$$

其中

$$\mathscr{F}_a=\tilde{f}_a+\frac{g}{\Gamma}\sum_{l=1}^{n-1}L_{ln}\mathscr{F}_{nl},$$

α 及 $\mathscr{F}_{nl}$ 是算符 a 及 F_{nl} 的经典对应量。注意到在线性区域内噪声算符的正规次序关联函数与经典对应量的关联函数一样[2,9]，并假定 $|1\rangle$，…，$|n-1\rangle$ 近简并($\Gamma\gg\omega_{ll'}$)，有：

$$\left.\begin{aligned}\langle 2D_{a^*a}\rangle\delta(t-t')&=\langle\mathscr{F}_a^*(t)\mathscr{F}_a(t')\rangle\\&=\frac{1}{2}\beta\sum_{l,l'=1}^{n-1}L_{ln}^*L_{l'n}\left[2\rho_{l'l}-\eta\rho_{ln}^*\rho_{l'n}\right]\delta(t-t')\\\langle 2D_{aa}\rangle\,\delta(t-t')&=\langle\mathscr{F}_a(t)\mathscr{F}_a(t')\rangle\\&=\frac{1}{2}\eta\beta\sum_{l,l'=1}^{n-1}L_{ln}L_{l'n}\rho_{ln}\rho_{l'n}\delta(t-t')\end{aligned}\right\} \tag{14}$$

3 激光运转及光子数压缩效应

为了便于分析光强 I 及位相 φ，作变数代换：$\alpha=\sqrt{I}\exp(i\varphi)$，即$(\alpha,\alpha^*)\to(I,\varphi)$。由方程(13)得 I 和 φ 的朗之万方程为：

$$\dot{I}=(A\beta\cos\psi-\gamma)I+2BS\sqrt{I}\sin(\Phi-\varphi)+F_I, \tag{15}$$

$$\dot{\varphi}=-(\omega-\nu)+\frac{1}{2}\beta A\sin\psi-\frac{BS}{\sqrt{I}}\cos(\Phi-\varphi)+F_\varphi. \tag{16}$$

其中

$$B\exp(i\Phi)=\sum_{l=1}^{n-1}L_{ln}\rho_{ln},\qquad A\exp(i\psi)=\sum_{l=1}^{n-1}L_{ln}\Big(\sum_{l'=1}^{n-1}\rho_{ll'}-\rho_{nn}\Big),$$

噪声算符 F_I 及 F_φ 为：

$$F_I=\sqrt{I}\left[\mathscr{F}_a\exp(-i\varphi)+\mathscr{F}_a^*\exp(i\varphi)\right],$$

$$F_\varphi=\left[\mathscr{F}_a\exp(-i\varphi)-\mathscr{F}_a^*\exp(i\varphi)\right]/(2i\sqrt{I}).$$

关联函数为

$$\left.\begin{aligned}\langle 2D_{II}\rangle\,\delta(t-t')&=\langle F_I(t)F_I(t')\rangle=4I\left[\langle D_{a^*a}\rangle+R_e\langle D_{aa}\exp(-i2\varphi)\rangle\right]\delta(t-t')\\&=2\beta I\Big[\sum_{l,l'=1}^{n-1}L_{ln}^*L_{l'n}\rho_{l'l}-\eta B^2\sin^2(\Phi-\varphi)\Big]\delta(t-t')\\\langle 2D_{\varphi\varphi}\rangle\,\delta(t-t')&=\langle F_\varphi(t)F_\varphi(t')\rangle=\frac{1}{I}\left[\langle D_{a^*a}\rangle-R_e\langle D_{aa}\exp(-i2\varphi)\rangle\right]\delta(t-t')\\&=\frac{\beta}{2I}\Big[\sum_{l,l'=1}^{n-1}L_{ln}^*L_{l'n}\rho_{l'l}-\eta B^2\cos^2(\Phi-\varphi)\Big]\delta(t-t')\end{aligned}\right\} \tag{17}$$

由(16)式可得锁相条件(满足稳定性条件 $\frac{d\dot{\varphi}}{d\varphi}<0$)

$$\varphi_0=\Phi-\frac{\pi}{2},$$

及频移

$$\omega-\nu=\frac{1}{2}\beta A\sin\psi,$$

在锁相条件下

$$\left.\begin{aligned}D_{II}&=\beta I\Big[\sum_{l,l'=1}^{n-1}L_{ln}^{*}L_{l'n}\rho_{l'l}-\eta\Big|\sum_{l=1}^{n-1}L_{ln}\rho_{ln}\Big|^{2}\Big],\\D_{\varphi\varphi}&=\frac{\beta}{4I}\sum_{l,l'=1}^{n-1}L_{ln}^{*}L_{l'n}\rho_{l'l}.\end{aligned}\right\}\tag{18}$$

由(18)式可见，泵浦统计只对光子数噪声有影响而对位相噪声没有影响。当 $\eta=1$ 时光子数噪声降低最多。由(15)可得平均光强为

$$\sqrt{I_0}=\sqrt{\langle a^{+}a\rangle}=2BS/(\gamma-A\beta\cos\psi),\tag{19}$$

由 $d\dot{I}/dI<0$ 可得(19)式稳定的条件为：

$$\gamma-A\beta\cos\psi>0,\tag{20}$$

由(15)、(16)式可得光场噪声起伏所满足的微分方程

$$\left.\begin{aligned}\frac{d}{dt}\langle:(\Delta\varphi)^2:\rangle&=-\frac{2BS}{\sqrt{I}}\langle:(\Delta\varphi)^2:\rangle+2\langle D_{\varphi\varphi}\rangle,\\\frac{d}{dt}\langle:(\Delta I)^2:\rangle&=-(\gamma-A\beta\cos\psi)\langle:(\Delta I)^2:\rangle+2\langle D_{II}\rangle.\end{aligned}\right\}\tag{21}$$

其中

$$\langle D_{\varphi\varphi}\rangle=\langle\Delta\varphi F_{\varphi}\rangle,\qquad\langle D_{II}\rangle=\langle\Delta IF_{I}\rangle,$$

::表示正规次序。由(21)式的定态解及

$$\langle(\Delta n)^2\rangle=\langle:(\Delta I)^2:\rangle+\langle I\rangle,\qquad\langle(\Delta\varphi)^2\rangle=\langle:(\Delta\varphi)^2:\rangle+\frac{1}{4\langle I\rangle},$$

可得光子数起伏 $\langle(\Delta n)^2\rangle$ 及位相起伏 $\langle(\Delta\varphi)^2\rangle$ 为：

$$\langle(\Delta n)^2\rangle=I_0\left[1+\frac{2\Big(\sum_{l,l'=1}^{n-1}L_{ln}^{*}L_{l'n}\rho_{l'l}-\eta\Big|\sum_{l=1}^{n-1}L_{ln}\rho_{ln}\Big|^2\Big)}{\frac{\gamma}{\beta}-\Big|\sum_{l=1}^{n-1}L_{ln}\Big(\sum_{l'=1}^{n-1}\rho_{ll'}-\rho_{nn}\Big)\Big|\cos\psi}\right],\tag{22}$$

$$\langle(\Delta\varphi)^2\rangle=\frac{1}{4I_0}\left[1+\frac{2\sum_{l,l'=1}^{n-1}L_{ln}^{*}L_{l'n}\rho_{l'l}}{\frac{\gamma}{\beta}-\Big|\sum_{l=1}^{n-1}L_{ln}\Big(\sum_{l'=1}^{n-1}\rho_{ll'}-\rho_{nn}\Big)\Big|\cos\psi}\right].\tag{23}$$

取 $\eta=1$，$\omega_{ln}-\nu\approx0$，并假定原子处在完全相干叠加态(不考虑原子的位相起伏)，则在一定条件下[16]有

$$\left.\begin{aligned}\sum_{l,l'=1}^{n-1}L_{ln}^{*}L_{l'n}\rho_{l'l}&\approx\rho_{11}\frac{\sin^2[(n-1)\theta/2]}{\sin^2(\theta/2)},\\\Big|\sum_{l=1}^{n-1}L_{ln}\rho_{nn}\Big|^2&\approx\rho_{11}\rho_{nn}\frac{\sin^2[(n-1)\theta/2]}{\sin^2(\theta/2)}.\end{aligned}\right\}\tag{24}$$

其中 $\theta=(\theta_{jn}-\theta_{ln})/(j-1)$。将(24)式代入(22)，可看到 $\langle(\Delta n)^2\rangle>I_0$，即光子数无压缩效应。若不考虑态 $|1\rangle$，$|2\rangle$，…，$|n-1\rangle$ 间的相干性，(原子位相起伏为无穷大)，但态 $|n\rangle\leftrightarrow|1\rangle$，

…，$|n-1\rangle$ 间仍处在完全相干叠加态，$|\rho_{jn}|^2=\rho_{jj}\rho_{nn}$，则

$$\sum_{l,l'=1}^{n-1}\rho_{l'l}=(n-1)\rho_{11}. \tag{25}$$

将此式代入(22)并取 $\theta=0$，有

$$\langle(\Delta n)^2\rangle=I_0[1+2(n-1)\rho_{11}[1-(n-1)\rho_{nn}]/(\gamma/\beta-|1-n\rho_{nn}|\cos\psi)] \tag{26}$$

由(26)式知若 $\rho_{nn}=1/(n-1)$，则$\langle(\Delta n)^2\rangle=I_0$，即光场呈泊松分布。若 $\rho_{nn}>1/(n-1)$，则$\langle(\Delta n)^2\rangle<I_0$，光子数有压缩效应。若$\gamma/\beta\gg|1-n\rho_{nn}|$，则压缩效应随简并的上能级数$(n-1)$的增加而增加。其它情形下，光子数压缩效应随$(n-1)$是增加还是减小依赖于$\gamma/\beta$及$\rho_{nn}$的取值。另一方面若 $\rho_{nn}>1/(n-1)$ 则激光增益

$$A=1-n\rho_{nn}<0 \tag{27}$$

但由于驱动项 BS 的存在，仍可得稳定激光输出[见(19)式]。在(19)式情况下，由(23)式可以看到位相噪声也有所降低，这主要是消除了上能级间的原子相干性对自发辐射噪声的贡献。

若 $|n\rangle$ 为激光上能级，$|n-1\rangle$，…，$|1\rangle$ 为激光下能级，则

$$\left.\begin{aligned}
&B\exp(i\Phi)=\sum_{l=1}^{n-1}L_{nl}\rho_{nl},\\
&A\exp(i\psi)=\sum_{l=1}^{n-1}L_{nl}\Big(\rho_{nn}-\sum_{l'=1}^{n-1}L_{l'l}\rho_{l'l}\Big),\\
&D_{II}=\beta I_0\Big(\sum_{l=1}^{n-1}|L_{nl}|^2\rho_{nn}-\eta\Big|\sum_{l=1}^{n-1}L_{nl}\rho_{nl}\Big|^2\Big),\\
&D_{\varphi\varphi}=\frac{\beta}{4I_0}\sum_{l=1}^{n-1}|L_{nl}|^2\rho_{nn}.
\end{aligned}\right\} \tag{28}$$

由(28)式容易证明，在任何条件下此体系都无光子数压缩效应。若 $n-1=2$，则(28)式就是文献[10]的结果。

结 论 本文利用朗之万方程证明，在激光振荡的线性区域，若近简并的 $(n-1)$ 个原子上能级间的相干性不存在(这相当于上能级间的原子位相起伏为无穷大)，但原子上能级和下能级间的相干性存在时，泵浦统计可导致多单光子关联辐射激光器的光子数压缩效应。

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Pump Statistics, Atomic Coherences and Photon-Number Squeezing in Many One-Photon Laser Field

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Abstract Using the Langevin equation theory, we show in this paper that pump statistics, atomic coherence and atomic phase fluctuations can result in photon-number squeezing in the linear region of operation of many one-photon lasers.

Key words many one-photon correlated-spontaneous-emission laser, pump statistics, atomic coherence, photon-number squeezing.