

# A New Method of Tolerating Rigid Body Motion for Digital Shearography

F. Chen    Y. Y. Hung    C. T. Griffen

(*Department of Mechanical Engineering, Oakland University, Rochester Hills, MI 48309 U. S. A*)

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**Abstract** One of the limitation of speckle correlation interferometric techniques including shearography is that the rigid body motion yields decorrelation of speckle patterns before and after deformation which results in the deterioration of fringe visibility. This paper proposes a method which can effectively tolerate rigid body motion. It involves rearranging the recorded speckles before and after deformation in order to enlarge the speckle size, averaging speckles to improve the quality of fringe patterns, and reconstructing fringe patterns and iteration of smooth to further suppress the speckle noise to obtain decent result.

**Key words** speckle rearranging, rigid body motion, digital shearography.

## 1 Introduction

As well known, rigid body motion produce decorrelation of speckle patterns which causes the deterioration of fringe visibility in speckle correlation interferometric techniques including shearography. Some investigations have been done<sup>[1, 2]</sup>. In general, the total decorrelation occurs when the rigid body motion orthogonal to the line of sight mapping to the image plane exceeds a speckle size on the image plane. The speckle size on the image plane is basically determined by the magnification, the  $F$  number and the size of individual photodetector of a video camera. Rigid body motion problem becomes more acute when speckle correlation interferometric techniques are used in the nondestructive evaluation of large structions, in the application of product line enviroment, and in the micromechnics field. As in the first two cases, a relative large rigid body motion is most likely to occur during the loading. For the last case, since the magification is larger, very small rigid body motion will cause the total decorrelation of speckle patterns. These rigid body motion is very difficulty to be effectively compensated or avoided simply by loading fixture design or by optical set up since usually the direction of rigid body motion is unknown and the changes of optical set up may further results in other kind of decorrelation. This paper proposes a algorithm which can tolerate rigid body motion in speckle correlation interferometry.

## 2 Description of the Method

One of remedy method to overcome rigid body motion is to increase the speckle size on the image

plane by using high  $F$  number of a video camera. The penalty for this is that large percentage of laser light is lost and make experiment very difficult. On the other hand, since the distance between the sensor and the lens of a video camera is so short that even using high  $F$  number, the speckle size on the image plane is still hard to reach the size of an individual photodetector or say a pixel size. However, the speckles on the image plane can be rearranged by algorithm. A algorithm is designed to rearrange speckles on the image plane before and after deformation. Assume that the speckle size on the image plane is determined by the size of pixel, in other words, the speckle size is just the size of pixel. This is the most common case in the application of digital speckle correlation interferometry. Therefore, if the rigid body motion mapping to the image plane is larger than the size of one pixel, fringes will generally not be formed. In order to resume the correlation between the two speckle patterns before and after deformation, the speckles are rearranged in such a way that for a line of pixels in  $x$  direction, the value of the second pixel is replaced by the value of the first pixel, the value of the fourth pixel is substituted by the value of the third pixel and so on. It is the same procedure in  $y$  direction. After this rearrangement of speckles on the image before and after deformation, the size of speckle which is originally one pixel becomes the size of four pixels. In this way, it can tolerate rigid body motion of one pixel size on the image plane in any direction. Following this procedure, two pixels rigid body motion or more can be tolerated. However, the procedure mentioned above will reduce the number of bright speckles which are basically to form bright fringe. This can be compensated by speckle averaging<sup>[3]</sup> as follows.

Assume that  $u_r = A_r(x, y) \exp(i\Phi_r)$  and  $u_0 = A_0(x, y) \exp(i\Phi_0)$  are the complex amplitudes of the reference and the object beams, respectively. Before deformation, the intensity of speckle pattern received and digitized by a video camera and computer system is

$$I_b = (u_r + u_0)(u_r^* + u_0^*) = I_r + I_0 + 2\sqrt{I_r I_0} \cos \Phi \quad (1)$$

Where  $\Phi = \Phi_r - \Phi_0$  is the random phase angle and  $I_r = u_r^2$ ,  $I_0 = u_0^2$ . After the object is deformed, the phase change  $\Delta\Phi$  is introduced into Eq. (1) and the intensity of speckle pattern becomes

$$I_a = (u_r + u_0)(u_r^* + u_0^*) = I_r + I_0 + 2\sqrt{I_r I_0} \cos(\Phi + \Delta\Phi) \quad (2)$$

Let us introduce phase shifts by  $\alpha_b$  and  $\alpha_a$  to the two speckle patterns instead of to fringe pattern. Thus, we have

$$I_b = I_r + I_0 + 2\sqrt{I_r I_0} \cos(\Phi + \alpha_b) \quad (3)$$

$$I_a = I_r + I_0 + 2\sqrt{I_r I_0} \cos(\Phi + \Delta\Phi + \alpha_a) \quad (4)$$

$$I = (I_b - I_a)^2 = 16I_r I_0 \sin^2[\Phi + \Delta\Phi/2 + (\alpha_b + \alpha_a)/2] [1 - \cos(\Delta\Phi + \alpha_b - \alpha_a)] \quad (5)$$

Eq. (5) can be expressed as

$$I_{ij} = B \sin^2(\Phi + \Delta\Phi/2 + \alpha_i) [1 - \cos(\Delta\Phi + \alpha_j)] \quad (6)$$

Where  $B = 16 I_r I_0$ ,  $\alpha_i = (\alpha_b + \alpha_a)/2$ ,  $\alpha_j = \alpha_b - \alpha_a$ .

If  $\alpha_j$  is kept constant and  $\alpha_i$  is kept changing from 0 to  $2\pi$ , we can have many fringe patterns with the same fringes but different speckles. If we integrate them, called speckle averaging, we can have

$$I_j = (B/2\pi) [1 - \cos(\Delta\Phi + \alpha_j)] \int_0^{2\pi} \sin^2(\Phi + \Delta\Phi/2 + \alpha_i) d\alpha_i \quad (7)$$

$$I_j = B [1 - \cos(\Delta\Phi + \alpha_j)] \quad (8)$$

Therefore, the speckle term is canceled in Eq. (8) after speckle averaging has been done.

If  $\alpha_i$  is not changed continuously but discretely, we will have

$$I_j = B[1 - \cos(\Delta\Phi + \alpha_j)] \frac{2}{n} \sum_{i=1}^n \sin^2\left(\Phi + \frac{\Delta\Phi}{2} + \alpha_i\right) \quad (9)$$

The speckle term can also be canceled by choosing  $n$  being even number and  $\alpha_i$  having  $\pi/2$  difference with  $\alpha_{i+1}$ . For example, let  $n = 2$ ,  $\alpha_i = 0$ ,  $\alpha_{i+1} = \pi/2$ , we have

$$I_j = B[1 - \cos(\Delta\Phi + \alpha_j)] [\sin^2(\Phi + \Delta\Phi/2) + \cos^2(\Phi + \Delta\Phi/2)] \\ I_j = B[1 - \cos(\Delta\Phi + \alpha_j)] \quad (10)$$

or  $n = 4$ ,  $\alpha_i = 0$ ,  $\alpha_{i+1} = \pi/2$ ,  $\alpha_{i+2} = \pi$ ,  $\alpha_{i+3} = 3\pi/2$ , we have

$$I_j = B[1 - \cos(\Delta\Phi + \alpha_j)] [2 \sin^2(\Phi + \Delta\Phi/2) + 2 \cos^2(\Phi + \Delta\Phi/2)] / 4 \\ I_j = B[1 - \cos(\Delta\Phi + \alpha_j)] \quad (11)$$

or  $n = 2$ ,  $\alpha_i = \pi/4$ ,  $\alpha_{i+1} = 3\pi/4$ , we have

$$I_j = B[1 - \cos(\Delta\Phi + \alpha_j)] [\sin^2(\Phi + \Delta\Phi/2 + \pi/4) + \cos^2(\Phi + \Delta\Phi/2 + 3\pi/4)] \\ I_j = B[1 - \cos(\Delta\Phi + \alpha_j)] \quad (12)$$

When  $j$  in Eq. (10) or Eq. (11) changes, phase shifted fringe patterns can be obtained. Eight speckle patterns, which the four with 90 degree phase shift with respect to each other before deformation and the four with 90 degree phase shift with respect to each other after deformation, are employed to do speckle averaging. Convolution reconstruction fringe procedure<sup>[4]</sup> can be used to further reduce speckle noise as the following. The four fringe patterns with 90 degree phase shift with each other are first smoothed and the four smoothed fringe are used to calculate the phase map. Then the phase map is employed to reconstruct and normalize four fringe patterns with 90 degree phase shift with respect to each other. These four reconstructed fringe patterns are smoothed again and then the smoothed four fringe patterns are used to calculate the phase map again. It forms an iterative way to suppress the speckle noise until the satisfied unwrapped phase map is obtained.

### 3 Experimental Demonstration and Conclusion

A rectangular plate with all its boundaries clamped and subjected to a uniform air pressure is chosen for the experimental demonstration. The optical set up is shown in Fig. 1. The speckle averaging uses 8 speckle patterns which 4 speckle patterns have 90 degree phase difference sequentially before loaded and 4 speckle patterns

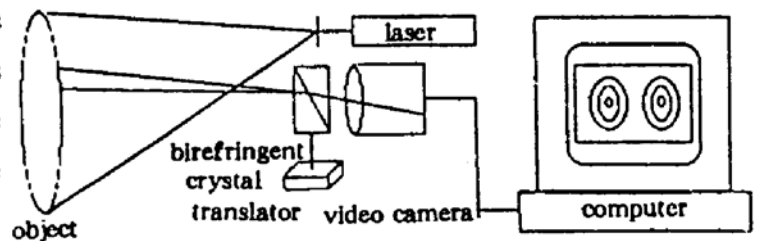


Fig. 1 Optical arrangement for electronic shearography

have 90 degree phase difference sequentially after loaded. During the loading, there is a rigid body motion in horizontal direction which makes the image of the specimen move one pixel on the image plane. The magnification is 12 and the pixel size in the video camera is 0.02 mm. Thus, the corresponding rigid body motion of the specimen is  $12 \times 0.02$ , namely, 0.24 mm.

The following is data processing. First, the speckles in the 8 speckles in the 8 speckle patterns are rearranged in such a way that for a line of pixels in  $x$  direction, the value of the second pixel is replaced by the value of the first pixel, the value of the fourth pixel is substituted by the value of the third pixel and so on. It is the same procedure in  $y$  direction. After this rearrangement of speckles on the image before and after deformation, the size of speckle which is originally one pixel becomes the

size of four pixels. Second, the 8 speckle patterns can be properly combined to generate 16 fringe patterns which 4 fringe patterns have 0 phase shift but the speckles have 90 degree phase shift sequentially, 4 fringe patterns have 90 phase shift but the speckles have 90 degree phase shift sequentially, 4 fringe patterns have 180 phase shift but the speckles have 90 degree phase shift sequentially, and 4 fringe patterns have 270 phase shift but the speckles have 90 degree phase shift sequentially. Thus, we have 4 group fringe patterns. We add 4 fringe patterns in its group to do speckle averaging to form one fringe pattern. Then we obtain four fringe patterns with 90 degree phase difference sequentially which are shown in Fig. 2. For comparison, there are four fringe patterns are shown in Fig. 3 which the upper left represents without rigid body motion and speckles rearrangement,

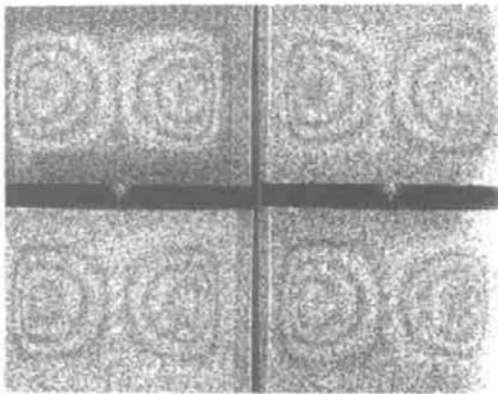


Fig. 2 Four fringe patterns with 90 degree phase shift obtained after speckle rearrangement and speckle averaging

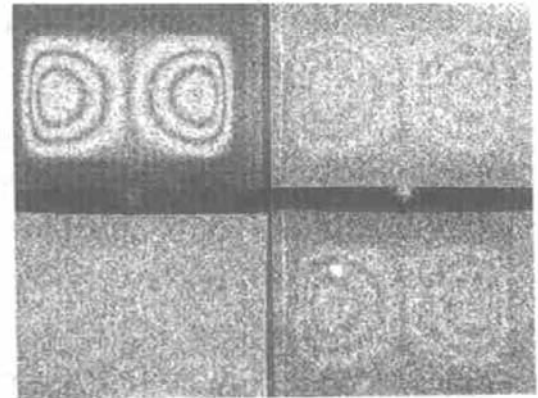


Fig. 3 Comparison among no rigid body motion fringe, speckle rearrangement only for deformed speckle pattern, fringe pattern without speckle rearrangement, and fringe pattern with speckle rearrangement

the upper right illustrates the fringe pattern obtained by only rearranging the speckles in the four speckle patterns after loading with rigid body motion, the lower right demonstrates the fringe pattern obtained by rearranging speckles in all eight speckle patterns in which four speckle patterns after loading with rigid body motion, and the lower left illustrates the fringe pattern obtained without speckles rearrangement with rigid body motion. It is obvious that there is almost no fringes in the lower left pattern since speckle patterns are totally decorrelation caused by rigid body motion. The third step is smooth reconstruction of fringe pattern procedure. The four fringe patterns shown in Fig. 2 are smoothed and then the four smoothed fringe patterns are used to calculate the phase map. The phase map is employed to reconstruct and normalize a new four fringe patterns which has 90 degree phase difference with each other. The new four fringe patterns are smoothed again and the smoothed four fringe patterns are used to calculate the phase map again. The phase map is employed to reconstruct and normalize the new four fringe patterns again. In all, there are three time smooths and three time reconstructions. The 2-D grey level phase map is shown in Fig. 4 and 3-D plot is in Fig. 5. Fig. 6 illustrates the comparison between the experimental result and the simulation. We can see that they are quite consistent. Fig. 7 shows the four fringe patterns with 90 degree phase shift sequentially which are reconstructed from the phase map in Fig. 4. From the experimental demonstration above, a conclusion can be drawn that this method can tolerate rigid body motion for speckle correlation interferometry including shearography.

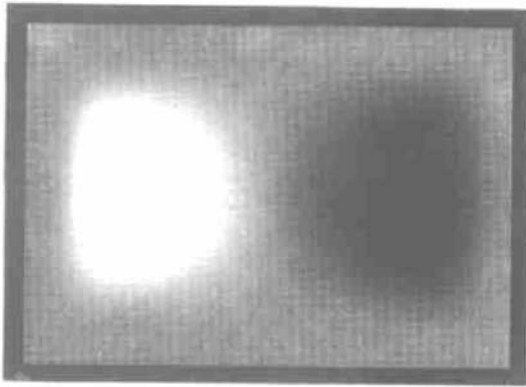


Fig. 4 2-D grey level phase map calculated from the fringe patterns in Fig. 2

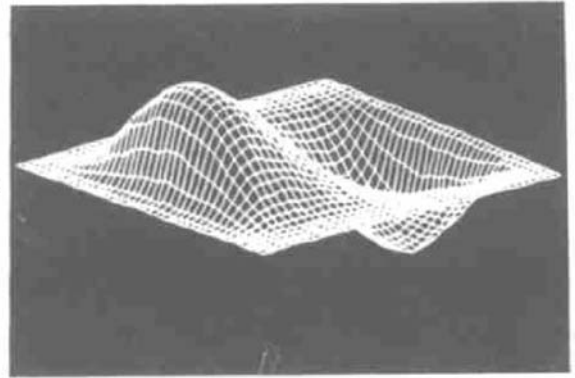


Fig. 5 3-D plot of phase map in Fig. 4

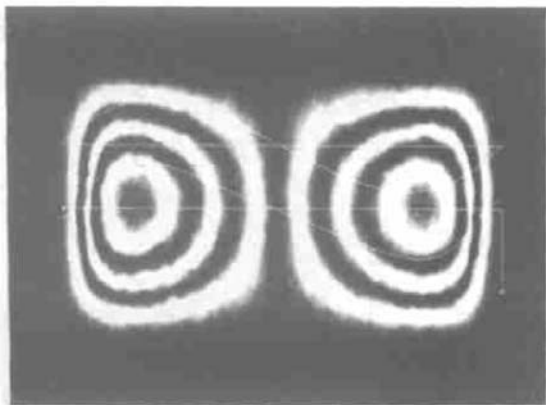


Fig. 6 Comparison between simulation and the result obtained from Fig. 4

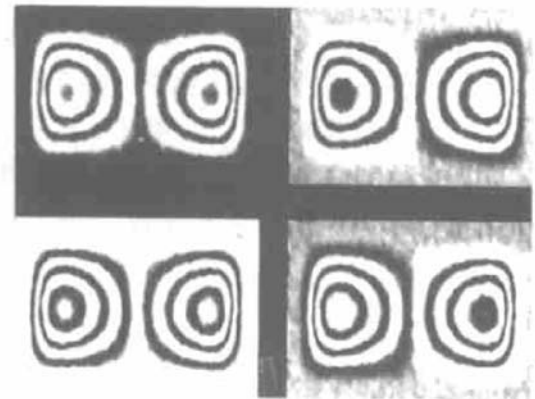


Fig. 7 Four fringe patterns with 90 degree phase shift sequentially obtained from Fig. 4

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## 数字剪切散斑干涉术中的 刚体位移补偿的新方法

陈方 Y. Y. Hung C. T. Griffen

(Department of Mechanical Eng., Oakland University, Rochester Hills, MI 48309, U. S. A)

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**摘 要** 给出一个数字剪切散斑干涉术中的刚体位移补偿的新方法。当刚体位移大于一个像素时,加载前后的散斑图将会由于位置的变化导致失相关。重新安排每一个像素将会克服此失相关。散斑平均、条纹重构以及迭代方法用来改善条纹质量,消除散斑噪声,最后得到可取结果。

**关键词** 散斑重新布局, 刚体位移, 数字剪切散斑干涉术。