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Long-Pulse Soliton in a Single Mode Optical Fiber*

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Abstract It is theoretically demonstrated that a new kind of soliton, having the waveform of $\operatorname{sech}^{1/2}(x)$, can propagate in a single mode optical fiber. For the conventional fiber, its temporal width is about one nanosecond when peak input power is about 1 Watt at the wavelength 1.3 micron.

Key words optical soliton, modified nonlinear schrödinger equation, inverse scattering method.

1 Introduction

The term soliton refers to a special kind of wave that can propagate undistorted over long distance. An optical soliton[1,2] is the nonlinear pulse packet propagating in a single mode optical fiber. It has drawn more and more attention of the scientists in the world because the optical soliton is not only of fundamental interest but also has wide potential applications, especially in the field of optical fiber communications[3]. According to a nonlinear Schrödinger (NLS) equation, so-called fundamental soliton is a hyperbolic-secant pulse with its pulse temporal width being about a few picosecond or less[2]. A self-steeping term[4] has to be included in the NLS equation to explain the asymmetry of the output puse spectrum. The NLS equation with the self-steeping term is called as the modified NLS (MNLS) equation in some literature[5~7], which also has a soliton solution proved by Anderson et al.[8]. In order to deal with the MNLS equation, one usually transforms it into the normalized perturbed NLS equation (the normalized NLS equation with the perturbed self-steeping term) by using a dimensionless (normalized) transformation[2,5~7].

In this letter, it is demonstrated theoretically that the MNLS equation can have another soliton solution, which is not only much different from the fundamental soliton described by the NLS equation, but also slight different from the soliton presented by Anderson. Instead of the old transformation mentioned above, a new one is introduces to reduce the MNLS equation to normalized MNLS (NMNLS) equation, in which no perturbed term is contained. The mathematical method used here is the newly-presented inverse scattering method[9] to integrate the NMNLS equation. As a result, one may surprisingly find that this new kind of soliton is in the nanosecond region which means an electrically modulated emitter may excite the soliton propagation in the optical fiber.

2 Result

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The MNLS equation is[2, 4~8]

$$i\frac{\partial A}{\partial z}-\frac{\beta''}{2}\frac{\partial^2 A}{\partial T^2}+\gamma(|A|^2A)+i\alpha\frac{\partial}{\partial T}(|A|^2A)=0, \tag{1}$$

where A is slowly varying packet function, $T=t-\beta' z$ is the movement coordinate of the packet, $\beta=n_0\omega_0/c$, and the coefficients γ and α are defined by

$$\gamma=\frac{n_2\,\omega_0}{cA_{eff}},\qquad \alpha=\frac{2\gamma}{\omega_0},$$

where A_{eff} is known as the effective core area of the fiber[2,3], n_2 is the nonlinear-index coefficient.

Using the transformation

$$\xi=\frac{|\beta''|}{2T_0^2}z,\qquad \tau=\frac{T}{T_0},\qquad q=2\left(\frac{\gamma T_0}{|\beta''|\omega_0}\right)^{1/2}A, \tag{2}$$

in the anomalous-dispersion regime ($\beta''<0$), Eq. (1) is reduced to the (dimensionless) NMNLS equation[9]

$$i\frac{\partial}{\partial\xi}q+\frac{\partial^2}{\partial\tau^2}q+2\rho|q|^2q+i\frac{\partial}{\partial\tau}(|q|^2q)=0, \tag{3}$$

where $\rho=\omega_0T_0/4$. Like the old transformation, the arbitrary time scale T_0 in Eq. (2) allows a pulse of standard duration in the dimensionless variable τ to correspond to a pulse of any desired duration in time T.

Recently, Chen[9] has presented a new method to solve Eq. (3) analytically. In her method, Eq. (3) is solved by an inverse scattering method out of a new Lax pair, and its soliton solution also corresponds to the discrete spectral parameter like that by Zakharov and Shabat[10]. By this method, the N-soliton solution of the NMNLS Eq. (3) (corresponding to the case of reflectionless) can be expressed as[9]

$$q(\xi,\tau)=-i2F_1(\xi,\tau)\,F_2(\xi,\tau). \tag{4}$$

In this solution above, F_1 and F_2 are respectively*:

$$F_1=\sum_{n=1}^{N}C_n^*\,\zeta_n^{*-1}\,\psi_{2n}^*\exp\left[-i(\zeta_n^*-\rho)\tau\right], \tag{5}$$

$$F_2=1+\sum_{n=1}^{N}C_n\,\zeta_n^{-2}\,\psi_{1n}\exp\left[i(\zeta_n-\rho)\tau\right], \tag{6}$$

where $\zeta_n(n=1,2,\cdots,N)$ are the discrete spectral parameters (also called as discrete eigenvalues) of the scattering equation of the Lax pair for Eq. (3), ψ_{1n} and ψ_{2n} are the values of Jost solutions of the scattering equation at $\zeta=\zeta_n$, which can be found from the system of $2N$ linear algebraic equations in $2N$ unknowns ($m=1,2,\cdots,N$)

$$\psi_{2m}^*\exp\left[i(\zeta_m^*-\rho)\tau\right]=1-\zeta_m^*\sum_{n=1}^{N}\frac{C_n\psi_{1n}\exp\left[i(\zeta_n-\rho)\tau\right]}{(\zeta_n-\zeta_m^*)\zeta_n^2}, \tag{7a}$$

$$\psi_{1m}\exp\left[-i(\zeta_m-\rho)\tau\right]=\zeta_m\sum_{n=1}^{N}\frac{C_n^*\,\psi_{2n}^*\exp\left[-i(\zeta_n^*-\rho)\tau\right]}{(\zeta_n^*-\zeta_m)\,\zeta_n^*}, \tag{7b}$$

where the asterisk denotes complex conjugate. In the equations above, C_n is the discrete scattering date which can be expressed as

$$C_n=C_{on}\exp\left[i\,4(\zeta_n-\rho)^2\xi\right],$$

* the symbol in the right hand side of the two formula in Eq. (51) of Ref. [9] should be "+", instead of "−". See, Chen, private communication; also see, Chen, "Explicit N-Soliton Solution of the Modified Nonlinear Schrödinger Equation by Means of the Inverse Scattering Transformation", (*Commun. Theor. Phys.*, Vol. 15, April 1991, PP. 421~426.

where C_{0n} is a constant determined by the initial condition.

When $N=1$, Eq. (7) reduces to the system of two linear equations of two unknowns, and can be solved easily. Substituting the solution of Eq. (7) for $N=1$ into Eq. (4), the single soliton solution of Eq. (3) is found to be

$$q = 2\sqrt{2\eta}\,\mathrm{sech}\,[4\eta(\tau - 4\rho\xi - \Omega)]\exp(i\phi), \tag{8}$$

where phase factor ϕ is

$$\phi = 2\rho\tau - 4(\rho^2 - \eta^2)\xi + 3\mu, \tag{9}$$

$\Omega = \ln[|C_{01}|^2/(4\eta^3)]/4\eta$, and $\mu = \arctan\{[|C_{01}|^2/(4\eta^3)]\exp[-4\eta(\tau - 4\rho\xi)]\}$. In order to obtain Eq. (8), we have supposed that the spectral parameter is located in the imaginary axis so that $\zeta_1 = i\eta$.

The solution (8) tells us that $|q|$ keeps its initial shape in propagation along the fiber with a delay, which has been found earlier[8]. The delay per unit length is $d\tau/d\xi = 4\rho$.

In particular, with $\Omega = 0$, the solution (8) is simplified to

$$q = 2\sqrt{2\eta}\,\mathrm{sech}\,[4\eta(\tau - 4\rho\xi)]\exp(i\phi). \tag{10}$$

The phase (9) does not change in form, but the factor μ in it is reduced to

$$\mu = \arctan\{\exp[-4\eta(\tau - 4\rho\xi)]\}. \tag{11}$$

Letting $\xi = 0$ in the Eq. (10) gives the input condition at the fiber input terminal

$$q_0 = 2\sqrt{2\eta}\,\mathrm{sech}\,(4\eta\tau)\exp(i\phi_0), \tag{12}$$

where ϕ_0 is obtained by letting $\xi = 0$ in the expression of ϕ (Eqs. (9) and (11)). To verify our calculation, we have numerically simulated the evolution of Eq. (3) under the initial condition (12) for different values of η by means of the split-step Fourier method[2]. The result concurs with Eq. (10) completely.

3 Discussion

Now, We can come back to the real world to find out the input power, the full width at half-intensity maximum (FWHM), and the other real quantities necessary for the propagation of the soliton solution (10). Without loss of generality, letting $\eta = 1/2$, Eq. (12) gives

$$|q_0| = 2\sqrt{\mathrm{sech}\,(2\tau)}.$$

For this standard pulse, $\tau_{FWHM} = 1.317$, and $|q_0|_{max} = 2$. Form the transformation (2), therefore, we have

$$P_0 = 2.1 \times 10^{-15}\frac{\lambda^2|D|A_{eff}}{n_2 T_{FWHM}}, \tag{13}$$

where P_0 is the peak of the input power (its unit is in W), λ is the wavelength (in μm) of the carrier wave, T_{FWHM} is the FWHM of the soliton pulse (in ps) equating $1.317\,T_0$, D is dispersion parameter ($D = -2\pi c\beta''/\lambda^2$, its unit is in ps/km/nm), the unit of A_{eff} is μm^2, and $n_2 = 3.2 \times 10^{-16}$ cm^2/W. For the conventional step-index single mode fiber at $\lambda = 1.3$ μm, $D \approx 2$ ps/km/nm, $A_{eff} \approx 50$ μm^2, so from Eq. (13) we have that $T_{FWHM} \approx 1.1$ ns when $P_0 = 1$ W. But for the fundamental soliton described by NLS equation, formula[2]

$$P_0 = 7.89 \times 10^{-13}\frac{\lambda^3|D|A_{eff}}{n_2 T_{FWHM}^2 c},$$

gives that $T_{FWHM} \approx 1.34$ ps if all of other parameters do not change, where the unit of c is km/s. It can also be obtained from Eq. (2) that the delay quantity per propagation length

$$\frac{d}{dz}T = 500\ \lambda|D|,$$

where the unit of delay per length is in ps/km, λ is in μm, and D in ps/km/nm. It turns out $dT/dz =$ 1. 3 ns if parameter values above are chosen, which means that the pulse will delay one pulse width after it propagates about one kilometer.

There are some differences between the soliton presented here and the fundamental soliton out of NLS equation. The most pronounced feature is that the former temporal width is much larger than the latter one. This is because the transformation coefficient in Eq. (2c), $2[\gamma T_0/(|\beta''|\omega_0)]^{1/2}$, is much smaller than that[2] in the old transformation which is $(\gamma/|\beta''|)^{1/2}T_0$. Second, the former width is inversely proportional to the square-root of its amplitude, which can be get from Eq. (2c) and (10), but the latter width is inversely proportional to the amplitude[2]. Third, the former has the form of the square-root of hyperbolic-secant, while the latter is the hyperbolic-secant pulse[2]. This soliton is also slight different from the soliton presented by Anderson[8] in their expressions.

4 Conclusion

In conclusion, we have theoretically demonstrated that the NLS equation with the self-steeping term can support the propagation of a new kind of optical siliton. The intensity of this soliton has the form of sech(x), and its pulse width is about one nanosecond when the input peak power is 1 W at λ =1. 3 μm for conventional single mode fiber. This result is not only the fundamental interests to study the contribution of the self-steeping term to the NLS equation, but also practical important. As well known, the soliton laser with photo- inject mode- lock[11] and CPM laser[12] can excite soliton in picosecond or sub-picosecond region. Our result shows that an electrically modulated emitter may also easily excite soliton propagation in the single mode optical fiber. It seems the this new soliton can not be used to the optical fiber communications because of its long-pulse feature, but we can be sure its new application will soon be suggested after it is experimentally demonstrated.

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单模光纤中传输的长脉冲孤子*

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摘　要　理论上证明了一种新的孤子可以在单模光纤中稳定传输，其波形为 $sech^{1/2}(x)$。对于普通光纤，在工作波长为 1.3 μm 处，当输入峰值功率为 1 W 时，这种孤子脉冲的脉宽大约为 1 ns。

关键词　光孤子，　修正的非线性薛定谔方程，　反散射法。

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