

## Phase Distribution Evaluation of Time Dependent Holographic Fringe Pattern by Dual Fringe Patterns Phase Shifting Interferometry

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**Abstract** This paper presents how a new and simple technique referred as to Dual Fringe Patterns Phase Shifting Interferometry to be used in holography interferometry for automated phase evaluation of time dependent problem. The novel feature of the technique is that two fringe patterns with exactly desired phase shift can be obtained simultaneously. Although, the phase distribution of the two fringe patterns could vary with respect to time due to the effect of environmental disturbance, the phase shift between them is fixed. The technique is based on the combination of splitting fringe pattern, polarization phase shifting and two dimensional digital FFT schemes.

**Key words** holographic, phase shifting interferometry.

### 1 Introduction

Phase evaluation from holographic interferograms is of increasing importance for variety of research and industrial measurements. For phase evaluation, the fringe pattern must be transformed into a meaningful and continuous surface. This process proves difficult in general case because of ambiguities in fringe data. With the matual of digital image processing system, the phase shifting interferometry, which recently is developed rapidly, are being used to automatically evaluate the phase distribution of fringe pattern. However, for conventional real time phase shifting interferometry<sup>[1, 2]</sup>, fringe patterns with mutual phase shift are obtained in different time. The phase shift between the fringe patterns will suffer a drift because of the disturbed air or the vibration isolation system is not high quality enough. Thus, it is difficult to capture the fringe pattern with the exactly desired amount of phase shift. This prevents the technique from being applied to on line production industrial. Furthermore, it is not practical to employ it to evaluate phase distribution of fringe pattern for a transient problem on account of requiring time to move optical elements to fulfill phase shifting. These limitations have drawn attention of some researchers who contributed some of worthwhile work<sup>[3-5]</sup>. Compared with these work, this paper presents a new and simple technique for automated phase evaluation of time dependent holographic fringe pattern referred to as Dual Fringe Patterns Phase Shifting Interferometry (DPPSI)<sup>[6]</sup>. The novel feature of the technique is that two fringe patterns with exactly desired amount of phase shift can be obtained simultaneously. Although the phase distribution of two fringe patterns could change with respect to time, the phase shift between them is fixed, in other words, is independent of time. The principle of this technique along with experimental

verification is presented.

## 2 Principle of the technique

Fig. 1 shows schematically a real time holographic interferometry optical system with dual images phase shifting arrangement. In Fig. 1, the object beam and the reference beam reaching the hologram plate are plane-polarized and the polarization direction of the reference beam is adjustable. The function of quarter wave plate  $Q_1$  and  $Q_2$  placed on the optical path is to convert a plane polarized beam to a circularly polarized one. Thus, the intensity of the beam through a polarizer does not depend on polarization axis of polarizer. The hologram plate is exposed while the object is not deformed and the polarization direction of the beams are vertical. The hologram plate is developed in place (or developed and then returned to its original position). The object is then deformed and the polarization of the reference beam is adjusted to the angle  $\theta = 0$  as shown in Fig. 1 and Fig. 2 by rotating polarizer  $P_1$ . By viewing the image of the deformed object through the birefringent crystal, the quarter wave plate  $Q_2$ , the hologram and the polarizer  $P_2$ , two live interferometric fringe patterns with  $\pi/2$  mutual phase shift are observed. Two of such fringe patterns are digitized into a computer with a frame grabber via a video camera. In Fig. 2, where  $W_1$  and  $W_2$  denote the two polarization directions of birefringent crystal which are perpendicular to each other.  $Q_f$  and  $Q_s$  are the fast and slow axes of quarter wave plate

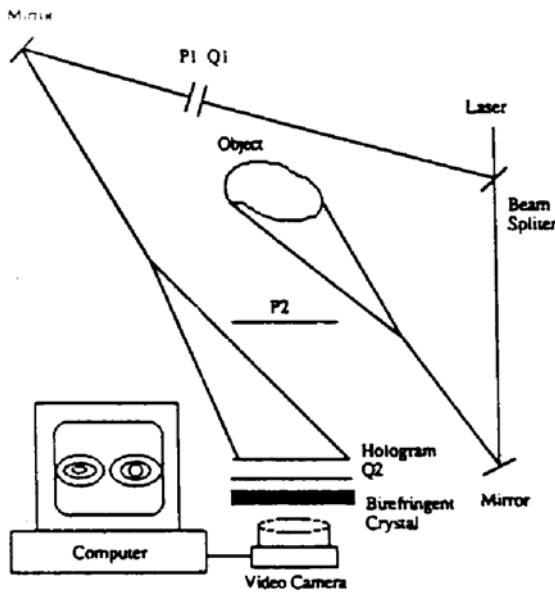


Fig. 1 Schematic diagram of the optical arrangement

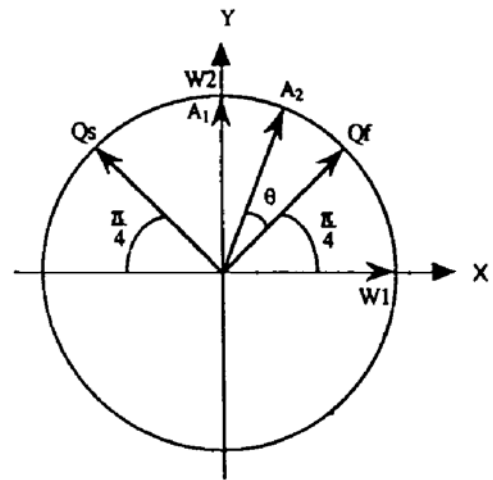


Fig. 2 Description of polarization directions for waves  $A_1$  and  $A_2$ , and the polarization axes of the birefringent crystal

$Q_2$  respectively.  $A_1$  is the object wave before deformation and  $A_2$  after deformation. Set

$$A_1 = a_1 \exp(i\phi_1), \quad A_2 = a_2 \exp(i\phi_2), \quad \Delta\phi = \phi_1 - \phi_2$$

According to Fig. 2, as the wave enters the quarter-wave plate, it resolved into components  $E_s$  and  $E_f$  with vibration directions parallel to the fast and slow axes, respectively. Thus the following results will be obtained

$$E_f = a_2 \cos \theta \exp(i\phi_2) + a_1 \cos(\pi/4) \exp(i\phi_1) \tag{1}$$

$$E_s = a_2 \sin \theta \exp[i(\phi_2 - \pi/2)] + a_1 \sin(\pi/4) \exp[i(\phi_1 - \pi/2)] \tag{2}$$

The above two wave emerging from the  $W_1$  and  $W_2$  directions of birefringent crystal can be expressed respectively by

$$\begin{aligned} E_{w1} &= \frac{\sqrt{2}}{2} E_f - \frac{\sqrt{2}}{2} E_s \\ &= \frac{\sqrt{2}}{2} \{ a_1 \exp [i(\phi_1 + \frac{\pi}{4})] + a_2 \exp [i(\phi_2 + \theta)] \}, \end{aligned} \quad (3)$$

$$\begin{aligned} E_{w2} &= \frac{\sqrt{2}}{2} E_f + \frac{\sqrt{2}}{2} E_s \\ &= \frac{\sqrt{2}}{2} \{ a_1 \exp [i(\phi_1 - \frac{\pi}{4})] + a_2 \exp [i(\phi_2 - \theta)] \}. \end{aligned} \quad (4)$$

Then, the intensities can be acquired as follows

$$I_{w1} = E_{w1}^2 = \frac{1}{2} [a_1^2 + a_2^2 + 2a_1a_2 \cos (\Delta\phi - \theta + \frac{\pi}{4})], \quad (5)$$

$$I_{w2} = E_{w2}^2 = \frac{1}{2} [a_1^2 + a_2^2 + 2a_1a_2 \cos (\Delta\phi + \theta - \frac{\pi}{4})]. \quad (6)$$

Where,  $I_{w1}$  is the intensity coming through  $W_1$  axis and intensity  $I_{w2}$  from  $W_2$  axis. It demonstrates that two fringe patterns occur at the same time, but with the different polarization direction. The two fringe patterns can be separated in space because they come through birefringent crystal with different refractive angles. From Eq(5) and Eq(6), the subsequent novel result can be achieved by letting  $\theta = 0$ .

$$I_{w1} = \frac{1}{2} [a_1^2 + a_2^2 + 2 a_1 a_2 \cos (\Delta\phi + \frac{\pi}{4})], \quad (7)$$

$$I_{w2} = \frac{1}{2} [a_1^2 + a_2^2 + 2 a_1 a_2 \cos (\Delta\phi - \frac{\pi}{4})]. \quad (8)$$

Eq(7) and Eq(8) indicate that there is a phase difference  $\pi/2$  between the two fringe patterns and it does not change with respect to time. Using FFT, the frequency domain expression of Eq(7) is given by

$$S(u, v) = A(u, v) + C(u, v). \quad (9)$$

Where  $S(u, v)$ ,  $A(u, v)$  and  $C(u, v)$  are the Fourier spectra of  $I_{w1}$ ,  $(a_1^2 + a_2^2)/2$  and  $a_1a_2 \cos (\Delta\phi + \pi/4)$  respectively. Selecting only spectra  $C(u, v)$  from frequency domain to separate the information term  $C(u, v)$  from background term  $A(u, v)$ , then computing the inverse 2-D FFT of the spectrum, the result can be written by

$$I_{w1}^* = a_1 a_2 \cos (\Delta\phi + \pi/4). \quad (10)$$

Similarly, processing the second fringe pattern represented by Eq(8) gives

$$I_{w2}^* = a_1 a_2 \cos (\Delta\phi - \pi/4). \quad (11)$$

After let

$$\phi = \Delta - \pi/4 \quad (12)$$

Then, the final results will be

$$\phi = \arctan (- I_{w1}^*/I_{w2}^*) \quad (13)$$

$$\Delta\phi = \arctan (- I_{w1}^*/I_{w2}^*) + \frac{\pi}{4} \quad (14)$$

From Eq(5) and Eq(6), the phase shift with  $0$ ,  $\pi/2$ ,  $\pi$  and  $3\pi/2$  between these two fringe patterns can also be obtained.

### 3 Experiment result

A rectangular plate is used in our experiment. The plate is clamped along its four boundaries and subjected to a uniform pressure. In the experiment, the surface displacement of the plate is dominated by the out of plane displacement. Two holographic fringe patterns with  $\pi/2$  phase shift between them, as shown in Fig. 3, are recorded simultaneously via the video camera connected with computer image processing system. The intensity distribution within a rectangular area on the these two fringe patterns as shown in Fig. 4(a) are transformed into frequency domain by a 2-D FFT algorithm. After the background intensity frequency is filtered, the two fringe patterns are transformed back from frequency domain to spatial domain as shown in Fig. 4(b). The phase at every pixel point within the rectangular area is calculated by Eq (14). The result directly obtained from Eq (14) is a wrapped phase distribution as shown Fig. 4(c). An unwrapped phase distribution image is final achieved by an unwrapped phase algorithm as shown in Fig. 4(d). Fig. 5 (a) shows the phase distribution along one line and (b) gives the 3-D plot.

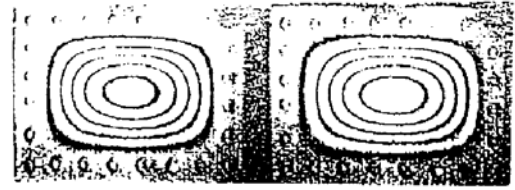


Fig. 3 Live fringe patterns with 90 degree phase difference

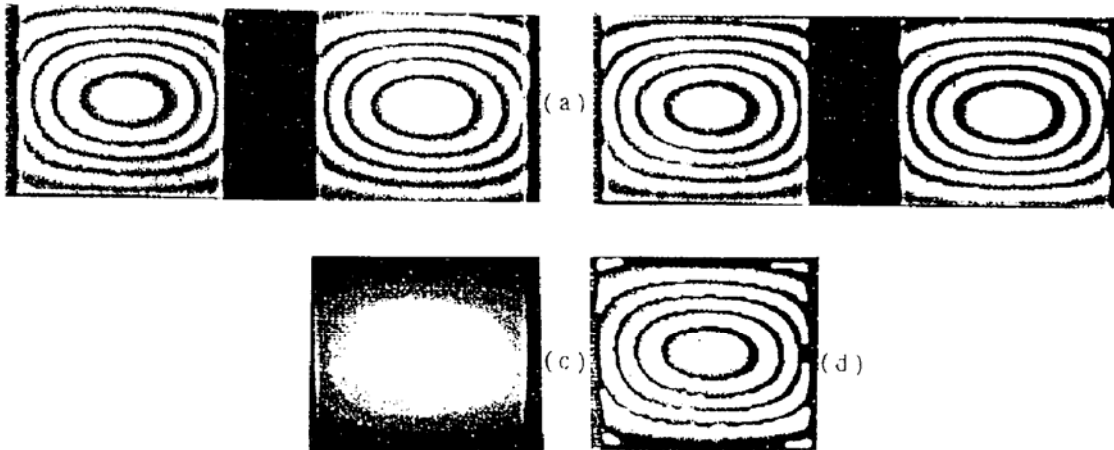


Fig. 4 Experimental results

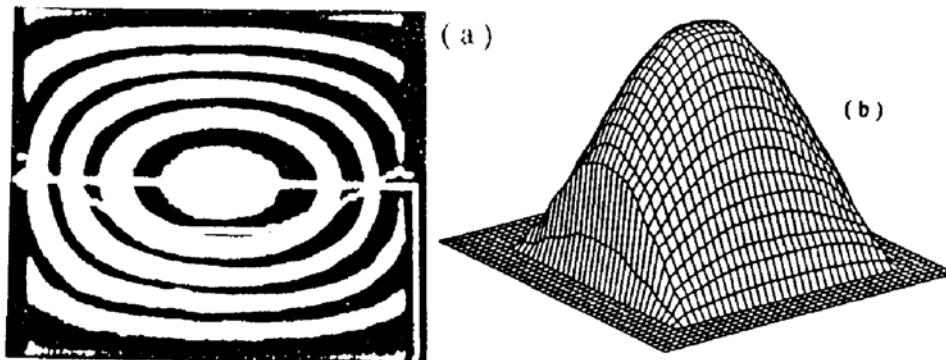


Fig. 5 The experimental results (b)

## 4 Conclusion

A novel and simple technique of Dual Fringe Patterns Phase Shifting Interferometry has been presented. It posses the major following merits:

1) The two fringe patterns with  $\pi/2$  phase difference can be obtained at the same time, thus it may be used in quantitatively evaluating phase distribution of fringe pattern for time dependent problem.

2) The phase difference is fixed between the two fringe patterns, therefore it eliminates the drifting of the phase difference between the two fringe patterns.

3) The phase shifting system is common path, so that it relaxes the optical elements quality and vibration isolation requirements.

4) Because of having two fringe patterns, the restrict on fringe shape can be removed compared with one fringe pattern FFT technique in which requires sinusoidal fringe and enough straight fringe to separate spectral orders.

5) The optical set-up is simple and compact in which a birefringent crystal and polarization system are employed to fulfill splitting one fringe pattern into two fringe patterns and the phase shifting. Here, birefringent crystal serves both splitting fringe pattern and shifting phase.

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# 用双图全息相移干涉法计算与时间有关的位相分布

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**摘 要** 提出一种双图全息相移干涉法. 该方法可以用来计算与时间有关的位相分布. 两幅全息干涉条纹图可以同时得到, 并且有固定的  $90^\circ$  位相差. 虽然两幅条纹图的位相可以随着时间变化, 如环境影响, 或载荷变化, 但他们之间的位相差恒定. 该方法利用二维傅氏变换, 结合偏振相移法以及分光技术.

**关键词** 全息, 相移干涉法.