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五孔剪切相机同时测量二维曲率、扭率和斜率*

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提 要

本文首次提出一种散斑剪切相机．该相机能把曲率、扭率和斜率的二维信息同时记录在一张双曝光散斑图上．对散斑图进行滤波分析时在频谱面上出现十三块互不重叠的衍射晕．在适当的衍射晕上滤波即可获得二维曲率、扭率和斜率的等值全场条纹．

关键词 五孔剪切相机， 曲率， 扭率， 斜率．

1 引 言

文献[1]提出利用三孔剪切干涉仪同时测量一维斜率和曲率；文献[2]又提出利用四孔剪切干涉仪同时测量一维斜率、曲率和离面位移．本文在上述研究的基础上提出一种五孔剪切相机，它能在一张双曝光散斑图上同时记录曲率、扭率和斜率的二维信息．

2 差分原理

考虑图 1 所示曲面 Γ 上某点 O_1 及邻域 O_2、O_3、O_4 和 O_5 四点．根据中心差分方法[3]，点 O_1 处曲面 Γ 的曲率、扭率和斜率分别为

$$\left.\begin{aligned}\frac{\partial^2 w}{\partial x^2}&=\frac{w_2+w_3-2w_1}{\Delta^2},\\ \frac{\partial^2 w}{\partial y^2}&=\frac{w_4+w_5-2w_1}{\Delta^2}\end{aligned}\right\}\tag{1}$$

$$\left.\begin{aligned}\frac{\partial^2 w}{\partial x'\partial y'}&=\frac{w_4+w_5-w_2-w_3}{2\Delta^2},\\ \frac{\partial^2 w}{\partial y'\partial x'}&=\frac{w_4+w_5-w_2-w_3}{2\Delta^2}\end{aligned}\right\}\tag{2}$$

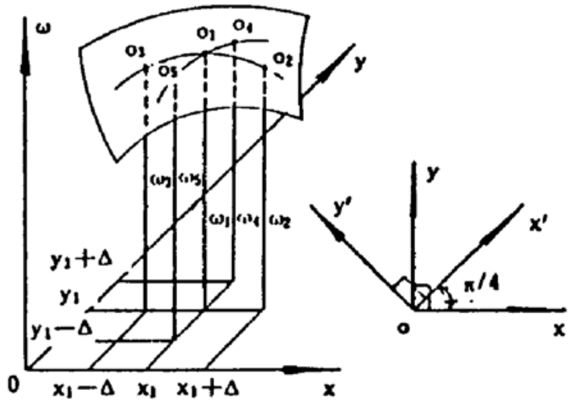

Fig. 1 Schematic representation for curvature, twist and slope from finite difference

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$$\frac{\partial w}{\partial x}=\frac{w_2-w_3}{2\Delta},\qquad \frac{\partial w}{\partial y}=\frac{w_4-w_5}{2\Delta}. \tag{3}$$

3　五孔剪切相机

图 2 是五孔离焦型剪切相机的光路系统. 五孔屏置于成像透镜前，五孔都是裸露的，对物体进行离焦照相. 这样像面上一点的场为来自物面上相邻五点场的叠加.

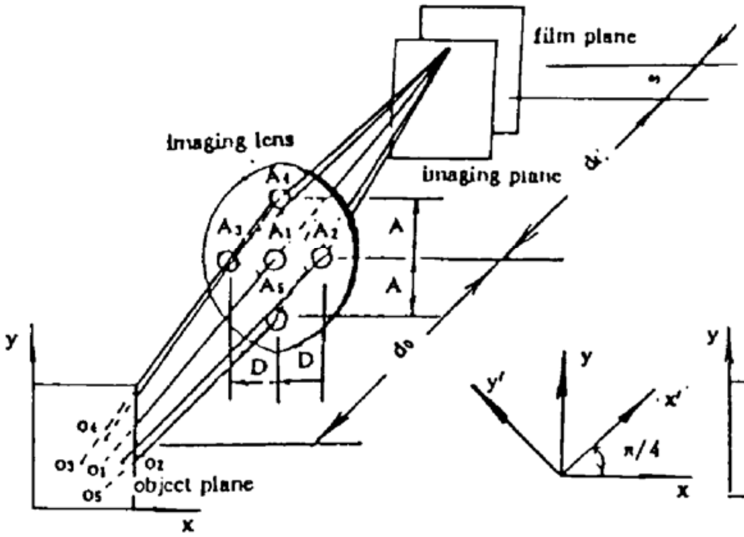

Fig. 2 Schematic of defocusing-type five-aperture speckle shearing camera

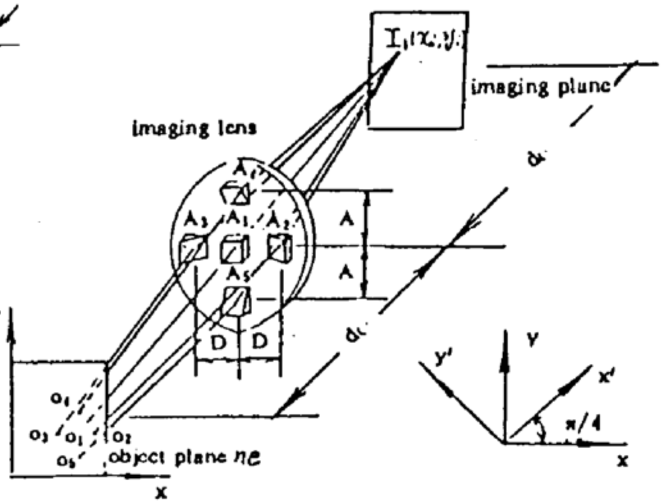

Fig. 3 Schematic of wedge-type five-aperture speckle shearing camera

图 3 是五孔光楔型剪切相机的光路系统. 同样的五孔屏置于成像透镜前，外面四孔各覆盖一块等倾角光楔，中间孔覆盖平晶玻璃片. 这样像面上一点的场为来自物面上相邻五点场的叠加，且在物面上外面四点相对于中间点的错位量均为

$$\Delta=(n-1)\,\alpha\, d_0 \tag{4}$$

式中 d_0 为物距，n 和 α 分别为光楔的折射率和顶角.

在物体变形前后进行双曝光，将所拍双曝光散斑图放入滤波分析系统中，在频谱面上出现 13 块互不重叠衍射晕，如图 4 所示. 其中衍射晕 (A_{12}, A_{13}) 是孔 A_1、A_2 的互相关频谱和孔 A_1、A_3 的互相关频谱的叠加，含曲率 $(\partial^2/\partial x^2)$ 信息；同理衍射晕 (A_{14}, A_{15}) 含曲率 $(\partial^2/\partial y^2)$ 信息；衍射晕 (A_{25}, A_{34}) 是孔 A_2、A_5 的互相关频谱和孔 A_3、A_4 的互相关频谱的叠加，含扭率 $(\partial^2/\partial x'\partial y')$ 信息；同理衍射晕 (A_{24}, A_{35}) 含扭率 $(\partial^2/\partial y'\partial x')$ 信息；衍射晕 A_{23} 是孔 A_2、A_3 的互相关频谱，含斜率 $(\partial/\partial x)$ 信息，同理衍射晕 A_{45} 含斜率 $(\partial/\partial y)$ 信息；$\sum_{i=1}^{5}A_i$ 是各孔的自相关频谱的叠加，是零频分量.

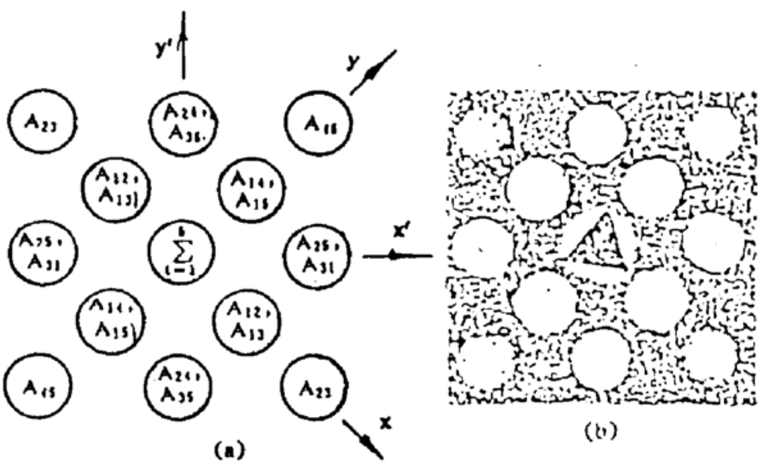

Fig. 4 Spectra plane halo distributions: (a) Schematic pattern; (b) Photograph

4　理论分析

物点 O_1、O_2、…、O_5 对同一像点 I_1 的场有贡献如图 2 和图 3 所示. 变形前像点 I_1 的复振幅

和光强分别为

$$U_1 = A\exp(i\phi_1) + A\exp[i(\phi_2+\delta_x)] + A\exp[i(\phi_3-\delta_x)] + A\exp[i(\phi_4+\delta_y)] + A\exp[i(\phi_5-\delta_y)] \tag{5}$$

$$I_1 = A^2[5 + 2\cos(\phi_{21}+\delta_x) + 2\cos(\phi_{13}+\delta_x) + 2\cos(\phi_{41}+\delta_y) + 2\cos(\phi_{15}+\delta_y) + 2\cos(\phi_{43}+\delta_x+\delta_y) + 2\cos(\phi_{25}+\delta_x+\delta_y) + 2\cos(\phi_{42}-\delta_x+\delta_y) + 2\cos(\phi_{35}-\delta_x+\delta_y) + 2\cos(\phi_{23}+2\delta_x) + 2\cos(\phi_{45}+2\delta_y)] \tag{6}$$

式中 $\phi_{ij}=\phi_i-\phi_j$ 是随机位相差，$\delta_x = kx_iD/d_i$，$\delta_y = ky_iD/d_i$，(x_i,y_i) 是像点坐标.

同理变形后像点 I_1 的复振幅和光强分别为

$$U_2 = A\exp[i(\phi_1+\delta_1)] + A\exp[i(\phi_2+\delta_x+\delta_2)] + A\exp[i(\phi_3-\delta_x+\delta_3)] A\exp[i(\phi_4+\delta_y+\delta_4)] + A\exp[i(\phi_5-\delta_y+\delta_5)] \tag{7}$$

$$I_2 = A^2[5 + 2\cos(\phi_{21}+\delta_{21}+\delta_x) + 2\cos(\phi_{13}+\delta_{13}+\delta_x) + 2\cos(\phi_{41}+\delta_{41}+\delta_y) + 2\cos(\phi_{15}+\delta_{15}+\delta_y) + 2\cos(\phi_{43}+\delta_{43}+\delta_x+\delta_y) + 2\cos(\phi_{25}+\delta_{25}+\delta_x+\delta_y) + 2\cos(\phi_{42}+\delta_{42}-\delta_x+\delta_y) + 2\cos(\phi_{35}+\delta_{35}-\delta_x+\delta_y) + 2\cos(\phi_{23}+\delta_{23}+2\delta_x) + 2\cos(\phi_{45}+\delta_{45}+2\delta_y) \tag{8}$$

式中 $\delta_{ij}=\delta_i-\delta_j$ 是因变形而引起的位相变化之差. 双曝光总光强为

$$I_T = I_1 + I_2 \tag{9}$$

4.1　曲率信息的提取

全场滤波时仅让衍射晕 (A_{12}, A_{13}) 通过，此时分析系统像面光强为

$$\begin{aligned} I &= |A^2\exp[i(\phi_{21}+\delta_x)] + A^2\exp[i(\phi_{13}+\delta_x)] + A^2\exp[i(\phi_{21}+\delta_{21}+\delta_x)] + A^2\exp[i(\phi_{13}+\delta_{13}+\delta_x)]|^2 \\ &= A^4\{4 + 4\cos[(d_{21}+\delta_{13})/2]\cos[(\delta_{21}-\delta_{13})/2] + 2\cos(\phi_{21}-\phi_{13}) \\ &\quad + 2\cos(\phi_{21}-\phi_{13}+\delta_{21}) + 2\cos(\phi_{21}-\phi_{13}-\delta_{13}) + 2\cos(\phi_{21}-\phi_{13}+\delta_{21}-\delta_{13})\} \end{aligned} \tag{10}$$

式中含 ϕ 的项为高频成分，含 δ 的项为低频成分. 文献[1]指出

$$\left.\begin{aligned} \delta_{21}+\delta_{13} &= k\left[\frac{u_2-u_3}{2\Delta}2\Delta\sin\theta_x + \frac{w_2-w_3}{2\Delta}2\Delta(1+\cos\theta_x) - \frac{2D}{d_0}u_1\right] \\ \delta_{21}-\delta_{13} &= k\left[\frac{u_2+u_3-2u_1}{\Delta^2}\Delta^2\sin\theta_x + \frac{w_2+w_3-2w_1}{\Delta^2}\Delta^2(1+\cos\theta_x)\right] \end{aligned}\right\} \tag{11}$$

把(1)式和(3)式代入(11)式，得

$$\left.\begin{aligned} \delta_{21}+\delta_{13} &= k\left[\frac{\partial u}{\partial x}2\Delta\sin\theta_x + \frac{\partial w}{\partial x}2\Delta(1+\cos\theta_x) - \frac{2D}{d_0}u_1\right] \\ \delta_{21}-\delta_{13} &= k\left[\frac{\partial^2 u}{\partial x^2}\Delta^2\sin\theta_x + \frac{\partial^2 w}{\partial x^2}\Delta^2(1+\cos\theta_x)\right] \end{aligned}\right\} \tag{12}$$

显然衍射晕 (A_{12}, A_{13}) 含因两斜率条纹的错位而产生的曲率$(\partial^2/\partial x^2)$ 条纹；同理衍射晕(A_{14}, A_{15}) 含曲率$(\partial^2/\partial y^2)$ 条纹.

4.2　扭率信息的提取

全场滤波时仅让衍射晕 (A_{25}, A_{34}) 通过，此时分析系统像面光强为

$$
\begin{aligned}
I = & |A^2 \exp[i(\phi_{13} + \delta_x + \delta_y)] + A^2 \exp[i(\phi_{25} + \delta_x + \delta_y)] \\
& + A^2 \exp[i(\phi_{43} + \delta_{43} + \delta_x + \delta_y)] + A^2 \exp[i(\phi_{25} + \delta_{25} + \delta_x + \delta_y)]|^2 \\
= & A^4[4 + 4\cos\frac{\delta_{43}+\delta_{25}}{2}\cos\frac{\delta_{43}-\delta_{25}}{2} + 2\cos(\phi_{13}-\phi_{25}) + 2\cos(\phi_{43}-\phi_{25}+\delta_{43}) \\
& + 2\cos(\phi_{43}-\phi_{25}-\delta_{25}) + 2\cos(\phi_{13}-\phi_{25}+\delta_{13}-\delta_{25})]
\end{aligned}
\tag{13}
$$

式中

$$
\left.
\begin{aligned}
\delta_{43} &= k\left[\frac{u_4' - u_3'}{\sqrt{2}\Delta}\sqrt{2}\Delta\sin\theta_{x'} + \frac{w_2 - w_3}{\sqrt{2}\Delta}\sqrt{2}\Delta(1+\cos\theta_{x'}) - \frac{\sqrt{2}D}{d_0}u_1'\right] \\
\delta_{25} &= k\left[\frac{u_2' - u_5'}{\sqrt{2}\Delta}\sqrt{2}\Delta\sin\theta_{x'} + \frac{w_2 - w_5}{\sqrt{2}\Delta}\sqrt{2}\Delta(1+\cos\theta_{x'}) - \frac{\sqrt{2}D}{d_0}u_1'\right]
\end{aligned}
\right\}
\tag{14}
$$

把(2)式和(3)式代入(14)式并整理，得

$$
\left.
\begin{aligned}
\delta_{43} + \delta_{25} &= k\left[\frac{\partial u'}{\partial x'}2\sqrt{2}\Delta\sin\theta_{x'} + \frac{\partial w}{\partial x'}2\sqrt{2}\Delta(1+\cos\theta_{x'} - \frac{2\sqrt{2}D}{d_0}u_1'\right] \\
\delta_{43} - \delta_{25} &= k\left[\frac{\partial^2 u'}{\partial x'\partial y'}2\Delta^2\sin\theta_{x'} + \frac{\partial^2 w}{\partial x'\partial y'}2\Delta^2(1+\cos\theta_{x'})\right]
\end{aligned}
\right\}
\tag{15}
$$

显然衍射晕(A_{25},A_{34})含因两斜率条纹的错位而产生的扭率($\partial^2/\partial x'\partial y'$)条纹；同理衍射晕($A_{24}$,$A_{35}$)含扭率($\partial^2/\partial y'\partial x'$)条纹.

4.3 斜率信息的提取

全场滤波时仅让衍射晕 A_{23} 通过，此时分析系统像面光强为

$$
\begin{aligned}
I &= |A^2\exp[i(\phi_{23} + 2\delta_x)] + A^2\exp[i(\phi_{23} + \delta_{23} + 2\delta_x)]|^2 \\
&= A^4(2 + 2\cos\delta_{23})
\end{aligned}
\tag{16}
$$

$$
\delta_{23} = k\left[\frac{\partial u}{\partial x}2\Delta\sin\theta_x + \frac{\partial w}{\partial x}2\Delta(1+\cos\theta_x) - \frac{2D}{d_0}u_1\right]
\tag{17}
$$

(17)式表明衍射晕 A_{23} 含斜率($\partial/\partial x$) 条纹；同理衍射晕 A_{45} 含斜率($\partial/\partial y$) 条纹.

5 实验结果

试件是直径 31 mm，厚度 1 mm，周边固定，中心集中加载的铝板. 光楔顶角 $\alpha = 30'$，$D =$ 15 mm，各孔直径 $d = 10$ mm. 20 mW He-Ne 激光器作照明光源，试件中心挠度 7 μm.

对双曝光散斑图进行全场滤波，结果如图 4(b)和图 5 所示. 图 4(b)是衍射晕照片；图 5 是从相应衍射晕中提取的信息条纹照片.

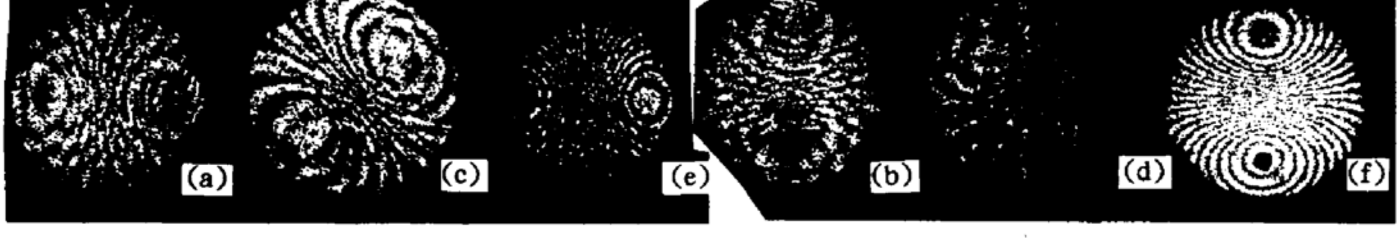

Fig. 5 (a) curvature ($\partial^2 w/\partial x^2$) fringes; (b) twist ($\partial^2 w/\partial x'\partial y'$) fringes; (c) slope ($\partial w/\partial x$) fringes; (d) curvature ($\partial^2 w/\partial y^2$) fringes; (e) twist ($\partial^2 w/\partial y'\partial x'$) fringes; (f) slope ($\partial w/\partial y$) fringes

6 结　论

本文提出的五孔剪切相机包括离焦型和光楔型用于二维曲率、扭率和斜率的同时测量是可行的.它为薄板弯曲问题的研究提供了一种重要手段和方法.但缺点是曲率条纹和扭率条纹的对比度不高.

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Simultaneous measurement of 2-D curvature, twist and slope with a five-aperture shearing camera

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Abstract

In this paper, a new speckle shearing camera is designed for simultaneous recording 2-D curvature, twist and slope on a double exposed specklegram. The specklegram yields thirteen distinct diffraction halos when subjected to filtering. The curvature, twist and slope fringes can be obtained by filtering via the appropriate halos.

Key words　five-shearing camera, curvature, twist, slope, central difference.