

# 群色散光纤中高强度光脉冲传输与压缩的研究

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## 提 要

考虑非线性四阶折射率, 推导了光脉冲在群色散单模光纤中传输的非线性方程, 并得到孤子型解析解. 同时指出在 Sech 型光纤脉冲压缩实验中, 光强度的阈值.

**关键词** 四阶折射率, 非线性, 脉冲压缩.

旨在获得飞秒脉冲的光纤脉冲压缩技术受到关注<sup>[1~4]</sup>. 由于脉冲的不断压缩, 脉冲光的功率变得很强, 因而光纤中的四阶非线性折射率已不可忽略. 本文研究色散光纤中, 同时考虑二阶和四阶非线性的光脉冲传输, 得到非线性薛定谔方程的解析解. 同时指出, 为得到压缩的 Sech 型脉冲, 光强度必须低于  $10^{11}$  W/cm<sup>2</sup>.

## 1 方程推导及求解

同时考虑二阶和四阶非线性折射率 ( $n_2$  和  $n_4$ ) 光纤中的折射率可表为

$$n(\omega, |E|^2) = n(\omega) + n_2|E|^2 + n_4|E|^4, \quad (1)$$

介质中的电场可设为

$$E(x, t) = \text{Re}\{u(x, t)\exp[i(k_0x - \omega_0t)]\}, \quad (2)$$

式中  $u(x, t)$  为缓变振幅,  $k_0 = \omega_0 n_0 / c$  为波数,  $\omega_0$  为频率, 而  $n_0 = n(\omega_0)$ . 如  $E(x, t) \sim \exp[i(kx - \omega t)]$ , 则  $u(x, t) \sim \exp\{i[(k - k_0)x - (\omega - \omega_0)t]\}$ . 而在色散光纤中

$$k = k_0 + \left. \frac{\partial k}{\partial \omega} \right|_{\omega_0} (\omega - \omega_0) + \frac{1}{2} \left. \frac{\partial^2 k}{\partial \omega^2} \right|_{\omega_0} (\omega - \omega_0)^2 + \frac{\omega_0 n_2}{c} |E|^2 + \frac{\omega_0 n_4}{c} |E|^4, \quad (3)$$

作算符代换:  $(k - k_0) \sim i(\partial/\partial X)$ ,  $(\omega - \omega_0) \sim i(\partial/\partial \tau)$ . 并设  $\zeta = x$ ,  $\tau = t - (x/v_g)$ ,  $v_g = (\partial k/\partial \omega)|_{\omega_0}$  为  $\omega = \omega_0$  处的群色散. 可得光脉冲在光纤中传播的非线性方程为下列形式:

$$\left. \begin{aligned} i \frac{\partial u}{\partial \zeta} + \alpha \frac{\partial^2 u}{\partial \tau^2} + \beta |u|^2 u + \gamma |u|^4 u &= 0, \\ \alpha = -\frac{1}{2} \left. \frac{\partial^2 k}{\partial \omega^2} \right|_{\omega_0}, \quad \beta = \frac{\omega_0 n_2}{c}, \quad \gamma = \frac{\omega_0 n_4}{c}. \end{aligned} \right\} \quad (4)$$

下面求方程(4)的解析解. 设  $u = \rho(\zeta, \tau)\exp[i\phi(\zeta, \tau)]$  代入方程(4), 并令实、虚部分别相等, 得

$$\left. \begin{aligned} \rho\phi_\zeta &= \alpha\rho_{\tau\tau} - \alpha\rho\phi_\zeta^2 + \beta\rho^3 + \gamma\rho^5, \\ \rho_\zeta &= -2\alpha\rho_\tau\phi_\zeta - \alpha\rho\phi_{\tau\tau} \end{aligned} \right\} \quad (5)$$

为了得到方程(5)的孤波解,进一步可设<sup>[5]</sup>

$$\rho = \rho(z), \quad \phi = \phi(z, \zeta) \quad (6)$$

式中  $z = \tau - M\zeta$ , 且  $\phi_\zeta = \cos nt$ .  $= k$ ,  $\phi'_z$  与  $\zeta$  无关. 常数  $M$  和  $K$  分别对应孤子速度的倒数和波数漂移. 把(6)式代入(5)式,得

$$\rho(K - M\phi'_z) = \alpha\rho'' - \alpha\rho\phi_z'^2 + \beta\rho^3 + \gamma\rho^5, \quad (7)$$

$$M\rho' = 2\alpha\rho'\phi'_z + \alpha\rho\phi_{zz}'' \quad (8)$$

(8)式两边同乘以  $\rho$ , 积分一次得

$$(1/2)M\rho^2 - \alpha\rho^2\phi'_z = \text{常数}. \quad (9)$$

考虑到在反常色散( $(\partial^2 K/\partial\omega^2)_{\omega_0} < 0$ )介质中, 只存在  $z \rightarrow \pm\infty$  时,  $\rho \rightarrow 0$  的孤子解, 故积分常数为零. 从而

$$\phi'_z = M/2\alpha. \quad (10)$$

将(10)式代入(7)式,有

$$\rho'' + (\gamma/\alpha)\rho^5 + (\beta/\alpha)\rho^3 - \left(\frac{K}{\alpha} - \frac{M^2}{4\alpha^2}\right)\rho = 0. \quad (11)$$

将(11)式积分一次得

$$\begin{aligned} (1/2)(\rho')^2 + \pi(\rho) &= 0, \\ \pi(\rho) &= (1/6\alpha)\gamma\rho^6 + (\beta/4\alpha)\rho^4 - \frac{1}{2}\left(\frac{K}{\alpha} - \frac{M^2}{4\alpha^2}\right)\rho^2 + c. \end{aligned} \quad (12)$$

积分常数  $c$ , 根据条件  $z \rightarrow \pm\infty$ ,  $\rho \rightarrow 0$ ,  $\rho' \rightarrow 0$  决定, 可知  $c = 0$ .

不失一般性, 可假设脉冲峰值位于  $z = 0$  处, 即  $\rho(0) = \rho_0$ ,  $\rho'(0) = 0$ . 通过下式

$$\int_{\rho_0}^{\rho} \frac{d\rho}{[-2\pi(\rho)]^{1/2}} = \pm z. \quad (13)$$

可求得

$$\left. \begin{aligned} \rho^2 &= \frac{\rho_0^2}{2-v} \left[ \cosh^2(uz) + \frac{v-1}{2-v} \right]^{-1}, \\ u^2 &= (\gamma/3\alpha)\rho_0^6 + (\beta/2\alpha)\rho_0^4, \quad v = (\beta\rho_0^2/2\alpha u^2). \end{aligned} \right\} \quad (14)$$

(10)式和(14)式即是非线性方程(4)的解.

## 2 分析和讨论

将(10)式变回到时间坐标, 可知在光纤脉冲压缩实验中, 自相位调制是线性的. 这符合不考虑四阶非线性折射率时光脉冲压缩实验结果<sup>[2]</sup>. 从推导中知道, 四阶非线性折射率不影响相位调制, 故在光纤脉冲压缩实验中总可以得到线性相位调制. 但如考虑光脉冲在光纤中长距离传输, 则非线性方程中必须加入微分项  $\partial(|u|^2u)/\partial\tau$  的贡献, 得到修正的非线性方程<sup>[6,7]</sup>, 这时自相位调制就不再是线性的了, 从(14)式知, 只要  $\gamma \neq 0$  则恒有  $v \neq 1$ . 也就是说, 如果考虑四阶非线性折射率的影响, 则得不到 sech 型脉冲. 这样, 为得到在测量方面应用广泛的 sech 型脉冲, 光脉冲强度必须低于一阈值. 下面对强度阈值作一数量级估计.  $n_2$  一般为  $10^{-21}$  (MKS),  $n_4 \sim 10^{-41}$  MKS<sup>[8]</sup>, 故对光强  $I > 10^{11}$  W/cm<sup>2</sup> 的光脉冲, 四阶非线性折射率的影响已不可忽略. 可以认为,  $I_t = 10^{11}$  W/cm<sup>2</sup> 就是 sech 型光脉冲压缩的强度阈值.

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### Propagation and compression of high intensity pulse in GVP optical fibers

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#### Abstract

Taking into account of the fourth-order nonlinear refractive index, we have obtained the soliton solution of the nonlinear equation which governs the propagation of optical pulse in group velocity dispersion (GVP) single-mode fibers. And we point out that there is a intensity threshold for the sech-type pulse compression.

**Key words** fourth-order refractive index, nonlinear and pulse compression, optical fiber.