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# Carrier phase shifting holographic interferometry

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## Abstract

A new and elegent technique for automated phase evaluation referred to as carrier phase shifting holographic interferometry is proposed. The key features of the technique are as follows. The fringe patterns with different phase shift can be reconstructed from a double exposures hologram. Thus it can be used for automated phase evaluation to time dependent problems. Although, the fringe patterns with different phase shift are readout from different places on the hologram, they can be exactly overlaped. This cancels the positioning problem faced by multiple channels phase shift methods. It provides live phase shift fringe patterns because the fringe patterns with continously phase shift can be observed as the reconstructing laser beam moves. Principle of the technique as well as its experimental demonstration is presented.

**Key words** phase shifting, holographic interferometry

## 1 Introduction

Nowadays, methods for automated interpretation of holographic interferograms have reached some degree of sophistication, such as phase shifting interferometry, phase locking method, heterodyne and quasi-heterodyne techniques and so forth. These techniques mainly use more than one fringe pattern to remove the ambiguity in the fringe data for interpretation. These fringe patterns are taken at different time with different phase shift[1~2], which may be called separated time methods. Separated time will basically prohibit them to be applied to time dependent problems and suffer phase shift due to the effect of enviromental disturbtance. Other fringe pattern methods either have some refrains about fringe pattern design, such as one fringe pattern FFT technique, or need some human interaction[3~4]. Some researchers developed multiple channels methods which may be referred to as separated space methods[5~8]. Using this kind of techniques, the fringe patterns with different phase shift can be obtained simultaneously, but through different optical channels. This causes another problem that the fringe patterns with different phase shift can not be overlaped exactly when they are being processed, which will add some errors to the final results. Two reference beams quasi-heterodyne technique can combine the standard double exposure holographic interferometry. However, it requires some sophisticated instruments and suffers misalignment. This paper presents a new and elegent method based on the marriage of merits from both the separated time and the separated space techniques. The fringe patterns with different phase shift can be reconstructed from a double exposure hologram, thus it can be applied for automated phase evaluation to time dependent problems. Although the fringe patterns with different phase shift are readout from different positions on the hologram, they can be exactly overlaped. This eliminates the positioning problem. This technique also provides us live

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phase shifting fringe patterns since the fringe patterns with continuously phase shift can be observed as the reconstructing laser beam moves.

## 2 Principle of the method

Fig. 1 schematically shows the optical set-up with carrier phase shift arrangement for holographic interferometry. Where, the object and the hologram plate are positoned at the front and the back Fourier spectral planes respectively. and the reference beam is collimated. Before the second exposure, the reference beam is tilted by a small angle. Then the second exposure is taken after the object is deformed. Assume $U_{01}(x, y)$ and $U_{02}(x,y)$ represent the object beam before and after deformation respectively, $U_{01}(x_f, y_f)$ and $U_{02}(x_f, y_f)$ be the object beam at the back Fourier spectral plane correspondingly. Let $R_1$ and $R_2$ represent the reference beams before and after the object is deformed respectively. Thus they can be expressed as follows.

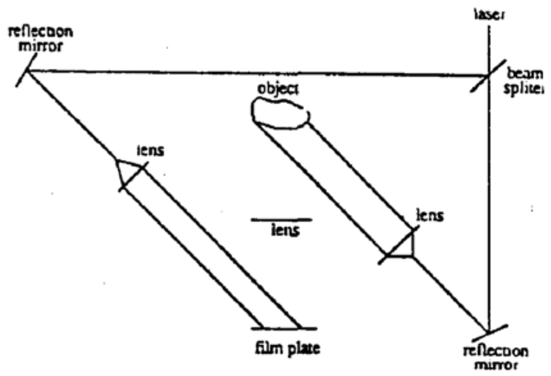

Fig. 1 Optical arrangement of the method

$$U_{01}(x,y) = a \exp\,(i\phi_1) \tag{1}$$

$$U_{02}(x,y) = a \exp\,[i(\phi_1 + \Delta\phi)] \tag{2}$$

$$U_{01}(x_f,y_f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_{01}(x,y) \exp\left[-i\frac{2\pi}{\lambda f}(xx_f + yy_f)\right] dxdy \tag{3}$$

$$U_{02}(x_f,y_f) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} U_{02}(x,y) \exp\left[-i\frac{2\pi}{\lambda f}(xx_f + yy_f)\right] dxdy \tag{4}$$

$$R_1(x_f,y_f) = r \exp\,[i(b_1x_f + cy_f)] \tag{5}$$

$$R_2(x_r,y_f) = r \exp\,[i(b_2x_f + cy_f)] \tag{6}$$

For the first exposure, the intensity of the beam at the back Fourier spectral plane is

$$\begin{aligned} I_1 &= [U_{01}(x_f,y_f) + R_1][U_{01}(x_f,y_f) + R_1]^* \\ &= U_{01}^2(x_f,y_f) + R_1^2(x_f,y_f) + R_1(x_f,y_f)U_{01}(x_f,y_f) + R_1(x_f,y_f)U_{01}^*(x_f,y_f) \end{aligned} \tag{7}$$

Similarly, for the second exposure, the intersity of the beam at the back Fourier spectral plane is

$$\begin{aligned} I_2 &= [U_{02}(x_f,y_f) + R_2][U_{02}(x_f,y_f) + R_2]^* \\ &= U_{02}^2(x_f,y_f) + R_2^2(x_f,y_f) + R_2(x_f,y_f)U_{02}(x_f,y_f) + R_2(x_f,y_f)U_{02}^*(x_f,y_f) \end{aligned} \tag{8}$$

The object image is reconstructed by illuminating the double exposure hologram with an unexpanded thin laser beam. This is equivenlant Fourier transformation when the reconstruction image plane is far enough from the hologram. Let $F$ repesent the Fourier transformation, the reconstructed beam is

$$E = F[I_1 + I_2] \tag{9}$$

There are eight terms in Eq(9). The first two terms, $F[U_{01}^2(x_f,y_)]$ and $F[U_{02}^2(x_f,y_f)]$, are self-autocorrelation functions, which only form the background halo in the reconstruction image plane. The third and the fourth terms, $F[R_1]$ and $F[R_2]$, are delta functions which form bright point at zero point in the reconstruction image plane. The fifth and the sixth terms are as follows

$$E_1 = F[R_1(x_f,y_f)U_{01}(x_f,y_f) + R_2(x_f,y_f)U_{02}(x_f,y_f)]$$
$$= \iint_\sigma R_1(x_f,y_f)U_{01}(x_f,y_f)\exp\left[i\frac{2\pi}{\lambda f}(xx_f + yy_f)\right]dxdy$$
$$+ \iint_\sigma R_2(x_f,y_f)U_{02}(x_f,y_f)\exp\left[i\frac{2\pi}{\lambda f}(xx_f + yy_f)\right]dxdy \quad (10)$$

Where $\sigma$ is a small area illuminated by an unexpanded thin laser beam. Since the area is small enough, the reference beams $R_1$ and $R_2$ can be treated as constants. When the coordinates $(x_f, y_f)$ are chosen at $(x_{f_0}, y_{f_0})$ and can be taken out of the integral in the above, the integral becomes

$$E = R_1(x_{f_0},y_{f_0})\iint_\sigma U_{01}(x_f,y_f)\exp\left[i\frac{2\pi}{\lambda f}(xx_f + yy_f)\right]dxdy$$
$$+ R_2(x_{f_0},y_{f_0})\iint_\sigma U_{02}(x_f,y_f)\exp\left[i\frac{2\pi}{\lambda f}(xx_f + yy_f)\right]dxdy \quad (11)$$

According to Eq(3) and (4), the above equation can be written as

$$E_1 = R_1U_{01}(x,y) + R_2U_{02}(x,y)$$
$$= r\exp[i(b_1x_{f_0} + cy_{f_0})]\,a\exp(i\phi_1) + r\exp[i(b_2x_{f_0} + cy_{f_0})]\,a\exp[i(\phi_1 + \Delta\phi)] \quad (12)$$

Hence, the intensity is

$$I_{e1} = E_1E_1^* = 2(ar)^2 + 2(ar)^2\cos(\Delta\phi + Bx_{f_0}) \quad (13)$$

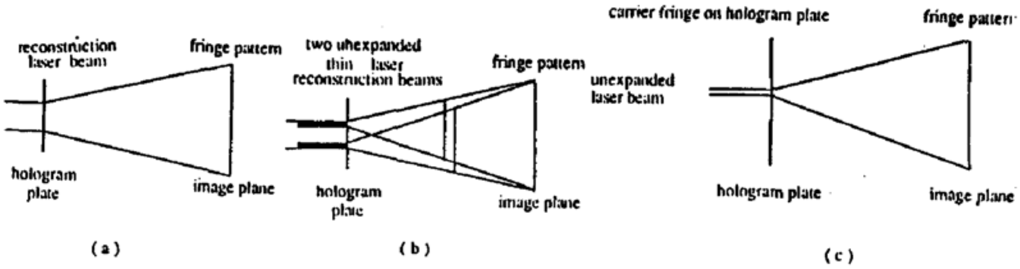


Fig. 2 Optical setup for reconstruction

Where $B = b_2 - b_1$, which is related to the tilted angle of reference beam before the second exposure. Similarly, the conjugate result can be obtained from the last two terms in the integral in Eq. (8). From Eq. (13) it is obvious that on the hologram plate there is a carrier $Bx_f$ and the corresponding paralled fringes can be seen. When the illuminated point $(x_{f_0}, y_{f_0})$ on the hologram is chosen, the phase shift is obtained in the fringe pattern at the reconstruction image plane. Thus, when using an unexpanded thin laser beam to scan the hologram plate, the fringe pattern with any phase shift can be acquired. The width of one carrier fringe should be at least four times the width of the unexpanded thin laser beam as shown in Fig. 2(c). The width of one carrier fringe is related to the tilted angle before the second exposure. It is relatively simple to obtain by experiment rather than calculation. In this way, four fringe patterns with different phase shift from 0 to 360 degree can be obtained. The fringe patterns with any phase shift image are at exactly the same place at the reconstruction image plane as shown in Fig. 2. Wherein (a), the reconstruction laser beam is exactly the same as the reference laser beam, and (b), the reconstruction laser beam scans from one position to another. However, the two reconstruction fringe patterns with different phase shift are exactly overlaped at the reconstruction image plane. Therefore, they can be exactly overlaped in the processing procedure without positioning problem. If the fringe patterns are captured at different plane, the positions of them depends on the illuminated spots on the hologram plate as shown in Fig. 2 (b). At these planes, the fringe patterns can still be seen clearly since the focal depth is increased by using an unexpanded thin laser beam. The optical setup for the former is Fourier holographic interferometry. For common holographic interferometry optical setup Eq. (13) can be directly obtained from Eq(1), (2), (5) and (6). While reconstructing, it is better to use a lens to image the fringe patterns. Once again, the fringe patterns with different phase shift can be exactly overlaped at the image plane.

## 3 Experimental demonstration

A rectangular plate is used in our experiment. The plate is clamped along its four boundaries and subject to a uniform pressure. In the experiment, the surface displacement of the plate is dominated by the plane displacement. The optical setup is shown in Fig. 1. Before the second exposure, the collimated reference beam is tilted by a small angle. Then the plate is defomed and the second exposure is taken. After the plate is developed, an unexpanded thin laser beam is used to reconstruct the object image with fringd pattern. When the thin laser beam is employed to scan the hologram plate, four fringe patterns with 0, 90, 180 and 270 degree phase shift respectivly can be obtained as shown in Fig. 3(a). These fringe patterns at the image plane are captured through a video camera connected with compter image processing system as shown in Fig. 4. The common four fringe patterns phase shifting technique is used to calculate the phase and displacement distributions. Fig. 3 shows the results in which (b) represents the displacement distribution on the line coinciding with the coordinate, and (c) depicts the phase distribution in the demonstrated rectangular area of the plate.

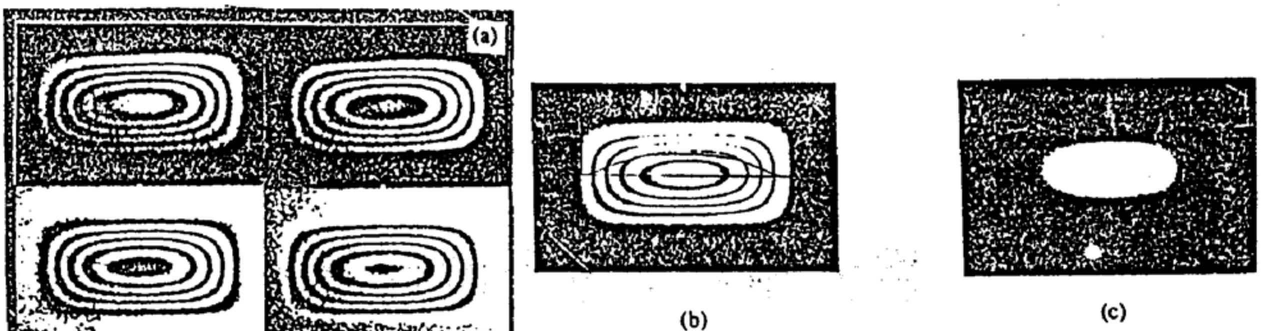


Fig. 3 Experimental results

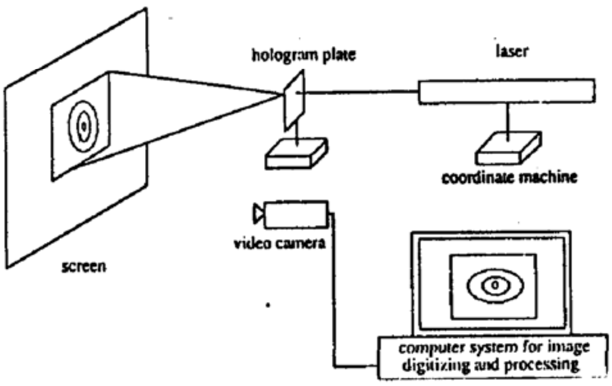


Fig. 4 Optical setup for reconstruction and image processing

## 4 Conclusion

An elegent and simple technique of carrier phase shifting holographic interferometry has been presented, which posses the following major merits:

1) The fringe patterns with different phase shift can be read outfrom double exposure hologram plate, thus it can be used for automated phase evaluation of time dependent problem.

2) Although the fringe patterns with different phase shift come from different positions on the hologram plate, they can be exactly overlaped because they are captured at the reconstruction image plane.

3) Live phase shifting fringe patterns can be obtained again at any time and any where in a similar optical setup after the experiment.

4) The thin laser beam reconstruction increases the focal depth, therefore, thus high quality fringe pattern can be obtained.

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# 载波相移全息干涉计量

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## 提 要

本文报道了一项称为载波相移全息干涉计量的自动相位估算的新技术.该技术的主要特点是不同相移的条纹图案可用双曝光全息重现.这样即可用来对与时间相关的问题进行自动相位估算.

**关键词** 相移, 全息干涉计量.