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A pulsed current solenoid wiggler

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Abstract

This paper analyses time character and the main parameters of a pulsed current solenoid system for wiggler field in free-electron laser and discusses the characteristics of the solenoid wiggler field.

Key words free-electron laser, wiggler field.

1 Introduction

Free-electron laser (FEL) ia a device for coverting the kinetic energy in a relativistic electron beam to optical radiation. Wiggler is a main part of this device. Usually there are two types about Wiggler. One is constructed by permanent magnetic material, Los Alamos National Laboratory[1] and MIT[2] have done many very good work. Another is a solenoid derived wiggler.

For the far infrared free-electron laser (FI-FEL) and high efficiency FEL, a novel wiggler has been designed and been buiding[3,4]. This wiggler construction is shown in Fig. 1.

A staggered array of high-permeability "poles" situated inside the bore of solenoid. The effect of the poles is to deflect the longitudinal field of solenoid thereby providing the periodic transverse field B_w. The value for B_w depend upon the solenoid field B_o, the magnetic characteristics of the poles and the dimensions of the array.

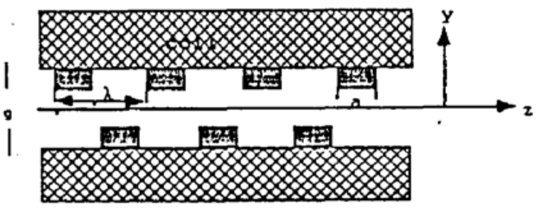

Fig. 1

In the bore of solenoid, there is not current, so that the field can be calculated by means of Laplace' s equation. Some simplifying assumptions are available.

(a) The permeability μ of the pole is very large and the material is not saturated so that on the surface of pole the field B is perpendicular. Then the longitudinal field $B_z = 0$ on the pole faces and, considering Ampere' s circuit law, the following boundary conditions exist,

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$$B_z = 0 \qquad (y = |g/2|,\ z \text{ is at pole face})$$
$$B_z = B_o/f,\ f = a/\lambda_w \quad (y = |g/2|,\ z \text{ is between poles}) \tag{1}$$

(b) The poles have infinite extent in the x- direction, so that B_z obeys the two-dimensional Laplace′ s equation

$$\partial^2 B_z/\partial z^2 + \partial^2 B_z/\partial y^2 = 0 \tag{2}$$

(c) The poles are arranged periodically in the bore of solenoid so that the field distribution is also periodic, that is $B_z(z) = B_z(z + \lambda_w)$ and $B_w(z) = B_w(z + \lambda_w)$.

According to the periodic field distribution, the boundary condition for B_z and Ampere′ s circuit law, the solution to eq(2) is

$$B_z = B_0 + \sum_{(n=odd)} C_n \sinh(2n\pi y/\lambda_w)\cos(2n\pi z/\lambda_w) + \sum_{(n=even)} D_n \cosh(2n\pi y/\lambda_w)\cos(2n\pi z/\lambda_w) \tag{3}$$
$$C_n = -2B_0\sin(nf\pi)/[(nf\pi)\sinh(n\pi g/\lambda_w)] \quad (n = odd)$$
$$D_n = -2B_0\sin(nf\pi)/[(nf\pi)\cosh(n\pi g/\lambda_w)] \quad (n = even)$$

The transverse field $B_w = B_y$, may be calculated from $\nabla \cdot B = 0$ and eq(3), giving

$$B_w = B_y = \sum_{(n=odd)} C_n \cosh(2n\pi y/\lambda_w)\sin(2n\pi z/\lambda_w) + \sum_{(n=even)} D_n \sinh(2n\pi y/\lambda_w)\sin(2n\pi z/\lambda_w) \tag{4}$$

This novel wiggler has some interesting advantages. For example, the presence of a strong longitudinal magnetic field confines the electrons along an axis. Secondly, the field is easily altered by changing the solenoid current, Thirdly, it is rather simple to fabricate the solenoid-driven wiggler, especially at short wiggler periods.

2 The characteristics of the solenoid wiggler field

On the axis of the solenoid, C_1 in eq. (4) is much larger than the other coefficients so that B_w may be represented as a simple sinusoid.

Comparing the amplitudes C_n, D_n of various terms with C_1 in eq. (4), for the parameters $g = 2\times 10^{-3}$m, $\lambda_w = 10^{-2}$ m, $f = 0.4$, we have that

$$C_n/C_1 = \sin(nf\pi)\sinh(\pi g/\lambda_w)/[n\sin(f\pi)\sinh(n\pi g/\lambda_w)]$$
$$D_n/C_1 = \sin(nf\pi)\sinh(\pi g/\lambda_w)/[n\sin(f\pi)\cosh(n\pi g/\lambda_w)] \tag{5}$$

Table 1

n	1	3	5	7	9
C_n/C_1	1.00000	−0.04285	−0.00004	0.00147	−0.00052
n	2	4	6	8	10
D_n/C_1	0.10931	−0.02702	0.00515	−0.00068	0.00000

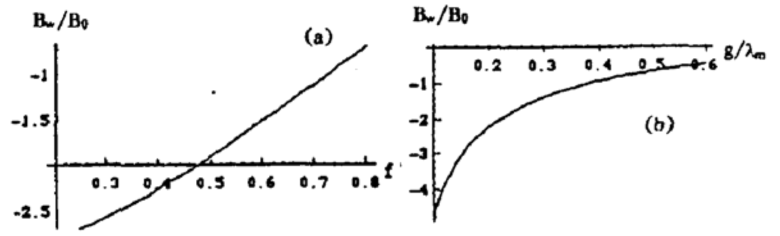
From table. 1, $C_n \ll C_1$ ($n = 3,5,7,9\cdots\cdots$), $D_n \ll C_1$ ($n = 2,4,6,8,10\cdots\cdots$) , so we could omit the high order harmonic term in eq(4). Then $B_w \approx C_1\cosh(2\pi y/\lambda_w)\sin(2\pi z/\lambda_w)$, B_w is nearly an harmonic function.

The amplitude $B_w(y)$ of the wiggler field is dependent upon the solenoid field B_0 and the dimensions f, g, λ_w. For the first term in eq. (4), the relationship is

$$B_w(y)/B_0 = -\{2\sin(f\pi)/[f\pi\sinh(\pi g/\lambda_w)]\}\cosh(2\pi y/\lambda_w) \quad (6)$$

The dependence $B_w|_{y=0}$ upon $f, g/\lambda_w$, are shown in Figs. 2(a), (b).

The relation between $B_w(y)$ and y is shown in Fig. 3. From Figs. 2(a), (b), we see that $|B_w/B_0|$ ($y = 0$, the field on the axis of the solenoid) increases with decreasing f and decreasing g/λ_w. Fig. 3 shows that $|B_w(y)/B_0|$ increases with increasing $|y|$.



In where, $g/\lambda_w = 0.2$ In where, $f = 0.4$

Fig. 2

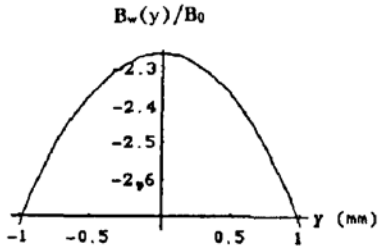


Fig. 3

We have noted the finite permeability of the pole, so that boundary condition (1) of field B_z in the bore is approximate. So the solution of Laplace' s eq (2) may deviates from real field in some case.

3 The pulsed current solenoid for the wiggler field

Superconducting solenoid is an ideal wiggler solenoid, but it is expensive, and it requires liquid helium equipment. In this part we consider a pulsed current solenoid, constructed with copper pipe coil, a water cooling system and a pulsed current supply circuit. From a simulation calculation, this method is a less expensive alternative to the superconducting magnet.

As shown at Fig. 4, the bore of the solenoid is cylindrical, its diameter is $h = 0.1$ m, its length is $D = 1.5$ m. A staggered array of high permeability poles are situated inside the bore. The coil is close wound at outside bore, the outside radius of copper pipe a is 0.01 m, and the inside radius is ka, where $k = 0.7$.

According to the solution of Laplace' s equation about the wiggler solenoid, the wiggler field B_w is determined by B_0 and the magnetic pole arrangement. In this paragraph we assume $B_0 = 1.0$ or 0.5 T, to calculate the coil current I, the thermal power P of solenoid and the solenoid inductance L.

B_0 is the field in the absence of the pole pieces, so that

$$I = B_0 D/\mu_0 N \quad (8)$$

where N is the number of turns and μ_0 is the permeability of free space.

For a pulsed current solenoid, the rise time should be short, but considering that the field B_0 is rather large, then the turns of the solenoid or the solenoid inductance L should be moderate. We select $B_0 = 1.0$ T, N is varied from 150 to 1500, and the figure I vs N is shown at figure 5. At $N = 300$, $I = 3981$ A, and for $B_0 = 0.5$ T, $N = 300$, $I = 1990$ A.

The copper pipe resistance R is given by

$$R = \rho l/s = \rho^* N^* (h + 4N^* a^2/D)/(1 - k^2)a^2, \qquad \rho = 1.6 \times 10^{-8}\,\Omega m \quad (9)$$

For $N = 300$, $R = 0.0169\ \Omega$. The thermal power of the solenoid P

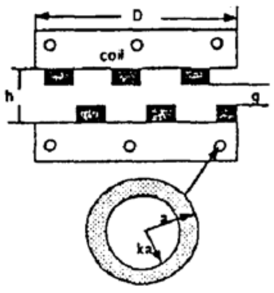


Fig. 4

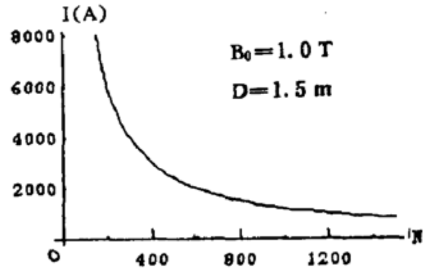


Fig. 5

$$P = I^2R = \rho(DB_0/\mu)^2(h + 4Na^2/D)/[N(1-k^2)*a^2] \quad (10)$$

The figure P vs N is shown in Fig. 6. For $B_0 = 1.0$ T, $N = 300$, $P = 268475$ W, and for $B_0 = 0.5$ T, $N = 300$, $P = 67119$ W. The solenoid inductance L is

$$L = N\Phi/I = \mu\pi[N(h + 4Na^2/D)]^2/4D \quad (11)$$

For $N = 300$, $L = 0.00192$ H, which does not include the contribution of magnetic poles in bore.

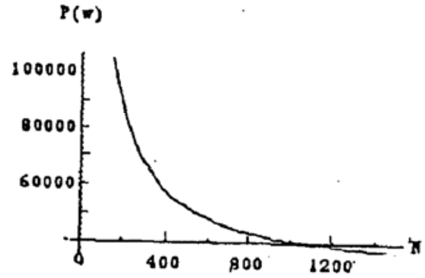


Fig. 6 (In where, $B_0 = 0.5$ T, $h = 0.1$ m $a = 0.01$ m, $K = 0.7$, $D = 1.5$ m)

4 Pulsed current supply circuit and water cooling system

The sketch of the pulsed current supply circuit for the wiggler solenoid shown at Fig. 7. The circuit includes two parts, one is the RC charging circuit, another is the RCL discharging circuit. The parameter of circuit is calculated as follows.

4.1 Discharging circuit

For the parameter values $N = 300$, $L = 0.00192$ H and $R = 0.0169\ \Omega$, then for $B_0 = 1.0$ T we have $I = 3918$ A and for $B_0 = 0.5$ T we have $I = 1990$ A. We will find the parameters of the discharging circuit C, Q_m, V_m, and the current I, Where Q_m is the initial charge on the capacitor, V_m is the voltage on the capacitor, C is the capacitance of the capacitor. The circuit equation is

$$L(d^2q/dt^2) + R(dq/dt) + C_q = 0 \quad (12)$$

This is a damped oscillatory circuit, and discharging is most rapid for critical damping. The critical damping condition is

$$\frac{R}{2}\left(\frac{C}{L}\right)^{1/2} = 1, \text{ or } C = 4L/R^2 = 26.7\ F \quad (13)$$

From eq. (12), making use of the initial value $q|_{t=0} = Q_m$ and $(dq/dt)|_{t=0} = 0$, we get

$$q(t) = Q_m(1 + Rt/2L)\exp[-Rt/2L] \quad (14)$$

$$I(t) = -(R^2Q_mt/4L^2)\exp[-Rt/2L] \quad (15)$$

R and L are known parameters, and the maximum current I_m must larger than or equal to 3981 A for $N = 300$, $B_0 = 1.0$ T. Then we could calculate I_m, Q_m and time width of current pulse. From $(dI/dt)|_{t=t_m} = 0$, we obtain $t_m = 2L/R$; so

$$I_m = -RQ_m/2Le\ (e = 2.718). \quad (16)$$

Then we get

$$Q_m = 2eLI_m/R \quad (17)$$

For $L = 0.00192$ H, $R = 0.0169\ \Omega$, $I_m = 3981$ A, $Q_m = 2448$ C; $V_m = Q_m/C = 91.65$ V. If we choose $B_0 = 0.5$ T, $I_m = 1990$ A. then $Q_m = 1224$ C; $V_m = 45.83$ V. The figure $I(t)$ vs t is shown in Fig. 8.

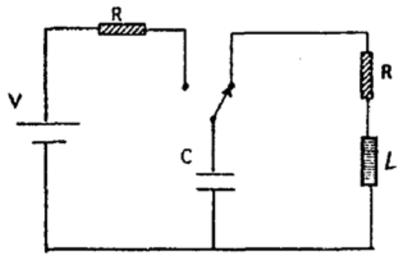


Fig. 7

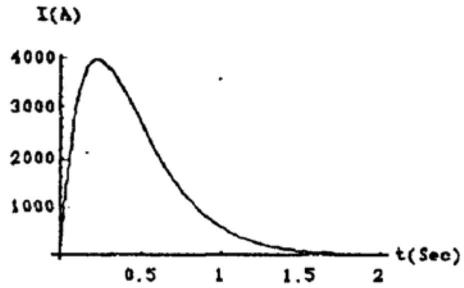


Fig. 8

The time width δ of pulse current can be obtained directly from Fig. 8, or from the formula $I_m/2 = -(R/2L)^2 Q_m t \exp[-Rt/2L]$. The result is $\delta \approx 0.58$ sec.

4.2 **Charging circuit**

The charging circuit is an RC circuit. The circuit equation is

$$V_0 = q/C + R_0(dq/dt) \tag{18}$$

The initial value $q|_{t=0} = 0$, so that

$$q = CV_0[1 - \exp(-t/CR_0)] \tag{19}$$

$$V = V_0[1 - \exp(-t/CR_0)] \tag{20}$$

$$I = (V_0/R_0)\exp[-t/CR_0], \qquad I_m = V_0/R_0 \tag{21}$$

The V_0 of power supply should be larger than or equal to the maximum electrical voltage V_m. For $B_0 = 1.0$ T, $V_0 \approx 91.65$ V; for $B_0 = 0.5$ T, $V_0 \approx 45.83$ V. The peak power P_m of the power supply will change following R_0, we assume $P_m = 40$ kW for $B_0 = 1.0$ T, so $R_0 = V_0^2/P_m = 0.21\ \Omega$. The rise time $\tau = CR_0 = 5.61$ sec, the maximum charging current $I_m = 436$ A. For $B_0 = 0.5$ T, we take $P_m = 20$ kW, so $R_0 = 0.105\ \Omega$, $\tau = 2.8$ sec, $I_m = 436$ A.

4.3 **Some results**

1) Power supply

For $B_0 = 1.0$ T, $V_0 = 91.65$ V, $P_m = 40$ kW, $r \leqslant R_0 = 0.21\ \Omega$, $I_m = 436$ A. For $B_0 = 0.5$ T, $V_0 = 45.83$ V, $P_m = 20$ kW, $r \leqslant 0.105\ \Omega$, $I_m = 436$ A.

2) Capacitor: $C = 26.7$ F, $V_m = 91.65$ V for $B_0 = 1.0$ T, or 45.83 V for $B_0 = 0.5$ T.

3) Solenoid coil: $L = 0.00192$ H, $R = 0.0169\ \Omega$, $I_m = 3981$ A for 1.0 T or 1990 A for 0.5 T.

4) Charging time: $\Delta t = 3\tau = 16.8$ sec for 1.0 T, $\Delta t = 8.4$ sec for 0.5 T.

5) Discharging time: $d = 0.58$ sec.

6) Discharging peak power: $P = 268.5$ kW for 1.0 T, or $P = 67.1$ kW for 0.5 T.

4.4 **Water cooling system**

Water colling system may be used to reduce the temperature of the solenoid. We know that the peak thermal power of solenoid is $P = 268.5$ kW for $B_0 = 1.0$ T or $P = 67.1$ kW for $B_0 = 0.5$ T, and the width of the current pulse $\delta = 0.58$ sec. Assuming that the period of current pulse is Λ, the average thermal power of the solenoid is

$$P_a \approx P^* \delta/\Lambda = 268.5\ \mathrm{kW} 0.58/\Lambda \tag{22}$$

At $\Lambda = 23.2$ sec, $P_a = 6712$ W. For $B_0 = 0.5$ T, $P_a = 67.1$ kW $0.58/\Lambda$. At $\Lambda = 12$ sec, $P_a = 3244$ W.

In the water cooling system, the water flow will remove heat by thermal conduction. The removed heat Q and the temperature increment of water $(T - T_0)$ obey following formula.

$$Q = cm\ (T - T_0) \tag{23}$$

where $c = 1$ cal $gm^{-1}K^{-1}$ is the specific heat of water and m is the mass.

In thermal equilibrium state,

$$dQ/dt = P_a/J = P_a/4.186 \tag{24}$$

so that

$$c(T - T_0)dm/dt = dQ/dt = P_a/4.186 \tag{25}$$

Assuming $T_0 = 20°C$, $P_a = 6712$ W, the flow volume of water in copper pipe is

$$dm/dt = 6712/4.186(T - 20)\ (g/s) \tag{26}$$

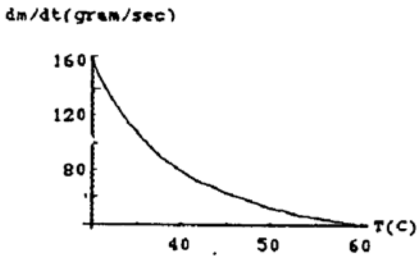


Fig. 9 In where, $T_0 = 20°$, Pa=6712 W)

The figure of dm/dt vs T is shown in Fig. 9. At $T = 40°C$, $dm/dt = 80.2$ g/s. The density of water $\sigma = 1$, the cross section of water flow $S = \pi(ka)^2$ so that the water flow velocity

$$v = (dm/dt)/\sigma\pi(ka)^2 = 6712/6.44(T - 20)\ cm/s \tag{27}$$

For $T = 40°C$, $v = 52.1$ cm/s, which is a very moderate rate of flow. For $B_0 = 0.5$ T, $P_a = 3244$ W, $T_0 = 20°C$, $T = 40°C$, the flow volume of water is $dm/dt = 38.8$ g/s; the water flow velocity is $v = 25.2$ cm/s.

5 Discussion about the time character of pulsed current and circuit parameter

Usually the relativistic electron beam is short pulse type, at Far- IR FEL of Stanford University[5], the micropulse width of electron beam is 10 ps; the average macropulse width is 3 μs, the macropulse space is 0.1 sec, From eq. (15) or Fig. 9, although the solenoid current $I(t)$ or solenoid field $B_0(t)$ is not square wave, the $I(t)$ or $B_0(t)$ vary with t slowly for short, fast pulse of electron beam. In the period of that the electron beam macropulse pass through the wiggler, the solenoid current I can be supposed as a constant current.

We make again some figure of $I(t)$ vs t (they are part figure of Fig. 8). Fig. 10(a) show that around peak current, as $\Delta t = 3000$ μs, $\Delta I/I = 0.01$ %. Fig. 10(b). Fig. 10(c) show that around half peak current, as $\Delta t = 3000$ μs, $\Delta I/I = 0.8\%$; but as Δt % $= 3$ μs, $\Delta I/I = 5 \times 10^{-4}\%$. Then we think that the enoid current I can be supposed as a constant current in the macropulse width range (3 μs) of electron beam; this pulsed current solenoid wiggler can fit in with Far-IR FEL.

For a solenoid field B_0 that is not too large, the pulsed current solenoid is less expensive. But there are some weakness in this method. From the simulation calculation, we know that the capacitor is large in the pulsed current supply circuit. For example at $B_0 = 0.5$ T, the capacitor is $v = 45$ V, $c = 27$ F, it needs 243 element capacitors (15 V, 1 F) to make such a large capacitor. At $B_0 = 1$ T, the capacitor is $v = 92$ V, $c = 27$ F. It needs 972 element capacitors (15 V, 1 F)).

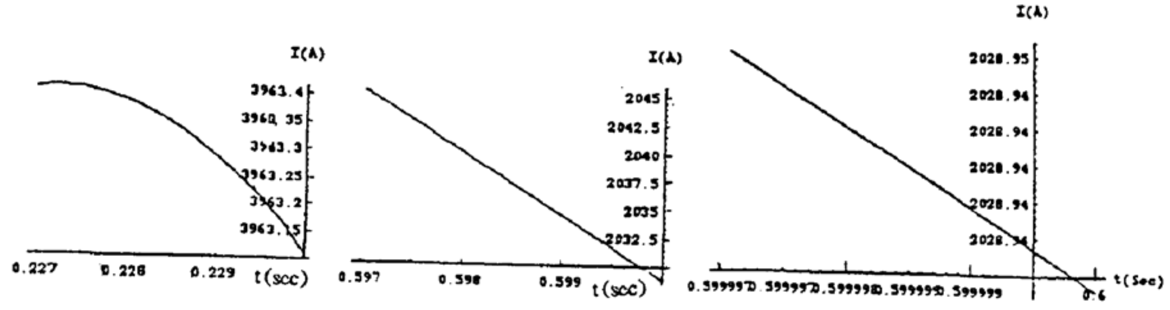


Fig. 10

Of course, we could raise the turns of solenoid to lower the capacity C, but this is not a good way. For maintaining the same solenoid field B_0, if the capacity is decreased, the voltage of the capacity will become very high.

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脉冲螺旋 Wiggler 磁体*

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提 要

本文讨论了 Stanford 大学所建造的自由电子激光器中所采用的新奇的螺旋管 Wiggler 磁场的有关特性，并着重分析了脉冲电流螺旋 Wiggler 磁体的主要参数和时间特性.

关键词 自由电子激光， Wiggler 磁场.

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