

部份相干光照明的随机孔屏衍射特性

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提 要

本文从部份相干理论出发,导出了随机孔屏的衍射强度期望分布及涨落方差表达式;对随机孔屏的衍射特性作了较深入的分析;讨论了光源的空间特性对衍射图形的影响;研究表明:衍射强度的光场分布不仅与孔和屏的几何特性有关,还与光源的形状和大小密切相关.文中还指出:在特定照明情况下,形状一定的小孔无论在数目还是尺寸上发生随机变化,都不会改变衍射场的强度期望分布和涨落方差分布.

关键词 随机孔屏, 强度期望分布.

1 引 言

随机孔屏是一种随机地分布着许多小孔的屏,屏上的小孔形状一定,数目、位置、尺寸都分别遵循某种随机分布.总结这种屏在光照下的衍射光强分布规律,在散斑技术、全息技术、表面测量及一些光学信息处理方面(如全息图的随机空间抽样技术中减弱散斑效应)等,都具有较重要的意义.自 Exner 首先开展这项研究以来,随机孔屏的远场衍射特性研究已受到重视. Barrat 对相干光照明的随机孔屏进行研究^[1],得出统计平均的衍射光强期望分布和反映各点强度涨落程度的光强方差公式.相干照明方法在使用上要求较高,颗粒噪声对于准确的测量非常不利.因此,本文将研究扩展到较普遍的情况,从部份相干理论出发,导出了适用于普通光源照明的衍射光强期望分布和光强涨落方差公式,为今后进行衍射特性的分析测量时,选择合适的光源形式,打下良好的基础.

2 理论推导及计算

如图1所示光学系统,设光源上坐标为 x_0, y_0 , 辐射强度分布为 $v(x_0, y_0)$;随机孔屏上坐标为 x, y , 屏上分布有 N 个形状和大小一样的透明小孔,整个屏的孔径函数为 $A(x, y)$;输出谱面的坐标为 u, v , 谱函数为 $F(u, v)$. 容易得到

$$A(x, y) = A_0(x, y) \cdot C(x, y) \quad (1)$$

$$F(u, v) = F_0(u, v) \sum_{j=1}^N \exp \left[\frac{ik}{f_1} (ux_j + vy_j) \right] \quad (2)$$

式中 $A_0(x, y)$ 为孔屏上单个小孔的孔径函数, $C(x, y)$ 为各孔中心的位置函数

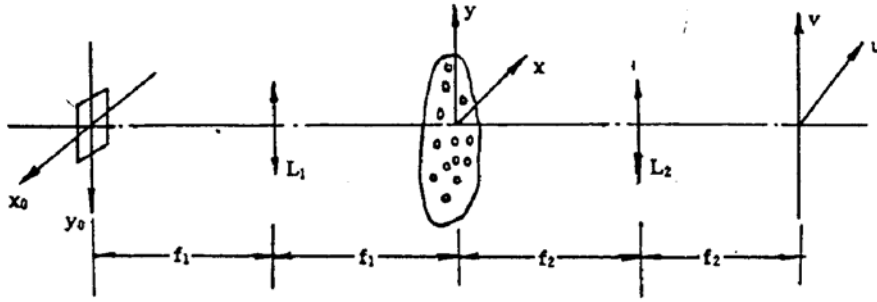


Fig. 1 Diffraction system of the screen with randomly-distributed holes with partially coherent illumination

$$C(x, y) = \begin{cases} \sum_{j=1}^N \delta(x - x_j, y - y_j) & \text{孔} \in \text{屏} \\ 0 & \text{孔} \notin \text{屏} \end{cases}$$

按部份相干理论,得输出面上相互强度为

$$J(u_1, v_1, u_2, v_2) = \iiint \iiint \iiint \gamma(x_0, y_0) \left\{ A(x_1, y_1) \cdot e^{-ik \left[r_1 \left(\frac{x_0}{f_1} + \frac{u_1}{f_2} \right) + y_1 \left(\frac{y_0}{f_1} + \frac{v_1}{f_2} \right) \right]} \right. \\ \left. \cdot A^*(x_2, y_2) e^{-ik \left[r_2 \left(\frac{x_0}{f_1} + \frac{u_2}{f_2} \right) + y_2 \left(\frac{y_0}{f_1} + \frac{v_2}{f_2} \right) \right]} \right\} dx_1 dy_1 dx_2 dy_2 dx_0 dy_0 \quad (3)$$

若令 $f_1 = f_2 = f$, 则得输出面归一化光强分布

$$\bar{I}(u, v) = \iint \gamma(u - x_0, v - y_0) \left| \frac{F_a(x_0, y_0)}{F_a(0, 0)} \right|^2 S_{10} S_{01} dx_0 dy_0 \quad (4)$$

$$S_{\alpha, \beta} = \sum_{j=1}^N p_j^{\alpha} q_j^{\beta}, \quad p_j = \exp \left[\frac{ik}{f} (x_0 x_j + y_0 y_j) \right], \quad q_j = \exp \left[-\frac{ik}{f} (x_0 x_j + y_0 y_j) \right]$$

考虑到每个小孔出现在屏上的位置是随机的,出现在某位置的概率为 $1/S$, 经整理后得期望光强为

$$\langle \bar{I}(u, v) \rangle = \iint \gamma(u - x_0, v - y_0) t_a(x_0, y_0) [N + N(N - 1) t_s(x_0, y_0)] dx_0 dy_0 \quad (5)$$

$$t_a(x_0, y_0) = \left| \frac{F_a(u, v)}{F_a(0, 0)} \right|^2, \quad t_s(x_0, y_0) = \left| \frac{F(u, v)}{F(0, 0)} \right|^2$$

经繁杂的推导,得到反映强度涨落的光强协方差表达式

$$\sigma^2 I = \langle \bar{I}^2 \rangle - \langle \bar{I} \rangle^2 = \left. \begin{aligned} & \iiint \iiint \gamma(u - x_0, v - y_0) \gamma(u - x'_0, v - y'_0) t_a(x_0, y_0) t_a(x'_0, y'_0) \\ & (N^3 A_s + N^2 B_s + N C_s) dx_0 dy_0 dx'_0 dy'_0 \\ & A_s = G_s - 4 t_s(x_0, y_0) t_s(x'_0, y'_0) \\ & B_s = t_s(x_0 + x'_0, y_0 + y'_0) + t_s(x_0 - x'_0, y_0 - y'_0) + 10 t_s(x_0, y_0) t_s(x'_0, y'_0) - 3 G_s \\ & C_s = 2 G_s - 6 t_s(x_0, y_0) t_s(x'_0, y'_0) - t_s(x_0 + x'_0, y_0 + y'_0) - t_s(x_0 - x'_0, y_0 - y'_0) \\ & G_s = F_s(x_0 - x'_0, y_0 - y'_0) F_s^*(x_0, y_0) F(x'_0, y'_0) + F(x_0 + x'_0, y_0 + y'_0) F(x_0, y_0) \\ & \quad \cdot F_s^*(x'_0, y'_0) + F_s^*(x_0 - x'_0, y_0 - y'_0) F_s(x_0, y_0) F_s^*(x'_0, y'_0) \\ & \quad + F_s^*(x_0 + x'_0, y_0 + y'_0) F_s(x_0, y_0) F_s(x'_0, y'_0) \end{aligned} \right\} \quad (6)$$

从 $\langle \bar{I}(u, v) \rangle$ 公式可以看出, 衍射图样的光强期望分布是光源辐射强度函数 $\gamma(u - x_0, v - y_0)$ 与一个与孔屏系统有关的函数 $t_a(x_0, y_0)[N + N(N - 1)t_s(x_0, y_0)]$ 的卷积, 其中一项依赖于光源及小孔函数, 另一项还依赖于孔屏函数

设光源的辐射强度为矩形函数.

$$\gamma(x_0, y_0) = \text{rect}\left(\frac{x}{a}\right) \cdot \text{rect}\left(\frac{y}{a}\right) \quad (7)$$

屏与小孔均为圆形, 孔屏尺寸之比 $\varepsilon = 0.05$, 小孔数目 $N = 200$, 按 $\langle \bar{I}(u, v) \rangle$ 公式, 计算出输出面上得到的孔屏衍射强度分布如图 2, 从图中可以看到不同光源尺寸时的衍射强度期望分布及强度起伏情况; 由图 3 可看到半强度点位置随光源尺寸的变化关系. 计算中归一化屏的半径为 1.

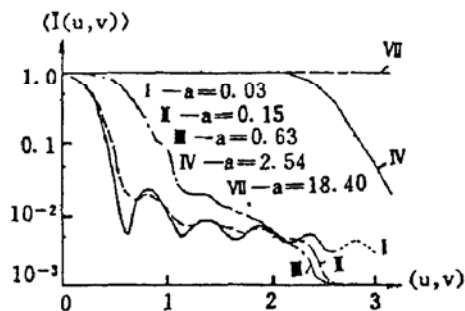


Fig. 2 Distribution of diffraction intensity vs u, v with different source sizes

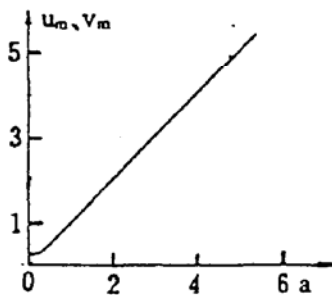


Fig. 3 Relationship of coordinates u_m, v_m of half-peak vs source radius a

3 光源大小对衍射特性的影响

3.1 部份相干照明

衍射强度分布为光源辐射强度分布和与孔屏有关的函数的卷积. 在远离衍射图形中心处, 由于 t_s 的衰减

$$\begin{aligned} \langle \bar{I}(u, v) \rangle &= N \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} t_a(x_0, y_0) \gamma(u - x_0, v - y_0) dx_0 dy_0 \\ \sigma^2 I(u, v) &= \iiint \gamma(u - x_0, v - y_0) \gamma(u - x'_0, v - y'_0) t_a(x_0, y_0) t_a(x'_0, y'_0) \\ &\quad \cdot (N^2 - N) dx_0 dy_0 dx'_0 dy'_0 \end{aligned}$$

由图 2 可以看出, 由于卷积的光滑作用, 随着光源的增大, 衍射强度的波动减小, $\sigma^2 I(u, v)$ 完全非相干 $< \sigma^2 I(u, v)$ 部份相干 $< \sigma^2 I(u, v)$ 相干, 部份相干情况下, 衍射图样的中央不是一个亮点, 而是一块亮斑, 该亮斑的大小不但与小孔几何量和小孔数目有关, 还与光源辐射分布有关.

3.2 点光源照明

$\gamma(x_0, y_0) = \delta(x_0, y_0)$, 此时对应于相干照明, 代入前面公式后得到的 $\langle \bar{I}(u, v) \rangle$ 和 $\sigma^2 I(u, v)$ 表达式与 R. Barakat 推得的完全一致, 这在很大程度上验证了繁杂公式的正确性. 在离衍射图中心较远处, $\langle \bar{I}(u, v) \rangle \sim N t_a(u, v)$, $\sigma^2 I(u, v) \sim N [1 - (1/N)]^2 t_a(u, v)$; 若小孔有均匀的瞳函数, 则可得 $\langle \bar{I}(0, 0) \rangle = N^2$, $\sigma^2 I(0, 0) = 0$. 即在衍射图样的中心可以观察到一个非常明亮的点, 该点强度为常量; 衍射强度的每一个样本实现均包含这个亮点.

3.3 当光源足够大且均匀分布

相当于完全非相干照明时,得

$$\langle \bar{I}(u, v) \rangle = \iint \gamma(-x_0, -y_0) \cdot t_a(x_0, y_0) [N + N(N-1)t_a(x_0, y_0)] dx_0 dy_0$$

$$\sigma^2 I(u, v) = 0$$

此时衍射强度无波动,输出面上衍射图形稳定不变.

4 随机孔屏对衍射特性的影响

假设随机孔屏上孔的分布没有重迭现象,屏上的 ε 和 N 均为随机数,它们相互独立, ε 的密度函数为 $f(\varepsilon)$, N 则按泊松分布

$$P(N) = \frac{\langle N \rangle^N \exp(-N)}{N!} \quad N = 0, 1, 2, \dots$$

当相干照明即 $\gamma(x_0, y_0) = \delta(x_0, y_0)$ 时,得强度期望值 $\langle I(u, v) \rangle$ 和强度波动 $\sigma^2 I(u, v)$ 分别为:

$$\langle I(u, v) \rangle \sim \begin{cases} \langle N \rangle^2 \langle t_a(0, 0) \rangle & u, v = 0 \\ \langle N \rangle \langle t_a(u, v) \rangle & u, v \neq 0 \end{cases} \quad (8)$$

$$\sigma^2 I(u, v) \sim \begin{cases} 4 \langle N \rangle^3 \langle t_a(0, 0) \rangle & u, v = 0 \\ \langle N \rangle^2 \langle t_a^2(u, v) \rangle & u, v \neq 0 \end{cases} \quad (9)$$

式中 $\langle N \rangle$ 为泊松分布的 N 的平均值,它满足

$$\sum_{N=0}^{\infty} N(N-1)\dots(N-L+1) \frac{\langle N \rangle^N \exp(-\langle N \rangle)}{N!} = \langle N \rangle^L \quad l = 0, 1, 2, \dots \quad (10)$$

$$\langle t_a(\varepsilon \rho^2) \rangle = \int_0^{\infty} t_a(\varepsilon \rho^2) f(\varepsilon) d\varepsilon$$

如果小孔是圆的,则 $t_a(\varepsilon \rho^2) = \left(\frac{2J_1(\varepsilon \rho^2)}{\varepsilon \rho^2} \right)$.

小结 点光源照明 $\gamma(x_0, y_0) = \delta(x_0, y_0)$ 时,随机和非随机孔屏的衍射特性列表如下:

u, v	$u, v = 0$		$u, v \neq 0$	
	random	non-random	random	non-random
average intensity $I(u, v)$	$\langle N \rangle^2 \cdot \langle t_a(0, 0) \rangle$	$N^2 t_a(0, 0)$	$\langle N \rangle \cdot \langle t_a(u, v) \rangle$	$N t_a(u, v)$
intensity fluctuation $\sigma^2 I(u, v)$	$4 \langle N \rangle^3 \cdot \langle t_a(0, 0) \rangle$	0	$\langle N \rangle^2 \cdot \langle t_a^2(u, v) \rangle$	$N^2 t_a^2(u, v)$

由表可见,在特定照明情况下,形状一定的小孔无论在数目还是尺寸上发生随机变化,都不会改变衍射场中各点的强度期望和涨落情况;衍射场的强度期望分布和涨落分布决定于小孔数目和尺寸的特征量.

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**Diffraction characteristics of screen with randomly-distributed holes
under a partially coherent illumination**

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Abstract

Based on the partial coherence theory, this paper derives the formula of intensity and variance of diffraction from a screen with randomly-distributed holes. The diffraction characteristics of the screen are analysed in detail, and the effect of spatial characteristics of the source on diffractive image is discussed. It has been shown that the light intensity distribution of diffraction field depends on geometric characteristics of the holes and the screen, and shape and size of the light source as well. This paper also shows that under certain illumination, both average intensity and variance distributions in diffraction field will not change if the number or size of definitely-shaped holes in the screen randomly changes.

Key words screen with randomly-distributed holes, intensity expectation distribution.