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# 偶奇相干态的q-类比

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提 要

在偶奇相干态的基础上,构造出偶、奇q-相干态,并讨论了它们的光统计特性.计算分析表明:偶奇q-相干态的光统计特性,是完全不相同的.

**关键词** 相干态, q-变形, 光统计特性.

## 1 构成偶奇q-相干态的方法

定义偶奇相干态分别为[1]

$$\left.\begin{aligned}|z\rangle_e &= N_e(z)\cosh(za^+)|0\rangle = N_e(z)\sum_{n=0}^{\infty}\frac{z^{2n}}{\sqrt{(2n)!}}|2n\rangle,\\ |z\rangle_0 &= N_0(z)\sinh(za^+)|0\rangle = N_0(z)\sum_{n=0}^{\infty}\frac{z^{2n+1}}{\sqrt{(2n+1)!}}|2n+1\rangle,\end{aligned}\right\}\tag{1}$$

于是偶奇q-相干态可分别地表示为

$$\left.|z\rangle_q^e = N_q^e(z)\sum_{n=0}^{\infty}\frac{z^{2n}}{\sqrt{[2n]_q!}}|2n\rangle_q,\qquad |z\rangle_q^0 = N_q^0(z)\sum_{n=0}^{\infty}\frac{z^{2n+1}}{\sqrt{[2n+1]_q!}}|2n+1\rangle_q.\right\}\tag{2}$$

利用偶奇q-相干态的归一化条件

$${}_q^e\langle z|z\rangle_q^e = 1,\qquad {}_q^0\langle z|z\rangle_q^0 = 1.\tag{3}$$

容易求出它们的归一化常数为

$$N_q^e(z) = [\cosh_q(z\bar{z})]^{-1/2},\qquad N_q^0(z) = [\sinh_q(z\bar{z})]^{-1/2}.\tag{4}$$

式中引入两个q-函数及q-指数函数.

$$\left.\begin{aligned}&\cosh_q X = \frac{1}{2}(e_q^X + e_q^{-X}]) = \sum_{n=0}^{\infty}\frac{X^{2n}}{[2n]_q!},\qquad \sinh_q X = \frac{1}{2}(e_q^X - e_q^{-X}]) = \sum_{n=0}^{\infty}\frac{X^{2n+1}}{[2n+1]_q!}\\ &e_q^X = \sum_{n=0}^{\infty}\frac{x^n}{[n]_q!},\end{aligned}\right\}\tag{5}$$

q-阶乘 $[n]_q! = [n]_q[n-1]_q\cdots\cdots[1]_q$,$[X]_q$ 的定义为 $[X]_q = (q^X - q^{-X})/(q - q^{-1})$.根据一般相干态理论,并定义相干态的 $q$-变形为 $q$-湮灭算符 $a_q$ 的本征态[2]

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$$a_q|z\rangle_q = z|z\rangle_q \tag{6}$$

(6)式的解为

$$\left.\begin{aligned} |z\rangle_q &= N_q(z)\sum_{n=0}^{\infty}\frac{z^n}{\sqrt{[n]_q!}}|n\rangle_q \\ |n\rangle_q &= \frac{(a_q^+)^n}{\sqrt{[n]_q!}}|0\rangle_q, \qquad 且\ a_q|0\rangle = 0 \end{aligned}\right\} \tag{7}$$

相干态体系的核心，是它的连续性和完备性. 容易证明，偶奇 q-相干态的集合，共同构成完备的希尔伯特(Hilbert)空间，它们的完备性关系为

$$\frac{[2]}{2\pi}\int d_q^2 z\, e_q^{-z\bar z}\{\cosh_q(z\bar z)|z\rangle_{qq}^{ee}\langle z| + \sinh_q(z\bar z)|z\rangle_{qq}^{00}\langle z|\} = I \tag{8}$$

式中 $d_q^2 z = rd_q r d\theta$, $z = re^{i\theta}$，对变量 $r$ 的积分是一个 q-积分[3]，而对 $d\theta$ 的积分为一个普通积分。偶奇 q-相干态能通过一般 q-相干态展开

$$\left.\begin{aligned} |z\rangle_q^e &= \frac{1}{2}N_q^{-1}(z)\,N_q^e(z)(|z\rangle_q + |-z\rangle_q), \\ |z\rangle_q^0 &= \frac{1}{2}N_q^{-1}(z)\,N_q^0(z)(|z\rangle_q - |-z\rangle_q) \end{aligned}\right\} \tag{9}$$

当 q→1 时，偶奇 q-相干态又复原为偶奇相干态. 此外还有

$$\begin{aligned} a_q|n\rangle_q &= \sqrt{[n]_q}\,|n-1\rangle_q, & a_q^+|n\rangle_q &= \sqrt{[n+1]_q}\,|n+1\rangle_q, \\ a_q|z\rangle_q^e &= z[\tanh_q(z\bar z)]^{1/2}|z\rangle_q^0, & a_q|z\rangle_q^0 &= z[\coth_q(z\bar z)]^{1/2}|z\rangle_q^e. \end{aligned} \tag{10}$$

## 2　压缩态和反聚束的 q-类比

若量子力学变量 $x_i$ 的集合，形成李代数形式 $[x_i, x_j] = a_{ij}^k x_k$ 当涨落满足 $\Delta x_i^2 \leqslant \frac{1}{2}|C_{ij}^k\langle x_k\rangle|$，或 $\Delta x_j^2 \leqslant \frac{1}{2}|C_{ij}^k\langle x_k\rangle|$ 时，我们就称这样的态为压缩态. 按标准方式定义 $x_i^q$ $(i=1,2)$ 为 $q$-电场正交位相的振幅算符[4]，它们可通过 $q$-湮灭算符 $a_q$，$q$-产生算符 $a_q^+$ 来表示

$$x_1^q = \frac{1}{2}(a_q^+ + a_q), \qquad x_2^q = \frac{1}{2}i(a_q^+ - a_q) \tag{11}$$

算符 $a_q$、$a_q^+$ 其 q-变形对易关系[5]

$$a_q a_q^+ - q a_q^+ a_q = q^{-N_q} \tag{12}$$

$N_q$ 为厄密数算符，且

$$a_q^+ a_q = [N_q]_q = \frac{q^{N_q} - q^{-N_q}}{q - q^{-1}} \tag{13}$$

$$[N_q, a_q] = -a_q \qquad [N_q, a_q^+] = a_q^+ \tag{14}$$

若变量 $x_1^q$ 满足如下关系式

$${}_q\langle(\Delta x_1^q)^2\rangle_q < \frac{1}{2}|{}_q\langle|[x_1^q, x_2^q]|\rangle_q|, \tag{15}$$

我们称它为 q-压缩，对于变量 $x_2^q$ 也是如此. 下面我们具体计算有关量在偶奇 q-相干态中的期待值

$${}_q^e\langle z|a_q^+ a_q|z\rangle_q^e = z\bar z\tanh_q(z\bar z) \qquad {}_q^e\langle z|a_q a_q^+|z\rangle_q^e = qz\bar z\tanh_q(z\bar z) + \frac{\cosh_q(q^{-1}z\bar z)}{\cosh_q(z\bar z)} \tag{16}$$

$$ {}_q^0\langle z|a_q^+ a_q|z\rangle_q^0 = z\bar{z}\coth_q(z\bar{z}) \qquad {}_q^0\langle z|a_q a_q^+|z\rangle_q^0 = qz\bar{z}\coth_q(z\bar{z}) + \frac{\sinh_q(q^{-1}z\bar{z})}{\sinh_q(z\bar{z})} \tag{17} $$

$$ {}_q^e\langle z|a_q^+|z\rangle_q^e = {}_q^0\langle z|a_q^+|z\rangle_q^0 = 0, \qquad {}_q^e\langle z|(a_q^2 + a_q^{+^2})|z\rangle_q^e = {}_q^0\langle z|(a_q^2 + a_q^{+^2})|z\rangle_q^0 = z^2 + \bar{z}^2 \tag{18} $$

于是

$$ \left.\begin{aligned} {}_q^e\langle z|(\Delta x_1^q)^2|z\rangle_q^e &= \frac{1}{2}r^2\{\cos 2\theta + \frac{1}{2}(1+q)\tanh_q r^2\} + \frac{1}{4}\frac{\cosh_q(q^{-1}r^2)}{\cosh_q r^2} \\ {}_q^0\langle z|(\Delta x_1^q)^2|z\rangle_q^0 &= \frac{1}{2}r^2\{\cos 2\theta + \frac{1}{2}(1+q)\coth_q r^2\} + \frac{1}{4}\frac{\sinh_q(q^{-1}r^2)}{\sin h_q r^2} \end{aligned}\right\} \tag{19} $$

$$ \left.\begin{aligned} {}_q^e\langle z|[x_1^q, x_2^q]|z\rangle_q^e &= \frac{1}{2}i\{(q-1)r^2\tanh_q r^2 + \frac{\cosh_q(q^{-1}r^2)}{\cosh_q r^2}\} \\ {}_q^0\langle z|[x_1^q, x_2^q]|z\rangle_q^0 &= \frac{1}{2}i\{(q-1)r^2\coth h_q r^2 + \frac{\sinh_q(q^{-1}r^2)}{\sinh_q r^2}\}. \end{aligned}\right\} \tag{20} $$

计算中已取 $z = r e^{i\theta}$，为了讨论 q-压缩的性质我们考虑

$$ \left.\begin{aligned} {}_q^e\langle z|(\Delta x_1^q)^2|z\rangle_q^e - \frac{1}{2}|{}_q^e\langle z|[x_1^q, x_2^q]|z\rangle_q^e| &= \frac{1}{2}r^2\{\cos 2\theta + \tanh_q r^2\}, \\ {}_q^0\langle z|(\Delta x_1^q)^2|z\rangle_q^0 - \frac{1}{2}|{}_q^0\langle z|[x_1^q, x_2^q]|z\rangle_q^0| &= \frac{1}{2}r^2\{\cos 2\theta + \coth_q r^2\}, \end{aligned}\right\} \tag{21} $$

(21)式中，$2\theta$ 取值在 $(\pi/2, \pi)$ 范围内. 由此式明显地可以看出，在有限的 $q$ 值下，由于 $\tan h_q r^2 < 1$, $\coth_q r^2 > 1$，进而给出

$$ \left.\begin{aligned} {}_q^e\langle z|(\Delta x_1^q)^2|z\rangle_q^e &< \frac{1}{2}|{}_q^e\langle z|[x_1^q, x_2^q]|z\rangle_q^e|, \\ {}_q^0\langle z|(\Delta x_1^q)^2|z\rangle_q^0 &> \frac{1}{2}|{}_q^0\langle z|[x_1^q, x_2^q]|z\rangle_q^0|. \end{aligned}\right\} \tag{22} $$

从(22)式明显地可以看出，偶 q-相干态总存在压缩，而奇 q-相干态不呈现压缩，所以偶奇 q-相干态，在压缩效应上的表现，是截然不同的. 若光场归一的二阶相关函数[6] $g^{(2)}(0) < 1$ 则光场呈现反聚束效应. 若引入 q-光场二阶 q-相关函数为

$$ g_q^{(2)}(0) = \frac{{}_q\langle z|(a_q^+)^2(a_q)^2|z\rangle_q}{|{}_q\langle z|a_q^+ a_q|z\rangle_q|^2} \tag{23} $$

容易算出，偶 q-相干态二阶相关函数和奇 q-相干态二阶函数分别为

$$ g_{qe}^{(2)}(0) = \coth_q r^2, \qquad g_{qo}^{(2)}(0) = \tan h_q r^2. \tag{24} $$

对于有限的 q-值，由于 $\tanh_q r^2 < 1$，而 $\coth_q r^2 > 1$，于是 $g_{qe}^{(2)}(0) > 1$，而 $g_{qo}^{(2)}(0) < 1$ 说明奇 q-相干态存在反聚束效应，而偶 q-相干态不呈现反射束效应. 从上面分析和计算得出，偶奇 q-相干态的光统计特性，是完全不同的.

## 参 考 文 献

[1] M. Hillery, Amplitude-squared squeezing of the electromagnetic field, *Phys. Rev. (A)*, 1987, **36**(8): 3796～3802

[2] L. C. Biedeharn, The quantum group $Su_q(2)$ and a q-analouge of the boson operators, *J. Phys. (A) Math. and Gen.*, 1989, **22**(17): L873～878

[3] R. W. Gray, C. A. Nelson, On the completeness relation of the q-harmonic coherent states, *J. Phys. (A) Math. and Gen.*, 1990 **23**(6): 954

[4] xia Yunjie, Guo Guangcan, Nonclassical properties of even and odd coherent states, *Phys. Lett. (A)*., 1989, **136**(6): 281～283

[5] Sun Changpu, Fu Hongchen, The q-deformed boson relization of the quantum group $Su_q(n)$ and its representations, *J. Phys. (A), Math. and Gen.*, 1989, **22**(21): 983～986

[6] D. F. Walls, Squeezing properties of the nonclassical light field, *Nature*., 1983, **306**(10): 141

# A q-analogue of the even and odd coherent states

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## Abstract

Based on the even and odd coherent states, the present paper is to construct the even and odd q-coherent states and study their important optical statistics properties (squeezing and antibunching). We have found that the squeezing may occur with the even q-coherent states but no antibunching effect. However, for the odd q-coherent states there is antibunching but no squeezing for all finite-q values.

**Key words**　coherent state, q-deformations, optical statistics properties.