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Influence of atomic coherence on collapse and revival phenomenon in the k-photon Jaynes-Cummings model

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Abstract

In this paper we have investigated the influence of atomic coherence on collapse and revival phenomenon in the k-photon Jaynes-Cummings model. It has shown that the atomic inversion exhibits the dependence on the relative phase between the atom and field.

Key words atomic coherence, collapse and revival effect, k-photon Jaynes-Cummings model.

The well-known Jaynes-Cummings model[1] of a two-level atom interacting with a single-mode quantized field is one of the simplest, exactly solvable, but by no means a trivial model in Quantum Optics and Laser Physics. Many studies have shown that this model has retained a great deal of interesting quantum features of the atom-field interactions such as the vacuum-field Rabi oscillations[2], collapse-revival effect of the atomic inversion[3], squeezing of the field[4], and so on. Recently, this model has been realized in the superconducting high-Q microwave resonantors, and the collapse-revival effect of the atomic inversion has been observed experimentally[5]. It has stimulated a number of authors to pay further attention to the quantum properties of the atom-field interactions, in particular regard to the collapse-revival phenomenon. Some time ago, this model was expanded by Sukumar and Buck[6] into the generalized Jaynes-Cummings model, involving the k-photon interaction between the atom and field. The collapse —

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revival effect has been theoretically predicted also in this k-photon Jaynes-Cummings model[7].

As we know, the collapso-revival effect of the atomic inversion is usually interpreted as the interfering of the incommensurate Rabi frequencies corresponding to different n values when the field is initially in a superposition of number states $|n\rangle$ and the atom in the excited or ground state. This effect, in this case, is very sensitive to intial photon statistics of the field, but not dependent on the phase of the field[3].

The question arises whether or not the atomic coherence also influences on the collapse-revival phenomenon. More recently, Zaheer et al[8] and Joshi et al[9] have studied the effects of the atom initially prepared in the superposition of its excited and ground states on the collapse-revival of the atomic inversion in the Jaynes-Cummings model with the field initially in a coherent state and a binomial state, respectively, and they have demonstrated that the influence is obvious. However, to the best of our knowledg, the influence of the atomic coherence on the collapses and revivals of the Rabi oscillations of the atomic inversion in the k-photon Jaynes-Cummings model has not been investigated. The purpose of this paper is to answer this question.

The system considered here is a two-level atom interacting with a single-mode quantized field via the k-photon transition mechanism. The Hamiltonian of this system in RWA is given by[6]

$$H=\omega a^{+}a+\omega_0 S_z+\varepsilon(S_{+}a^{k}+a^{+k}S)\quad(\hbar=1). \tag{1}$$

where a^+ and a are the creation and annihilation operators of the field mode of frequency ω, S_z and $S_\pm$ the atomic inversion and transition operators, respectively, ω_0 the atomic transition frequency, ε the atom-field coupling constant, k the absorbing or emitting photon numbers per atomic transition.

This model, like the one-photon Jaynes-Cummings model, is also exactly solvable. If we assume the atom initially in the coherent superposition of the excited and ground states

$$|s\rangle=\cos(\theta/2)\,|+\rangle+\exp(-i\phi)\sin(\theta/2)\,|-\rangle \tag{2}$$

and the field in a coherent state

$$|\alpha\rangle=\exp(-|\alpha|^2/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}|n\rangle, \tag{3}$$

where $0\leqslant\theta\leqslant\pi$ denotes the atomic distribution, $\alpha=\bar{n}^{1/2}\exp(i\psi)$, $\bar{n}$ is the initial mean photon number, ϕ and ψ are the phase, which may be chosen at will, of the atom and field, respectively. The general time-dependent state of the system on resonance ($k\omega=\omega_0$), in Schrödinger picture, can be expressed as[10]

$$|\psi(t)\rangle=\exp(-iHt)\,|s\rangle\otimes|\alpha\rangle=\exp(-\bar{n}/2)\sum_{n=0}^{\infty}\frac{\alpha^n}{\sqrt{n!}}\Big\{\cos(\theta/2)\exp\Big[-i\Big(n+\frac{k}{2}\Big)\omega t\Big]$$

$$\times\left[\cos\sqrt{\frac{(n+k)!}{n!}}\,\varepsilon t\,|+,\,n\rangle-i\sin\sqrt{\frac{(n+k)!}{n!}}\,\varepsilon t\,|-,\,n+k\rangle\right]$$

$$+\sin(\theta/2)\exp\left[-i\left(\phi+\left(n-\frac{k}{2}\right)\omega t\right]\left[\cos\sqrt{\frac{n!}{(n-k)!}}\,\varepsilon t\,|-,\,n\rangle\right.\right.$$

$$\left.\left.-i\sin\sqrt{\frac{n!}{(n-k)!}}\,\varepsilon t\,|+,\,n-k\rangle\right]\right\} \tag{4}$$

Then, the atomic inversion $\langle S_z\rangle$, with the help of $S_z|\pm,\,m\rangle=\pm\frac{1}{2}|\pm,\,m\rangle$, is easily evaluated from eq. (4) to be

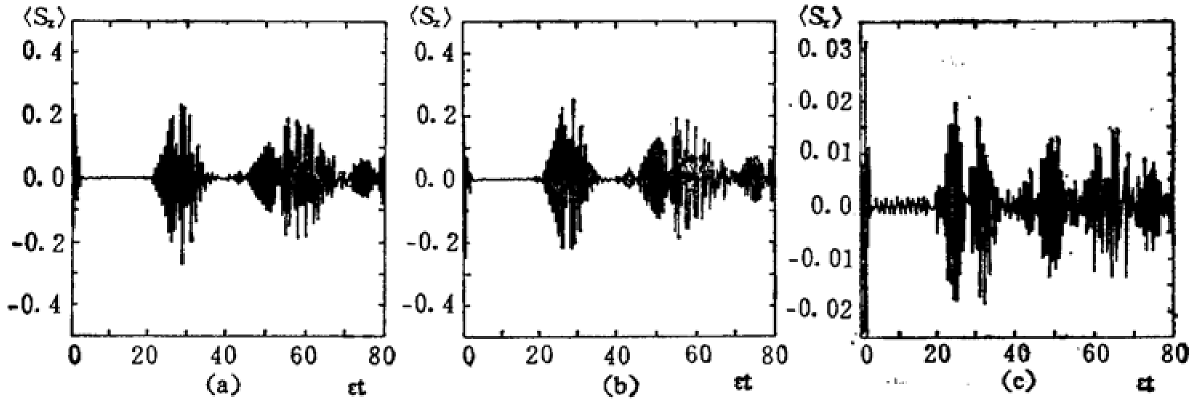
$$\langle S_z\rangle=\langle\psi(t)|S_z|\psi(t)\rangle=\frac{1}{2}\exp(-\bar{n})\sum_{n=0}^{\infty}\frac{\bar{n}^n}{n!}\left[\cos^2(\theta/2)\cos\sqrt{\frac{(n+k)!}{n!}}\,2\varepsilon t\right.$$

$$-\sin^2(\theta/2)\cos\sqrt{\frac{n!}{(n-k)!}}\,2\varepsilon t+\sin(\theta)\bar{n}^{k/2}\sqrt{\frac{n!}{(n+k)!}}$$

$$\left.\times\sin\sqrt{\frac{(n+k)!}{n!}}\,2\varepsilon t\,\sin(k\psi-\phi)\right] \tag{5}$$

In the above equation, the first and second terms respectively represent the contributions of the excited and ground states, the last term reflector the interference of both. It is clearly shown that the atomic inversion depends on not only the statistics of the field, but also the phases of the field and atom. It is worthy to point out that our results is suitable for the general case, and when $k=1$, eq. (5) becomes the result of Ref.[8]

The numerical calculation results about eq. (5), which are plotted in part by Fig. 1 and Fig. 2, exhibit that the influence of the parameters θ, ϕ and ψ on the collapse and revival behavior of the atomic inversion is not very obvious when $k\psi-\phi\neq0$, i.e. The dynamics of the atom is the essential same as that for the initially excited (ground) case, see Figs. 1(a), 1(b), 2(a) and 2(b), whereas, when

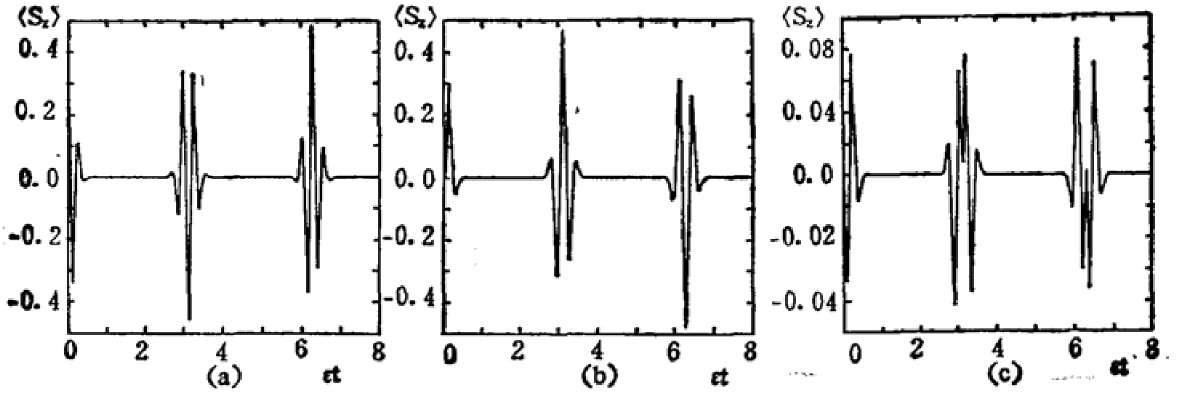


(a) The atom initially in the excited state $\theta=0$

(b) The atom initially in the superposition state with $\theta=\pi/2$ and $\psi-\phi=\pi/2$

(c) The atom initally in the superposition state with $\theta=\pi/2$ and $\psi-\phi=0$

Fig. 1 Time evolutions of the atomic inversion $\langle S_z\rangle$ in the one-photon Jaynes-Cummings model ($k=1$) for the mean photon number $\bar{n}=20$, of the initial coherent state field.



(a) The atom initially in the excited state $\theta=0$

(b) The atom initially in the superposition state with $\theta=\pi/2$ and $2\psi-\phi=\pi/2$

(c) The atom initially in the superposition state with $\theta=\pi/2$ and $2\psi-\phi=0$

Fig. 2 Time evolutions of the atomic inversion $\langle S_z\rangle$ in the two-photon Jaynes-Cummings model ($k=2$) for the mean photon number $\bar{n}=10$, of the initial coherent state field.

$k\psi-\phi=0$, θ denoting the initial atomic distribution evidently effects the amplitude of the Rabi oscillations of the atomic inversion, and the amplitude of the oscillations becomes extremely small for $\theta=\pi/2$, but the collapse-revival phenomenon still appears, see Fig. 1(c) and Fig. 2(c). On the other hand, This result can be easily found also from eq. (5) that, in the case of $k\psi-\phi=0$, the contribution of the third term which indicates the interference between the excited and ground states disappears. And the dynamics behavior of the atom depends on the difference of the contributions between the excited and ground sates to the atomic inversion. when the two levels are equally populated ($\theta=\pi/2$), this difference is very small.

In conclusions, it is clearly shown that, from the above discusses, the collapse and revival behavior of the atomic inversion does not drastically depend on the initial coherence of the atom, but this coherence clearly influences on the amplitude of the oscillations.

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k 光子 Jaynes-Cummings 模型中原子的相干性对崩溃和再生现象的影响*

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提　要

研究了 k 光子 Jaynes-Cummings 模型中原子的相干性对崩溃和再生现象的影响, 结果表明, 原子的反转依赖原子和场的初始相位。

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