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斜入射时孔径衍射旋量理论

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提 要

利用电磁场的旋量体系方法对斜入射平面波的平面孔径的电磁衍射进行了分析。获得了任意入射情况下孔径的衍射强度公式,并以特例说明与前人的公式是一致的.

关键词 衍射,旋量,电磁波.

1 引 言

麦克斯韦方程由于正确使用边界条件比较困难, 仅能解决一部分衍射问题; 基于惠更斯-菲涅耳(Huygen-Fresnel)原理的克希霍夫(Kirchhoff)理论代替麦氏理论是一种有力的工具,然而克希霍夫理论也是有局限性的.本文试图利用旋量理论[1~5].讨论衍射问题,当然此法象麦克斯韦方程一样,也不能在实际的边界条件下获得严格的衍射理论,但是在任何平面边界条件下,能获得唯一有限的旋量方程的解.

电磁旋量理论首先由 Hillion 提出[1], 之后, 他和他的同事们将其推广应用到几何光学[2],导出程函方程与输运方程. 在文献[3]中,他们又将直角坐标系推广到球坐标系,获得了惠更斯-菲涅耳原理的表达式, 并将其应用于湍流大气光波传播[4], 对于圆和矩形孔径的衍射问题也进行了初步研究[5],求得了特殊情况下的衍射方向图. 本文应用这一理论, 将电磁衍射问题从垂直入射情况推广为任意入射方向,得到了更广泛的孔径衍射方向图公式.

2 理 论 分 析

稳定旋量场用两个复旋量 $\psi(\boldsymbol{r})$来描述, 并且它的厄米特共轭用 $\psi^{+}(\boldsymbol{r})$表示, 它们分别满足如下方程

$$\left.\begin{aligned}(i\sigma^{j}\partial_{j}+k_{0}n)\psi(\boldsymbol{r})=0,\\ \psi^{+}(\boldsymbol{r})(-i\sigma^{j}\partial_{j}+k_{0}n)=0,\end{aligned}\right\}\tag{1}$$

$$\sigma^{j}\partial_{j}=\sigma_{1}\frac{\partial}{\partial x}+\sigma_{2}\frac{\partial}{\partial y}+\sigma_{3}\frac{\partial}{\partial z},\tag{2}$$

$$\sigma_{1}=\begin{bmatrix}0&1\\1&0\end{bmatrix},\quad\sigma_{2}=\begin{bmatrix}0&-i\\i&0\end{bmatrix},\quad\sigma_{3}=\begin{bmatrix}1&0\\0&-1\end{bmatrix},\tag{3}$$

式中 $\sigma^{j}\partial_{j}$ 为爱因斯坦求和符号, k_0 为波数, $\sigma_j(j=1, 2, 3)$ 为泡利(Pauli)矩阵, 为了确定 $\psi(\boldsymbol{r})$与物理量的关系,特别对波印亭矢量 S 和光强 $I(\boldsymbol{r})$有

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$$\left.\begin{aligned} S_j&=\psi^+(\boldsymbol{r})\sigma_j\psi(\boldsymbol{r}),\quad (j=1,\ 2,\ 3)\\ I(\boldsymbol{r})&=\psi^+(\boldsymbol{r})\psi(\boldsymbol{r}),\ I(\boldsymbol{r})=|\boldsymbol{S}|. \end{aligned}\right\} \tag{4}$$

在旋量体系等效到麦克斯韦理论中，有如下条件需满足：

(1) 电、磁能量密度相等；(2) 矢量 $\boldsymbol{E}$、$\boldsymbol{H}$、$\boldsymbol{S}$ 彼此正交.

对于正弦电磁场，电磁场与旋量体系有如下关系[3]

$$\left.\begin{aligned} \sqrt{\varepsilon}E_j(\boldsymbol{r})+\sqrt{\mu}H_j(\boldsymbol{r})&=(\sqrt{n}/2|\psi(\boldsymbol{r})|)\psi^T(\boldsymbol{r})\tau_j\psi(\boldsymbol{r}),\ (j=1,\ 2,\ 3)\\ |\psi(\boldsymbol{r})|&=|\psi^T(\boldsymbol{r})\psi(\boldsymbol{r})|^{1/2}, \end{aligned}\right\} \tag{5}$$

式中 E_j, H_j 为电磁场的分量，ε, μ 分别为介电常数和磁导率，$\psi^T(\boldsymbol{r})$ 为旋量的转置. 定义 τ_j 为

$$\left.\begin{aligned} &\tau_j=-i\sigma_2\sigma_j,\ (j=1,\ 2,\ 3)\\ &\tau_1=\begin{bmatrix}-1&0\\0&1\end{bmatrix},\ \tau_2=\begin{bmatrix}-i&0\\0&-i\end{bmatrix},\ \tau_3=\begin{bmatrix}0&1\\1&0\end{bmatrix}. \end{aligned}\right\} \tag{6}$$

设在某一平面 $z=f(x,\ y)$，有 $E_z=H_z=0$，则在此平面内，由(5)式知有

$$\psi_1(\boldsymbol{r})\psi_2(\boldsymbol{r})=0,$$

在此可选择 $\psi_2(\boldsymbol{r})=0$，应用边界条件

$$\psi_1[x,\ y,\ f(x,\ y)]=f_0(x,\ y). \tag{7}$$

则对方程(1)进行傅里叶变换，得

$$\left.\begin{aligned} \frac{\partial}{\partial z}\tilde{\psi}_1(k_x,\ k_y,\ z)&=ik_0n\tilde{\psi}_1(k_x,\ k_y,\ z)+(ik_x+k_y)\tilde{\psi}_2(k_x,\ k_y,z),\\ \frac{\partial}{\partial z}\tilde{\psi}_2(k_x,\ k_y,\ z)&=-ik_0n\tilde{\psi}_2(k_x,\ k_y,\ z)+(-ik_x+k_y)\tilde{\psi}_1(k_x,\ k_y,\ z), \end{aligned}\right\} \tag{8}$$

式中 $\tilde{\psi}_1(k_x,\ k_y,\ z)$, $\tilde{\psi}_2(k_x,\ k_y,\ z)$ 分别为 $\psi_1(x,\ y,\ z)$, $\psi_2(x,\ y,\ z)$ 的傅里叶变换，由(8)式得

$$\left.\begin{aligned} \tilde{\psi}_2&=A_1\exp(ik_0nsz)+A_2\exp(-ik_0nsz),\\ \tilde{\psi}_1&=-k_0n(k_x+ik_y)^{-1}[(s+1)A_1\exp(ik_0nsz)+(1-s)A_2\exp(-k_0nsz)], \end{aligned}\right\} \tag{9}$$

式中 A_1, A_2 均为 k_x, k_y 的函数

$$S^2=1-[(k_x^2+k_y^2)/k_0^2n]$$

由边界条件

$$\left.\begin{aligned} \psi_1[x,\ y,\ f(x,\ y)]&=f_0(x,\ y),\\ \psi_2[x,\ y,\ f(x,\ y)]&=0, \end{aligned}\right\} \tag{10}$$

对(9)式进行逆傅里叶变换

$$\left.\begin{aligned} \psi_1(x,\ y,\ z)=&\frac{1}{2\pi}\iint[-k_0n(k_x+ik_y)^{-1}][(s+1)A_1\exp(ik_0nsz)\\ &+(1-s)A_2\exp(-ik_0nsz)]\cdot\exp(-ik_xx-ik_yy)dk_x\,dk_y,\\ \psi_2(x,\ y,\ z)=&\frac{1}{2\pi}\iint[A_1\exp(ik_0nsz)+A_2\exp(-ik_0nsz)]\\ &\times\exp(-ik_xx-ik_yy)dk_x\,dk_y. \end{aligned}\right\} \tag{11}$$

对于通过原点的平面，可设 $z=ax+by$, a、b 为任意常数. 为了求以上积分，令

$$k_0nsb-k_y=-\lambda_2,\ k_0nsa-k_x=-\lambda_1, \tag{12}$$

则

$$\iint A_1 \exp(ik_0nsz)\exp(-ik_xx-ik_yy)\,dk_x\,dk_y$$

$$=\iint A_1'\exp(-i\lambda_1x-i\lambda_2y)\begin{vmatrix}\dfrac{\partial k_x}{\partial\lambda_1} & \dfrac{\partial k_x}{\partial\lambda_2}\\ \dfrac{\partial k_y}{\partial\lambda_1} & \dfrac{\partial k_y}{\partial\lambda_2}\end{vmatrix}d\lambda_1\,d\lambda_2$$

$$=\iint A_1\frac{k_0ns}{(ak_x+bk_y+k_0ns)}\exp(-i\lambda_1x-i\lambda_2y)\,d\lambda_1\,d\lambda_2. \tag{13}$$

同理可得

$$\iint A_2\exp(-ik_0nsz)\exp(-ik_xx-ik_yy)\,dk_x\,dk_y$$

$$=\iint A_2\left(\frac{-k_0ns}{ak_x+bk_y-k_0ns}\right)\exp(-i\lambda_1x-i\lambda_2y)\,d\lambda_1\,d\lambda_2 \tag{14}$$

将(13)、(14)式代入(11)式得

$$\left.\begin{aligned}\psi_2(x,\ y,\ z)&=\frac{1}{2\pi}\iint\left[A_1\frac{k_0ns}{ak_x+bk_y+k_0ns}-A_2\frac{k_0ns}{ak_x+bk_y+k_0ns}\right]\\&\quad\times\exp(-i\lambda_1x-i\lambda_2y)\,d\lambda_1\,d\lambda_2\\ \psi_1(x,\ y,\ z)&=\frac{1}{2\pi}\iint[-k_0n(k_x+ik_y)^{-1}]\left[(s+1)A_1\frac{k_0ns}{ak_x+bk_y+k_0ns}\right.\\&\quad\left.+(s-1)A_2\frac{k_0ns}{ak_x+bk_y-k_0ns}\right]\exp(-i\lambda_1x-i\lambda_2y)\,d\lambda_1\,d\lambda_2\end{aligned}\right\} \tag{15}$$

对边界条件(10)式进行傅里叶变换

$$\left.\begin{aligned}\tilde{\psi}_1&=\frac{1}{2\pi}\iint f_0(x,\ y)\exp(i\lambda_1x+i\lambda_2y)\,dx\,dy=V(\lambda_1,\ \lambda_2),\\ \tilde{\psi}_2&=0.\end{aligned}\right\} \tag{16}$$

将(15)式的傅里叶变换与(16)式相等有

$$\left.\begin{aligned}A_1&=\frac{-V(\lambda_1,\ \lambda_2)(ak_x+bk_y+k_0ns)(k_x+ik_y)}{2k_0^2n^2s^2},\\ A_2&=\frac{-V(\lambda_1,\ \lambda_2)(ak_x+bk_y+k_0ns)(k_x+ik_y)}{2k_0^2n^2s^2}.\end{aligned}\right\} \tag{17}$$

于是将(17)式代入(11)式便得

$$\psi_2(\boldsymbol{r})=-\frac{1}{2\pi}\iint\frac{V(\lambda_1,\ \lambda_2)(k_x+ik_y)}{k_0^2n^2s^2}[(ak_x+bk_y)\cos(k_0nsz)+ik_0ns\sin(k_0nsz)]$$
$$\times\exp(-ik_xx-ik_yy)\,dk_x\,dk_y, \tag{18}$$

$$\psi_1(x,\ y,\ z)=\frac{1}{2\pi}\iint\left[\left(1+\frac{ak_x+bk_y}{kns^2}\right)\cos(k_0nsz)+i\left(\frac{ak_x+bk_y}{k_0ns}+\frac{1}{s}\right)\sin(k_0nsz)\right]$$
$$\times V(\lambda_1,\ \lambda_2)\exp(-ik_xx-ik_yy)\,dk_x\,dk_y, \tag{19}$$

$$\left.\begin{aligned}S^2&=1-\frac{k_x^2+k_y^2}{k_0^2n^2}=\frac{(k_x-\lambda_1)^2}{k_0^2n^2a^2}=\frac{(k_y-\lambda_2)^2}{k_0^2n^2b^2},\\ k_x&=\lambda_1+\frac{-(a\lambda_1+b\lambda_2)\pm\sqrt{(1+a^2+b^2)\cdot k_0^2n^2-(\lambda_1^2+\lambda_2^2)-(a\lambda_2-b\lambda_1)^2}}{1+a^2+b^2}\,a\\ k_y&=\lambda_2+\frac{-(a\lambda_1+b\lambda_2)\pm\sqrt{(1+a^2+b^2)k_0^2n^2-(\lambda_1^2+\lambda_2^2)-(a\lambda_2-b\lambda_1)^2}}{1+a^2+b^2}\,b\end{aligned}\right\} \tag{20}$$

由已知某一面元上的旋量函数或其傅里叶变换，便可利用(18)、(19)式求得任意点 $\boldsymbol{r}$ 的旋量函数 $\psi(\boldsymbol{r})$，则光强 $I(\boldsymbol{r})$ 可由(4)式求得.

对于衍射问题，只要将边界条件代入(18)式、(19)式，便可求得衍射场的强度分布 $I(\boldsymbol{r})$.

当那已知平面为 $z=0$(即 $a=b=0$)时，本文结果与文献[5]结果一致，即

$$\left.\begin{aligned}\psi_1(\boldsymbol{r})=&\frac{1}{2\pi}\iint[\cos(k_0nsz)+(i/s)\sin(k_0nsz)]V(\lambda_1,\ \lambda_2)\\&\times\exp(-i\lambda_1x-i\lambda_2y)d\lambda_1\,d\lambda_2,\\\psi_2(\boldsymbol{r})=&-\frac{i}{2\pi}\iint(\lambda_1+i\lambda_2)[\sin(k_0nsz)/k_0ns]V(\lambda_1,\ \lambda_2)\\&\times\exp(-i\lambda_1x-i\lambda_2y)d\lambda_1\,d\lambda_2.\end{aligned}\right\}\qquad(21)$$

因此，这也就证明了本文的正确性.

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Spinor theory of diffraction from aperture in slant illumination

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Abstract

In this paper, diffraction from plane aperture with plane electromagnetic wave slant illumination is analyzed using the spinor formulism of electromagnetism. A formula of diffraction intensity is obtained with arbitrary illumination and which is agreement with the formula of reference in the special case.

Key words diffraction, spinor, electromagnetic wave.