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A simple method used to deal with the strong signal in free electron laser

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Abstract

A simple method for dealing with the strong signal in free electron laser with helical magnetic field was deduced based upon the energy model of FEL. The process how the saturation happens and the mechanism how the original energy dispersion affects the saturation were discussed. This method can also be used with the harmonic wave radiation in FEL.

Key words: the energy model of FEL; strong signal treatment.

§1. Introduce

The strong signal theory of free electron laser has been already set up[1], the analyses of which relied upon the coupling of Maxwell equations to the relativistic collisionless Boltzmann equations. According to this theory, the longitudinal part of the Boltzmann distribution function, $h(p_z, z, t)$, was expressed as the harmonic expansion in consideration of the space bunch effect of the relativistic electron beam, substituting this expansion into Boltzmann equation resulted in a set of generalized Bloch equations, from which some important results, such as the mechanism of the saturation etc., were obtained. Unfortunately, although this theory was verified to be quite successful, the way used was so complicated that its application was somewhat limited. Not long ago, we deduced a simple method for calculating the energy transfer rate of FEL with helical magnetic field under conditions of small signal where the amplitude of the radiation field could be asumed approximately as a constant[2]. However, when a strong signal is injected into the laser cell, the variation in the amplitude of the radiation field is no longer negliable, thus this method is no longer vaild. For this reason, we develop a new method in this paper to deal with the strong signal throng of FEL, based upon our former calculations[3]. The outline of this method is described as follows:

(1) The variation in the amplitude of the radiation field is related to that in the relativistic electron energy by means of energy conservation.

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(2) The transverse and the longitudinal velocities of the relativistic electron can all be expressed as the functions of the energy transfer rate $\delta\gamma/\gamma_0$, thus this is the only variable present in our basic equation.

(3) When it is injected into the laser cell, the strong signal will bring about the energy to transfer from the electron beam to the radiation field, which will in turn make the signal be stronger. The amplified radiation field will continue to interact with the electron beam and consequently furtherenergy transference will happen. This process will last until the equilibrium is attained, i.e. the saturation happens.

The advantage of adopting this new method is that the calculations become much easier when the vector equations, i.e. the dynamic equations of the relativistic electron are replaced by the scalar equation, i.e. the energy transfer rate equation. Also the physical meaning is clear. This method can also be used to deal with the harmonic wave radiation in free electron laser.

§ 2. The Energy Model Of Strong Signal For FEL

In this section, we'll set up the energy balance model for FEL with the helical magnetic field under conditions of strong signal where the effect caused by the variation in the amplitude or the radiation field on the interaction between the relativistic electron beam and the radiation field must be taken into considerations. This is realized with the help of the energy conservation condition. The analysis presented here is based upon the single particle model.

(1) The motion equations of the relativistic electron

We suppose that the wiggler $\boldsymbol{B}_w$ and the radiation field $\boldsymbol{E}_r$ and $\boldsymbol{B}_r$ can be expressed respectively as follows:

$$\boldsymbol{B}_w = B_w\{\hat{e}_x \cos\xi_w + \hat{e}_y \sin\xi_w\}. \tag{1}$$

where $\boldsymbol{B}_w$ is the amplitude of the wiggler, $\xi_w = K_w z + \phi_w$, $K_w = 2\pi/\lambda_w$, λ_w is the wiggler period and ϕ_w is the initial phase of the electron under discussicn when it enters the wiggler. Also

$$\boldsymbol{E}_r = E_r\{\hat{e}_x \cos\xi_r - \hat{e}_y \sin\xi_r\}, \tag{2.1}$$

$$\boldsymbol{B}_r = \frac{D_r}{C}\{\hat{e}_x \sin\xi_r + \hat{e}_y \cos\xi_r\}. \tag{2.2}$$

where $\xi_r = K_r z - w_r t + \phi_r$, $w_r = CK_r$ is the angular frequency of the radiation field, ϕ_r is the corresponding initial phase, and E_r and $\frac{E_r}{C}$ are the amplitudes of the electric and magnetic vectors of the radiation field respectively where E_r is the function of axial coordinate L.

The energy equation and the dynamic equation of the single relativistic electron are as follows:

$$\frac{d}{dt}\gamma=-\frac{|e|}{mc^2}\boldsymbol{v}\cdot\boldsymbol{E}_r, \tag{3}$$

$$\frac{d}{dt}\gamma\boldsymbol{v}=-\frac{|e|}{m}\{\boldsymbol{E}_r+\boldsymbol{v}\times(\boldsymbol{B}_r+\boldsymbol{B}_w)\}. \tag{4}$$

X and Y components cf Eq. (4) are expressed respectively as:

$$\frac{d}{dt}\gamma\dot{x}=-\frac{|e|E_r}{m}(1-\beta_z)\cos\xi_r+\frac{|e|B_w}{m}-\dot{z}\sin\xi_w, \tag{5}$$

$$\frac{d}{dt}\gamma\dot{y}=\frac{|e|E_r}{m}(1-\beta_z)\sin\xi_r-\frac{|e|B_w}{m}\dot{z}\cos\xi_w. \tag{6}$$

Integrating both sides of Eq. (5) results in:

$$\gamma\dot{x}=\frac{|e|E_r}{mw_r}\sin\xi_r-\frac{|e|B_w}{mK_w}\cos\xi_w-\frac{|e|}{mw_r}\int\sin\xi_r\,dE_r\,. \tag{7}$$

where the integration constant is neglected since it won't affect the final results essentially.

Similarly we get:

$$\gamma\dot{y}=\frac{|e|E_r}{mw_r}\cos\xi_r-\frac{|e|B_w}{mK_w}\sin\xi_w-\frac{|e|}{mw_r}\int\cos\xi_r\,dE_r \tag{8}$$

Substituting Eqs. (7) and (8) into Eq. (3) gives:

$$\begin{aligned}\gamma\frac{d}{dy}\gamma&=-\frac{|e|E_r}{mc^2}\{\gamma\dot{x}\cos\xi_r-\gamma\dot{y}\sin\xi_r\}\\&=\left(\frac{|e|}{mc}\right)^2\frac{E_rB_w}{K_w}\cdot\cos(\xi_w+\xi_r)+\left(\frac{|e|}{mc}\right)^2\frac{E_r}{w_r}\{\cos\xi_rI_s-\sin\xi_rI_c\}.\end{aligned} \tag{9}$$

where

$$I_s\equiv\int\sin\xi_r\,dE_r,\quad I_c\equiv\int\cos\xi_r\,dE_r. \tag{10}$$

Let $\delta\beta_z=\beta_{z_0}-\beta_z$ and $\delta\gamma=\gamma_0-\gamma$. Considering that $\beta_\perp\ll\beta_z\approx1$, we can derive the expression for $\delta\beta_z$ from the definition for the relativistic energy parameter γ:

$$\delta\beta_z\approx\frac{1}{\gamma_0^2}\left(\frac{\delta\gamma}{\gamma_0}\right). \tag{11}$$

then

$$\begin{aligned}\frac{dz}{dt}&=c\beta_{z_0}-c\delta\beta_z=u-\frac{c}{\gamma_0^2}\left(\frac{\delta\gamma}{\gamma_0}\right),\\z&=z_0+ut-\frac{c}{\gamma_0^2}\int\frac{\delta\gamma}{\gamma_0}dt.\end{aligned} \tag{12}$$

where u and z_0 are the original axial velocity and coordinate of the relativistic electron under discussion. Thus we have expressed the motion state of the relativistic electron as the function of its energy variation $\delta\gamma$. In the following paragraph we'll deduce the relation between the variation in the amplitude of the electric vector of the radiation field and that of the electron energy.

(2) The expression for the amplitude of the radiation field

Let the energy densities of the radiation field and the relativistic electron beam be denoted as W_r and W_e respectively, then W_r and W_e may be written as follows:

$$W_r = \varepsilon_0 E_r^2, \quad W_e = \rho_e mc^2 \bar{\gamma}. \tag{13}$$

where ρ_e is the macroscopic density of the relativistic electron beam. The total energy density W then is:

$$W = W_r + W_e. \tag{14}$$

Supposing there exists no other energy loss except the energy exchange between the electron beam and the radiation field, thus according to the energy conservation law we get:

$$dW = \varepsilon_0\, d(E_r^2) + \rho_e mc^2\, d\bar{\gamma} = 0. \tag{15}$$

integrating Eq. (15) gives:

$$E_r^2 - E_{r_0}^2 = -\frac{\rho_e mc^2}{\varepsilon_0}(\bar{\gamma} - \gamma_0) = \frac{\rho_e \gamma_0 mc^2}{\varepsilon_0}\left(\overline{\frac{\delta\gamma}{\gamma_0}}\right),$$

$$E_r^2 = E_{r_0}^2\left\{1 + \frac{\rho_e \gamma_0 mc^2}{\varepsilon_0 E_{r_0}^2}\left(\overline{\frac{\delta\gamma}{\gamma_0}}\right)\right\}. \tag{16}$$

where E_{r_0} is the original amplitude of the electric vector of the radiation field, and $\bar{\gamma}$ and $\delta\bar{\gamma}$ denote the average energy and average energy variation of the relativistic electron respectively.

Considering that the original energy density of the relativistic electron beam is much less than that of the radiation field under conditions of strong signal, we get the following approximate expression for E_r:

$$E_r \approx E_{r_0}\left\{1 + \frac{\rho \gamma_0 mc^2}{2\varepsilon_0 E_{r_0}^2}\left(\overline{\frac{\delta\gamma}{\gamma_0}}\right)\right\}, \tag{17}$$

Let:

$$E_r = E_{r_0} + \delta E_r, \tag{18}$$

then

$$\delta E_r = \alpha\left(\overline{\frac{\delta\gamma}{\gamma_0}}\right), \tag{19}$$

where

$$\alpha \equiv \frac{\rho_e \gamma_0 mc^2}{2\varepsilon_0 E_{r_0}}. \tag{20}$$

Eq. (17) describes how the radiation field intensity is affected by the variation in the electron beam energy. In the following we'll discuss the reverse process in the interaction between the relativistic electron and the radiation field, i.e. how the feedback of the radiation field works, as well.

(3) The basic equation

In this paragraph we'll deduce the basic equation to describe the energy transfer process of free electron laser with the helical magnetic field under conditions of strong signal.

Substituting Eqs. (12) and (19) into Eq. (9), approximately we have:

$$\frac{d}{dt}\left(\frac{\delta\gamma}{\gamma_0}\right) \approx -\left(\frac{|e|}{\gamma_0 mc}\right)^2 \frac{B_w}{K_w}\left(E_{r_0} + \alpha\overline{\frac{\delta\gamma}{\gamma_0}}\right)\cos(\Delta\omega t + \phi_0)$$
$$-\left(\frac{|e|}{\gamma_0 mc}\right)^2 \frac{B_{w}v_w\omega}{K_w \gamma_0^2}\left(E_{r_0} + \alpha\overline{\frac{\delta\gamma}{\gamma_0}}\right)\sin(\Delta\omega t + \phi_0)\int\frac{\delta\gamma}{\gamma_0}\,dt$$

$$
\begin{aligned}
&-\left(\frac{|e|}{\gamma_0 mc}\right)^2 \frac{\alpha}{\omega_r}\left(E_{r_0}+\alpha\frac{\overline{\delta\gamma}}{\gamma_0}\right)\\
&\times\left\{\cos(\xi_{r_0}+\delta\xi_r)\int\sin(\xi_{r_0}+\delta\xi_r)\alpha\left(\frac{\overline{\delta\gamma}}{\gamma_0}\right)\right.\\
&\left.-\sin(\xi_{r_0}+\delta\xi_r)\int\cos(\xi_{r_0}+\delta\xi_r)\alpha\left(\frac{\overline{\delta\gamma}}{\gamma_0}\right)\right\}.
\end{aligned}
\tag{21}
$$

where:

$$
\begin{aligned}
&\Delta\omega=K_w u-\omega_r(1-\beta_{z_0}),\ \beta_{z_0}=\frac{u}{c};\\
&\phi_0=(K_w+K_r)z_0+\phi_r+\phi_w,\quad \omega=c(K_r+K_w);\\
&\xi_{r_0}=-\omega_r(1-\beta_{z_0})t+\phi_r,\\
&\delta\xi_r=-\frac{\omega_r}{\gamma_0^2}\int\frac{\delta\gamma}{\gamma_0}dt.
\end{aligned}
\tag{22}
$$

We have seen that thevariable $\frac{\delta\gamma}{\gamma_0}$ is presen tinthe both sides of Eq. (2), but the physical meanings represented by it are quite different: On the left of the equation it denotes the energy transfer rate of the relativistic electron, however on the right, it denotes, according to Eqs. (12) and (19), the effects upon the energy transfer rate of the electron beam caused by the space charge bunching of the electron beam, and the variation in the intensity of the radiation field. In this sense, we say that Eq. (21) describes correctly the energy transfer process and the influence due to the feedback of the radiation field upon this process in the interaction between the relativistic electrons and the radiation field.

Neglecting the non-resonant terms in Eq. (21), we get:

$$
\begin{aligned}
\frac{d}{dt}\left(\frac{\delta\gamma}{\gamma_0}\right)\approx&-\left(\frac{|e|}{\gamma_0 mc}\right)^2\frac{B_w}{K_w}\left(E_{r_0}+\alpha\frac{\overline{\delta\gamma}}{\gamma_0}\right)\\
&\times\left\{\cos(\Delta\omega t+\phi_0)+\frac{\omega}{\gamma_0^2}\sin(\Delta\omega t+\phi_0)\int\frac{\delta\gamma}{\gamma_0}dt\right\}
\end{aligned}
\tag{23}
$$

Eq. (23) is the basic equation in this paper.

(4) The solution of the basic equation

When it is injected into the laser cell, the strong signal will act with the relativistic electron beam. This interaction will cause the energy to transfer from the electrons to the radiation field under some specific conditions. Having gained the energy from the electrons, the signal will become stronger and continue to interact with the electron beam, which will in turn induce further energy transference, and further amplification of the radiation field. Such a process will last until saturation happens. In this sense we express the amplitude of the radiation field E_r and the relativistic energy parameter γ of the electron with the following expansions respectively:

$$
\begin{aligned}
&E_r=E_{r_0}+\delta E_r=E_{r_0}+\delta E_{r_1}+\delta E_{r_2}+\cdots,\\
&\gamma=\gamma_0-\delta\gamma=\gamma_0-(\delta\gamma_1+\delta\gamma_2+\cdots).
\end{aligned}
\tag{24}
$$

Thus, as mentioned above, the process of the energy transference caused by the interaction between the radiation field and the relativistic electron beam can be specified with the following relations:

$$
\begin{aligned}
&E_{r_0}\to\frac{\delta\gamma_1}{\gamma_0}, \quad \delta E_{r_1}=\alpha\left(\overline{\frac{\delta\gamma_1}{\gamma_0}}\right);\\
&\delta E_{r_1}\to\frac{\delta\gamma_2}{\gamma_0}, \quad \delta E_{r_2}=\alpha\left(\overline{\frac{\delta\gamma_2}{\gamma_0}}\right);\\
&\quad\vdots \qquad\qquad\qquad \vdots\\
&\delta E_{r_n}\to\frac{\delta\gamma_{n+1}}{\gamma_0}, \quad \delta E_{r_{n+1}}=\alpha\left(\overline{\frac{\delta\gamma_{n+1}}{\gamma_0}}\right).\\
&\quad\vdots \qquad\qquad\qquad \vdots
\end{aligned}
\tag{25}
$$

where the sign "$\to$" represents the inducement.

The presences of the average values of the various order energy transfer rates just denote such a well known fact: Having entered the laser cell, some of the relativistic electrons will be decelerated, i. e. to lose energy, and others will be accelerated, i.e. to obtain energy when acting with the helical magnetic field and the radiation field. Whether they lose or obtain energy is determined by their original phases, thus the average over the original phase distribution range must be taken in order to estimate the effect of the relativistic electron beam as a whole.

According to Eqs. (23), (24) and (25), we get a set of coupled equations describing various orders of energy transfer rates, with different intensities of the radiation field.

(a) Let

$$
E_r=E_{r_0},\ \delta\gamma=\delta\gamma_1, \text{ and } \delta E_{r_1}=\alpha\left(\overline{\frac{\delta\gamma_1}{\gamma_0}}\right);
\tag{26}
$$

$$
\begin{aligned}
\frac{d}{dt}\left(\frac{\delta\gamma_1}{\gamma_0}\right)=&-\left(\frac{|e|}{\gamma_0 mc}\right)^2\frac{E_{r_0}B_w}{K_w}\cos(\Delta\omega t+\phi_0)\\
&-\left(\frac{|e|}{\gamma_0 mc}\right)^2\frac{E_{r_0}B_w\omega}{K_w\gamma_0^2}\sin(\Delta\omega t+\phi_0)\int\frac{\delta\gamma_1}{\delta_0}\,dt.
\end{aligned}
\tag{27}
$$

The average value for $\frac{\delta\gamma_1}{\gamma_0}$ can be obtained by averaging the solution of Eq. (27) over the range $0\sim2\pi$:

$$
\begin{aligned}
\left(\overline{\frac{\delta\gamma_1}{\gamma_0}}\right)\equiv\left\langle\frac{\delta\gamma_1}{\gamma_1}\right\rangle_{\phi_0}&=\frac{1}{2\pi}\int_0^{2\pi}\frac{\delta\gamma_1}{\gamma_0}\,d\phi_0\\
&=\frac{A_1B_1}{\Delta\omega^3}\left\{1-\cos\Delta\omega t-\frac{1}{2}\Delta\omega t\sin\Delta\omega t\right\}.
\end{aligned}
\tag{28}
$$

where

$$
A_1\equiv\left(\frac{|e|}{\gamma_0 mc}\right)^2\frac{E_{r_0}B_w}{K_w}, \quad B_1\equiv A_1\frac{\omega}{\gamma_0^2}.
\tag{29}
$$

Thus we get the following expression for δE_{r_1}:

$$
\delta E_{r_1}=\alpha\left(\overline{\frac{\delta\gamma_1}{\gamma_0}}\right)=\frac{\alpha A_1B_1}{\Delta\omega^3}\left\{1-\cos\Delta\omega t-\frac{1}{2}\Delta\omega t\sin\Delta\omega t\right\}
\tag{30}
$$

(b) Let

$$\delta E_r=\delta E_{r_1},\ \frac{\delta\gamma}{\gamma_0}=\frac{\delta\gamma_2}{\gamma_0},\quad \text{and } \delta E_{r_1}=\alpha\left(\overline{\frac{\delta\gamma_2}{\gamma_0}}\right); \tag{31}$$

$$\frac{d}{dt}\left(\frac{\delta\gamma_2}{\gamma_0}\right)=-\frac{A_1\delta E_{r_1}}{E_{r_0}}\cos(\Delta\omega t+\phi_0)-\frac{B_1\delta E_{r_1}}{E_{r_0}}\sin(\Delta\omega t+\phi_0)\int\frac{\delta\gamma_2}{\gamma_0}dt. \tag{32}$$

Let

$$\delta\gamma_2=\delta\gamma_2'+\delta\gamma_2''+\cdots,\ \text{where } \delta\gamma_2'\propto\delta E_{r_1},\ \delta\gamma_2''\propto(\delta E_{r_1})^2\cdots. \tag{33}$$

In this way, Eq. (32) is dissociated into a series of subequations:

$$\frac{d}{dt}\left(\frac{\delta\gamma_2'}{\gamma_0}\right)=-\frac{A_1\delta E_{r_1}}{E_{r_0}}\cos(\Delta\omega t+\phi_0)=-\frac{A_2}{\Delta\omega^3}\left\{1-\cos\Delta\omega t-\frac{\Delta\omega t}{2}\sin\Delta\omega t\right\}\cos(\Delta\omega t+\phi_0). \tag{34}$$

$$\frac{d}{dt}\left(\frac{\delta\gamma_2^{(n)}}{\gamma_0}\right)=-\frac{B_1\delta E_{r_1}}{E_{r_0}}\sin(\Delta\omega t+\phi_0)\int\frac{\delta\gamma_2^{(n-1)}}{\gamma_0}\cdot dt=-\frac{B_2}{\Delta\omega^3}\left\{1-\cos\Delta\omega s-\frac{\Delta\omega t}{2}\sin\Delta\omega t\right\}\times\sin(\Delta\omega t+\phi_0)\int\frac{\delta\gamma_2^{(n-1)}}{\gamma_0}dt,\quad (n\neq1). \tag{35}$$

where

$$A_2\equiv\frac{\alpha A_1^2B_1}{E_{r_0}},\quad B_2\equiv\frac{\alpha A_1B_1^2}{E_{r_0}}. \tag{36}$$

Integrating the both sides of Eq. (34) gives:

$$\frac{\delta\gamma_2'}{\gamma_0}=-\frac{A_2}{\Delta\omega^4}\{\sin(\Delta\omega t+\phi_0)-\sin\phi_0\}+\frac{5A_2}{16\Delta\omega^4}\{\sin(2\Delta\omega t+\phi_0)-\sin\phi_0\}+\frac{A_2t}{2\Delta\omega^3}\cos\phi_0-\frac{A_2t}{8\Delta\omega^3}\cos(2\Delta\omega t+\phi_0)-\frac{A_2t^2}{8\Delta\omega^2}\sin\phi_0, \tag{37}$$

And

$$\left\langle\frac{\delta\gamma_2'}{\gamma_0}\right\rangle_{\phi_0}=\frac{1}{2\pi}\int_0^{2\pi}\frac{\delta\gamma_2'}{\gamma_0}d\phi_0=0. \tag{38}$$

Substituting Eq. (37) into Eq. (35), and meanwhile letting $(n)\to''$ we derive the expression for $\left\langle\frac{\delta\gamma_2''}{\gamma_0}\right\rangle_{\phi_0}$ as follow:

$$\left\langle\frac{\delta\gamma_2''}{\gamma_0}\right\rangle_{\phi_0}=\frac{A_2B_2}{\Delta\omega^9}\Big\{1-\cos\Delta\omega t-\frac{11}{16}\Delta\omega t\sin\Delta\omega t+\frac{3}{16}(\Delta\omega t)^2\cos\Delta\omega t+\frac{5}{128}(\Delta\omega t)^2+\frac{1}{48}(\Delta\omega t)^3\sin\Delta\omega t+\frac{1}{192}(\Delta\omega t)^4-\frac{49}{256}(1-\cos2\Delta\omega t)+\frac{29}{128}\Delta\omega t\sin2\Delta\omega t$$

$$-\frac{7}{64}(\Delta\omega t)^2\cos 2\Delta\omega t-\frac{5}{192}(\Delta\omega t)^3\sin 2\Delta\omega t$$
$$+\frac{1}{384}(\Delta\omega t)^4\cos 2\Delta\omega t\Big\}. \tag{39}$$

And

$$\delta E_{r_2}=\alpha\left\langle\frac{\delta\gamma_2}{\gamma_0}\right\rangle_{\psi_0}\approx\alpha\left\langle\frac{\delta\gamma_2''}{\gamma_0}\right\rangle_{\psi_0}. \tag{40}$$

By using the same method, we are able to keep on solving Eq. (23) to an arbitrary order when it is required. However, in this paper the solutions which have been obtained are sufficient already for the following discussions.

§3. Discussion

In this section, we'll specify how the saturation happens in FEL under condition of the strong signal and discuss how the original energy dispersion of the relativistic electron beam affects this saturation as well.

(1) For the sake of simplicity in the discussions, we expand the triangle functions, $\cos\Delta\omega t$ and $\sin\Delta\omega t$, as Taylor series and keep the terms only up to $(\Delta\omega t)^6$. This approximation is quite reasonable since $\Delta\omega t\ll 1$. Thus, according to Eqs. (28) (30), (39) and (40), we have:

$$\delta E_{r_1}\approx\frac{\alpha A_1B_1}{\Delta\omega^3}\,\frac{(\Delta\omega t)^4}{24}\left\{1-\frac{1}{15}(\Delta\omega t)^2\right\}, \tag{41}$$

$$\delta E_{r_2}\approx\frac{\alpha A_2B_2}{\Delta\omega^9}\,\frac{1}{160}(\Delta\omega t)^6. \tag{42}$$

The ratio of δE_{r_1} and δE_{r_2} is then given as follow:

$$\frac{\delta E_{r_1}}{\delta E_{r_2}}\approx\frac{\varepsilon_0^2\gamma_0^4c^4K_w^4}{W_e^2B_w^4K_r^2\left(\dfrac{|e|}{\gamma_0mc}\right)^8}(\Delta\omega t)^4\{15-(\Delta\omega t)^2\}. \tag{43}$$

Obviously, δE_{r_2} is much less than δE_{r_1} as known from Eq. (43). This result reflects the fact that the stronger the signal becomes, the less energy the radiation field will obtain from the relativistic electron beam. In other words, there exists a limit over the amplification rate of the radiation field, which is characteristic of the saturation.

From Eq. (25) and the definitions for $\delta\beta_z$ and δE_r, we know that the axial velocity of the relativistic electron will decrease with the amplification of the radiation field, when the velocity decreases to such a degree that the conditions for the energy transference are no longer satisfied, the energy exchange between the radiation field and the electron beam will stop, i.e. the saturation happens.

(2) In the above calculations, we assumed that the original energy of the various electrons is all the same. However, such an ideal condition is by no means

realizable in the practice. Although is is undesired, the energy dispersion in unavoidable. For this reason, we must revise our former computation results in order to take the effect of the energy dispersion into consideration.

As is well known, the resonant parameter $\Delta\omega$ is the main factor affecting the energy transfer rate. Therefore we'll restrict our consideration to the influence of the deviation of this resonant parameter which is caused by the original energy dispersion.

We suppose that the resonant parameter $\Delta\omega$ may be written as follow:

$$\Delta\omega=\Delta\omega_s+\delta\omega. \tag{44}$$

where $\Delta\omega_s=[K_w u-\omega_r(1-\beta_{z0})]_S$, where the subscript S denotes the values corresponding to that of the central energy distribution, and $\delta\omega$ is the deviation of $\Delta\omega$ caused by the original energy dispersion $\delta\gamma_s$ of the relativistic electron beam. According to the formula for the radiation wavelengths of an FEL, $\delta\omega$ can be expressed approximately as follow:

$$\delta\omega=d(\Delta\omega)\approx\frac{\omega_r}{\gamma_s^4}\left(\frac{\delta\gamma_s}{\gamma_0}\right). \tag{45}$$

Substituting Eq. (44) into Eq. (30) and considering $\delta\omega\ll\Delta\omega_s$, approximately we get:

$$\begin{aligned}\delta E_{r_1}(\delta\omega)\approx&\frac{\alpha A_1B_1}{\Delta\omega_s^3}\left\{1-3\left(\frac{\delta\omega}{\Delta\omega_s}\right)+6\left(\frac{\delta\omega}{\Delta\omega_s}\right)^2\cdots\right\}\\&\times\left\{\left(1-\cos\Delta\omega_st-\frac{1}{2}\Delta\omega_st\sin\Delta\omega_st\right)\right.\\&\left.+\frac{1}{2}\delta\omega t\sin\Delta\omega_st-\frac{1}{2}(\delta\omega t)^2\cos\Delta\omega_st\right\}.\end{aligned} \tag{46}$$

Supposing that the original energy distribution of the relativistic electron beam is homogeneous where the maximum deviation of the original energy, $(\delta\gamma_s)_{max}=|\gamma_0\ \ \gamma_s|_{max}$, is denoted as $\Delta\gamma_s$, we derive the revised expression for δE_{r_1}:

$$\begin{aligned}\delta E_{r_1}\approx&\frac{\alpha A_1B_1}{\Delta\omega_s^3}\left\{\frac{1}{24}(\Delta\omega_st)^4-\frac{1}{2}\left[1-\frac{1}{3}(\Delta\omega_st)^2\right]\frac{\omega_rt^2}{\gamma_s^3}\left(\frac{\Delta\gamma_s}{\gamma_s}\right)^2\right\}\\=&\delta E'_{r_1}-\delta E''_{r_1}.\end{aligned} \tag{47}$$

where $\delta E'_{r_1}$ is the first order increment of the amplitude of the radiation field when nooriginal energy dispersion exists and $\delta E''_{r_1}$ denotes the decline caused by this dispersion. Letting $\eta_1=\delta E''_{r_1}/\delta E'_{r_1}$, we have:

$$\eta_1\approx\frac{12\omega_rt^2}{\gamma_s^8(\Delta\omega_st)^4}(\Delta\gamma_s)^2. \tag{48}$$

Similarly we get:

$$\begin{aligned}\delta E_{r_2}\approx&\frac{\alpha A_2B_2}{(\Delta\omega_s)^9}\left\{\frac{1}{160}(\Delta\omega_st)^6-\frac{5}{128}\left[1+\frac{2}{3}(\Delta\omega_st)^2\right]\frac{\omega_rt^2}{\gamma_s^3}\left(\frac{\Delta\gamma_s}{\gamma_s}\right)^2\right\}\\=&\delta E''_{r_2}-\delta E''_{r_2}.\end{aligned} \tag{49}$$

And

$$\eta_2=\frac{\delta E''_{r_2}}{\delta E'_{r_2}}\approx\frac{25\omega_rt^2}{4\gamma_s^5(\Delta\omega_st)^6}(\Delta\gamma_s)^2. \tag{50}$$

What we can conclude from the above equations is listed as follows:

(a) The existence of the original energy dispersion of the relativistic electron beam has a deleterious effect upon the energy transfer rate of free electron laser under conditions of strong signal. Since the decline in caused by the original energy dispersion is directly proportional to $t=\frac{L}{u}$, we believe that the longer the interaction length is, the more serious this effect will be.

(b) Since $\eta_1 \ll \eta_2$, we think that this effect is more pronounced upon δE_{r_2} than upon δE_{r_1}. This means that the energy dispersion will induce saturation.

(c) There exists a limit over the original energy dispersion above, which the radiation field will no longer be amplified.

Conclusions

In this paper, we deduce a simple method for dealing with the strong signal in FEL with helical magnetic field. By means of this we discuss how the energy dispersion affects the saturation in FEL as well as how this saturation happens. The advantage of adopting the energy model of FEL to deal with the strong signal lies that it not only makes the calculation quite simple but also makes the physical meaning quite clear. It also can be used to deal with the harmonic wave radiation in FEL when the non-resonant terms in Eq. (21) are not neglected.

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处理自由电子激光器强信号的一种简单方法

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提 要

基于自由电子激光器的能量模型，在具有螺旋磁场自由电子激光器中，本文采用一种简单的方法处理了强信号条件下能量转换过程。讨论了饱和产生的过程和初始能量色散对饱和影响的机制。这种处理方法也适用于自由电子激光器的谐波辐射。

关键词：自由电子激光器能量模型；强信号处理。