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OPTIMIZATION OF GRATING MULTI-BEAMSPLITTERS

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Abstract

We derive a numerical method of calculating binary grating structures, which have predetermined spatial spectra with a typical accuracy of less than one percent. As an experimental verification, we present test results of a grating that converts a plane wave into a central block of 11×11 uniformly diffracted waves.

Imroduction

Modulation of the periodic structure of a binary diffraction grating to shape the distribution of intensities of diffracted orders has acquired considerable interest in recent years. This kind of generalized (holographic) gratings can be conveniently used as image multipliers and multi-beam beamsplitters in optical information processing and fiber-optic communication systems. Several authors have considered the problem of calculating structures, which give a central block of equally intense diffracted beams/1, 2, 3, 4, 5/. Perhaps the most successfull of the various approaches is the one developed by Killat et. al. in Ref. /5/, in which several one-dimensional structures are presented, with diffraction efficiencies of 70~85% and nonuniformities of 0.3~1.7%.

In this investigation, we derive a considerably simple gradient-type optimization algorithm, which gives results comparable to those of Ref. /5/. Our method can, in addition, be used to design grating structures with nonuniform, predetermined spectra. We also analyze the accuracy required in the fabrication process of the grating structure, and fabricate a 11×11-beam twodimensional uniform beamsplitter.

Spectra of binary diffraction gratings

A general binary amplitude grating is a periodic structure, which has only two

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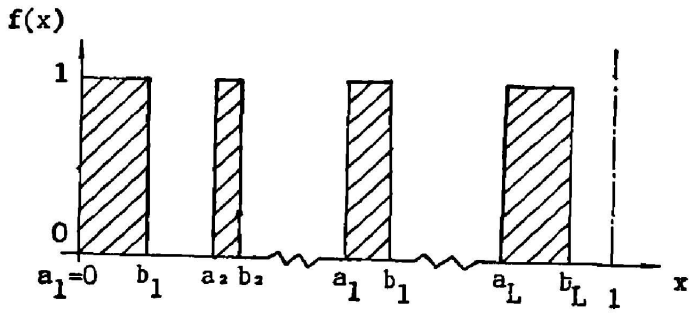


Fig. 1 Amplitude transmittance $f(x)$ of a general binary amplitude grating over one period. Length of the period is normalized to unity

different values of amplitude transmission $f(x)$, as illustrated in Fig. (1). Clearly, the trating and its spectrum are completely determined by the set $\{a_l, b_l\}$, $l=1, \cdots, L$, of ransition point positions of $f(x)$. Since $f(x)$ is periodic, it can be represented in the form of Fourier series,

$$f(x)=\sum_{m=-\infty}^{\infty} F(m)\exp\{2\pi imx\}. \tag{1}$$

By straighforward calculation, the Fourier coefficients of the grating structure are found in terms of the transition point positions:

$$F(0)=\sum_{l=1}^{L}(b_l-a_l), \tag{2}$$

for order $m=0$ and

$$F(m)=\frac{1}{2\pi m}[F_R(m)+iF_I(m)], \tag{3}$$

for higher orders. In Eqn. (3),

$$\begin{cases} F_R(m)=\sum_{l=1}^{L}[\sin(2\pi mb_l)-\sin(2\pi ma_l)], \\ F_I(m)=\sum_{l=1}^{L}[\cos(2\pi mb_l)-\cos(2\pi ma_l)]. \end{cases} \tag{4}$$

The power spectrum $\{P(m)\}$, $m=-\infty, \cdots, \infty$, of the structure, defined as

$$P(m)=|F(m)|^2, \tag{5}$$

gives the intensities to various diffraction orders. It can now be calculated from the known transition point positions with the aid of Eqs. (2)～(4). Note that $P(-m)=P(m)$, which indicates that the spectrum of a binary diffraction grating in symmetrical with respect to the undiffracted beam.

A binary phase grating is a periodic structure, which has a uniform amplitude transmission and only two different values, say $\pi/2+\theta$ and $\pi/2-\theta$, of phase retardation. The complex amplitude transmittance function $g(x)=f(x)\exp\{-i\theta(x)\}$ is then of the form

$$g(x)=\pm\sin\theta+i\cos\theta, \tag{6}$$

for all $x \in [0, 1]$. The power spectrum of this phase grating is simply related to the power spectrum of the corresponding amplitude grating (which has the same transition point positions). Straightforward calculation, where use is made of the identity

$$\int_0^1 \exp\{-2\pi i m x\}\, dx = 0,$$

and linearity of sum and integral operators, gives the Fourier coefficients $\{G(m)\}$ of a general binary phase grating. We get

$$G(0) = [2F(0) - 1]\sin\theta + i\cos\theta, \tag{7}$$

for order $m = 0$ and

$$G(m) = 2\sin\theta F(m), \tag{8}$$

for higher orders. Now the power spectrum $\{P(m)\}$ of the binary phase grating, defined as $P(m) = |G(m)|^2$, can be calculated from the transition point positions with the aid of Eqs. (2)$\sim$(4), (7) and (8).

Optimization of the grating structure

Our aim is to find a set $\{a_l, b_l\}$, $l = 1, \cdots, L$, of transition point positions (and a phase angle θ if it is a free variable) in such a way that the power spectrum $\{P(m)\}$, $|m| = 0, \cdots, M$, closely approximates some desired power spectrum $\{\hat{P}(m)\}$, with good diffraction efficiency

$$P_E = P(0) + 2\sum_{m=1}^{M} P(m). \tag{9}$$

This optimization problem can be analytically solved only in the case of a three-beam ($m = 0, \pm 1$) phase grating. For example, a uniform set of three diffracted beams is given by a structure $a_1 = 0$, $b_1 = 1/2$ and $\theta = \arctan(\pi/2) \approx 57.52°$. Diffraction efficiency into three central orders is $P_E = 12/(\pi^2 + 4) \approx 86.5\%$.

In more complicated cases, numerical optimization techniques have to be used. As a measure of the progress of optimization, we define a merit or error function by the expression

$$E^2 = \alpha\left\{[P(0) - P_E\hat{P}(0)]^2 + 2\sum_{m=1}^{M}[P(m) - P_E\hat{P}(m)]^2\right\} + (1-\alpha)(1-P_E)^2, \tag{10}$$

where α is a parameter within the range [0, 1]. The fist term in the merit function is a measure of the similarity of distributions $\{P(m)\}$ and $\{P_E\hat{P}(m)\}$, and the second term is proportional to the diffraction efficiency of the structure.

Due to the considerable complexity of the dependence of the merit function on the transition point positions $\{a_l, b_l\}$ and on phase angle θ, it generally has a large number local minima. Starting from arbitrary initial transition point positions, one of these minima can be found by the following gradient-type iteration technique.

Let the transition points in the beginning of n: th iteration step be $\{a_{l,n}, b_{l,n}\}$ and the value of the error function be E_n. A small trial change is made in each $a_{l,n}$ and $b_{l,n}$, and corresponding changes of the error function, $\Delta E_{a_{l,n}}$ and $\Delta E_{b_{l,n}}$, are calculated by using the formalism of the previous section. A new gradient in the $2L$-dimensional space of transition point positions is determined by equations

$$\begin{cases} \Delta a_{l,n} = -\gamma \Delta E_{a_{l,n}} / |\Delta E_{\max,n}| \\ \Delta b_{l,n} = -\gamma \Delta E_{b_{l,n}} / |\Delta E_{\max,n}|, \end{cases} \tag{11}$$

where γ is a free parameter and

$$\Delta E_{\max,n} = \max\{\Delta E_{a_{l,n}}, \Delta E_{b_{l,n}}\}.$$

With the aid of these equations, the error E_n can be interpreted as a function of γ. As illustrated in Fig. 2, the function $E_n(\gamma)$ is well behaved around the minimum, and only a few simple evaluations of the merit function are required to find $\gamma_{\min,n}$, witha sufficient degree of accuracy. Once $\gamma_{\min,n}$ is found, new values for the transition point positions are found by equations

$$\begin{cases} a_{l,n+1} = a_{l,n} - \gamma_{\min,n} \Delta E_{a_{l,n}} / |\Delta E_{\max,n}| \\ b_{l,n+1} = b_{l,n} - \gamma_{\min,n} \Delta E_{b_{l,n}} / |\Delta E_{\max,n}|. \end{cases} \tag{12}$$

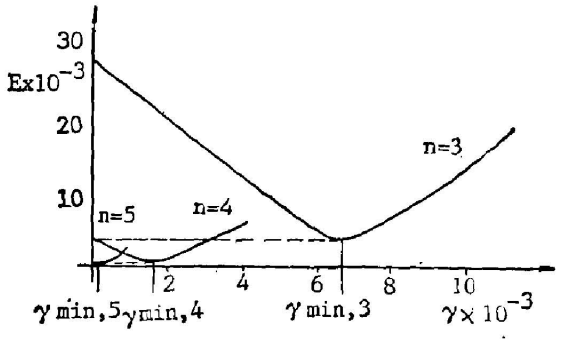


Fig. 2 Illustration of the progress of iteration in a realistic case of a uniform five-beam grating. Starting from randomly chosen transition point positions, five iteration steps were needed to obtain an acceptable optimum; the calculated dependence $E_n(\gamma)$ is shown for the last three steps

If the set $\{a_{l,n+1}, b_{l,n+1}\}$ is not an acceptable solution, a new iteration step is performed.

If the phase angle is a free variable, it can be treated in the iteration process just as any ond of the transition point positions. However, change of phase angle does not affect the rela-tive intensities of the diffraction orders $|m|>0$, as can be deduced fron Eqn. (8). Hence optimization of phase angle can be done algebraically. For example, if the aim distribution is a central block of uniformly diffracted beams, the intensity of the zero-order beam can be equated to the average intensity of the higher-order beams by the formula

$$\sin\theta_{n+1} = \sin\theta_n [1 + P_{\text{exc},n} - P_n(0)]^{1/2}, \tag{13}$$

in the end of each iteration step.

Free phase angle adds an additional degree of freedom in optimization. This is valuable especially when the number of diffracted orders in the aim distribution is low. For example, if the phase angle of a uniform three-beam grating is fixed to $\pi/2$, the maximum efficiency that can be obtained is $P_E = 66\%$, compared with 87% if the phase angle is a free variable. For a large number of diffracted orders, the optimal

phase angle is often close to $\pi/2$.

The iteration procedure outlined in this section gives a local minimum of error function from almost any starting distribution in less than ten iteration steps. If the number of free parameters in the grating structure is adequately large ($L \geqslant M$, see Ref. /2/) and $\alpha=1$ is chosen in merit function, the maximum relative deviation from the normalized aim distribution

$$\eta=\max_m\left\{\left|1-\frac{P(m)}{P_E\hat{P}(m)}\right|\right\} \tag{14}$$

is often less than 1%. However, when the desired number of diffraction orders becomes large, many randomly chosen starting distributions may be required to find a structure with good effi-ciency, and there seems to be no simple method to ascertain that the minimum obtained is the global minimum. If the local minimum closest to the initial transition point positions yields low efficiency, it is often possible to force the iteration toward another, better local minimum by choosing $\alpha<1$ for a while. The phase-retrieval algorithm of Ref. /4/, which as such does not generally give solutions with $\eta<10\sim20\%$, may also be employed to find suitable starting systems.

Spectrum shaping examples

In this section, we give some examples of the use of the optimization algorithm to design grating structures with specific spectra. We begin by defining some practically important aim distributions. An ideal uniform distribution is defined as

$$\hat{P}_U(m)=\frac{1}{2M+1}, \quad |m|=0, \cdots, M \tag{15}$$

and $\hat{P}_U(m)=0$ if $|m|>M$. Triangle distribution is defined by the requirement that the difference between intensities to two adjacent diffraction orders remains constant up to order M, after which intensities are equal to zero. This condition is equivalent with $\hat{P}_T(m)-\hat{P}_T(m+1)=\hat{P}_T(0)/(M+1)$, or $\hat{P}_T(m)=\hat{P}_T(0)[1-m/(M+1)]$. By requiring that the efficiency to M central orders is unity, and using the properties of arithmetic series, we obtain for the triangle distribution an expression

$$\hat{P}_T(m)=\frac{1}{M+1}\left(1-\frac{m}{M+1}\right), \quad |m|=0, \cdots, M \tag{16}$$

and $\hat{P}_T(m)=0$ if $|m|>M$. Another interesting nonuniform distribution, which we call power distribution, is characterized by the requirement that the ratio of two adjacent orders is a constant ϵ. Algebraically, $\hat{P}_P(m)=\hat{P}_P(0)\epsilon^m$. Taking the efficiency to $2M+1$ central orders to be unity, and using properties of geometric series, we get

$$\hat{P}_P(m)=\frac{1-\epsilon}{1+\epsilon-2\epsilon^{M+1}}\epsilon^m, \quad |m|=0, \cdots, M \tag{17}$$

and $\hat{P}_P(m)=0$ if $|m|>M$.

A five-beam power distribution with $\epsilon=1/2$, $P_E\approx75.9\%$ and $\eta\approx0.18\%$ is given by the structure

$$a_1=0 \qquad b_1=0.3540$$
$$a_2=0.73900 \qquad b_2=0.81125$$

and phase angle $\theta\approx57.55°$. A five-beam triangle distribution with $P_E\approx72.1\%$ and $\eta\approx0.08\%$ is given by the structure

$$a_1=0 \qquad b_1=0.05794$$
$$a_2=0.52423 \qquad b_2=0.81299$$

and phase angle $\theta\approx66.30°$.

As an example of uniform distribution, we design a two-dimensional 11-beam beamsplitter. Fabrication problems strongly suggest that a two-dimensional phase grating should have only two levels of phase modulation, which sets the restriction that the phase angle has to equal $\pi/2$. A solution

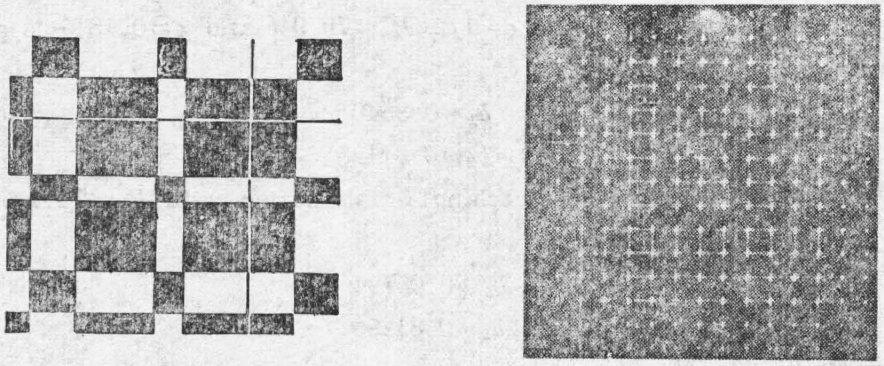
$$a_1=0 \qquad b_1=0.06857$$
$$a_2=0.20885 \qquad b_2=0.44467$$
$$a_3=0.52930 \qquad b_3=0.72101$$
$$a_4=0.7\square854 \qquad b_4=0.86437$$

was found. The maximum deviation from uniformity was $\eta\approx0.47\%$. Diffraction efficiency to 11 central orders was $P_E\approx76.4\%$ for one-dimensional structure and $P_E^2\approx58.4\%$ for twodimensional structure. We also analyzed the accuracy required in the fabrication process of the grating. If the transition points were rounded to four decimals, uniformity was $\eta\approx0.54\%$. Rounding to three decimals gave $\eta\approx2.69\%$, and two decimals gave $\eta\approx16.8\%$.

Experiment

As an experimental verification, we fabricated the uniform 11×11 beamsplitter designed in previous section. The structure shown in Fig. 3*a* was written, photoreduced and multiplied; the grating consisted of 10×10 periods, length of one period was $633\,\mu$m. A phase grating corresponding to the structure shown in Fig. 3*a* was prepared by ion beam etching. The farfield diffraction pattern of the grating, photographed in the focal plane of a Fourier-transform lens, is shown in Fig. 3*b*.

Because of an error in phase angle, the zero-order spot is more intense than the others, which show reasonable uniformity. This error is partly due to the finite number of grating periods, which causes noticeable structure (see Fig. 3*b*) especially around the zero-order spot. Maximum deviation from the uniform aim distribution in the upper row of uniformly diffracted beams in Fig. 3*b* was measured to be 9.5%. The difference between measured and theoretical uniformities is due to manufacturing



(a) (b)

Fig. 3

(a) A grating structure yielding a uniform central block of 11×11 diffraction orders
(b) The far-field diffraction pattern of the grating, photographed in the focal plane of a Fourier-transforming lens

inaccuracies in locations of the phase transition points. These inaccuracies were, according to the discussion in the end of the previous section, estimated to be a little less than 1% of the grating period, or of the order of 5 μm. Better uniformity could naturally be obtained by more sophisticated fabrication techniques, like electron beam lithography.

Conclusions

The optimization method presented in this paper was found to be successfull in design of holographic beamsplitter structures, which give arbitrary predetermined spectra. The optimization program we prepared is efficient enough to be run in a personal computer, and it gives the transition point positions with an accuracy that is beyond most practical fabrication techniques, which therefore set the limit to the accuracy of the spatial spectra.

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References

[1] H. Damman and K. Görtler, "High-efficiency in-line multiple imaging by means of multiple phase holograms", *Optics Communications*, Vol. 3, p. 312, 1971.
[2] H. Dammann and E. Klotz, "Coherent optical generation and inspection of two-dimensional periodic structures", *Optica Acta*, Vol. 24, p. 505, 1977.
[3] Wai-Hon Lee, "High-efficiency multiple beam gratings", *Applied Optics*, Vol. 18, p. 2153, 1979.
[4] Wu Zhouling, Lu Da, and Jin Guofan, "Application of an iterative method in the design of high-efficiency binary diffraction gratings", *Acta Optica Sinica*, Vol. 6, p. 567, 1986.
[5] U. Killat, G. Rabe, and W. Rave, "Binary phase gratings for star couplers with high splitting ratio", *Fiber and Integrated Optics*, Vol. 4, p. 159, 1982.

多光束分光光栅的优化设计

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提　　要

本文描述了一种设计计算二元光栅结构参数的数值方法，这种方法可以把光栅衍射谱的分布误差控制在 1% 以内。作为实验验证，本文给出了一个二元光栅谱分布的测试结果，该光栅将入射平面波转换成在中央 11×11 级次内均匀分布的衍射波。

国际光电子科学及工程学术会议（'89 ICOESE）将在北京召开

由中国光学学会主办，中国科学技术协会、中国测量及控制联合会和国际光学委员会支持的“国际光电子科学及工程学术会议”，将于 1989 年 7 月 5～8 日在我国北京市召开。

会议主要议题为：新型光电、电光、声光、磁光元器件及其集成化，光电检测、传感、测量及控制，红外光学，微光技术，光纤通讯，光盘，光信息显示、存贮及处理，激光技术，光学双稳态和光学计算等。在我国召开这样一次综合性的国际光电会议旨在吸取国际上先进的光电技术，促进光电子科学领域中的国际间学术交流。

会议期间拟组织一个小型展览会。

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