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Holographic tomography in determining refractive index of three-dimensional transparent object

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I. Introduction

Holographic tomography is a powerful tool to study refractive index of a phase object in plasma diagnostics, aerodynamics, and measurement of density or deformation of a transparent object. Several authors [1~4] have studied various types of 3-D fields employing different methods. The major advantage of utilizing holographic technique is the ability of capturing the complex wave field of an object. Incorporation of holographic and tomographic concepts may offer some advantages in studying the complex transmitted field of a 3-D object. By means of this holographic tomography concept, the conventional holographic multiplexing technique could be used to record various projected or transmitted wave fields from a 3-D object. Multidirectional interferograms provide a set of measured values of line intergrals through the transparent medium. The interferograms can be reconstructed with appropriate coherent illuminations, and can then be sequentially detected by 2-D or 1-D CCD array detector for computer post-processing. Therefore, various cross sections of 3-D object fields may be reconstructed.

In our study, a double exposure holographic technique is used to record the path length difference of probing rays in a phase field. The refraction effect in measured phase object may be negligible in the reconstructed interferograms [5]. In the case, rays remain straight lines and the path intergral becomes a line intergral. An image sensing device is used to detect reconstructed interferograms. The detected data are inputted to a computer for post-processing.

In the following sections, we shall explain the analog recording and reconstruction systems as well as the principle of the digital reconstruction of the phase field. Some preliminary experimental results are presented. At the last section, we shall discuss the advantages and disadvantages of the present method.

II. Principle

A double exposure holographic technique is employed to record the distribution of refractive index of a transparent object. The schematic diagram of the holographic interferometer is depicted in Fig. 1.

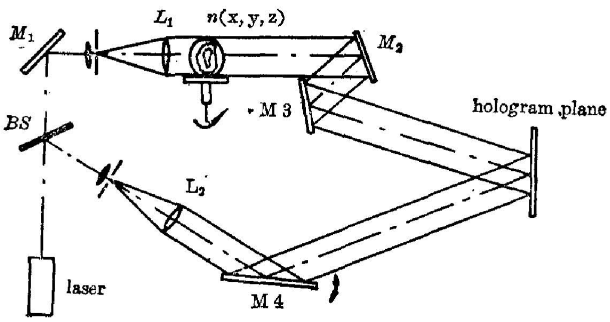


Fig. 1 Schematic diagram of the holographic interferometer

Object platform and mirror M4 in the interferometer are rotated synchronously to perform multi-directional holograms. A double exposure hologram is taken for each direction.

During the first exposure, a plane wave probs the phase object. Then the phase object is taken away before second exposure. Within the phase object the distribution refractive index is $n(x, y, z)$ during the initial holographic exposure and n_0 during the second. Ofter n_0 is a constant. For a new direction, the phase object is rotated to a certain orientation while the mirror M4 is rotated to a relevant orientation to ensure the separation of reconstructed multidirectional interferograms. When the developed hologram is illuminated by reference beam, two object waves will be reconstructed simultaneously in the same direction [5]:

$$\left.\begin{aligned} U_f &= a_f(x, y)\exp\left[i\frac{2\pi}{\lambda}\Phi_f(x, y)\right], \\ U_0 &= a_0(x, y)\exp\left[i\frac{2\pi}{\lambda}\Phi_0(x, y)\right], \end{aligned}\right\} \tag{1}$$

where

$$\Phi_f(x, y) = \int n(x, y, z)dz,$$

$$\Phi_0(x, y) = \int n_0\, dz.$$

U_f and U_0 represent the complex amplitudes of the phase object and the plane wave respectively.

The fringe pattern is the irradiance of the sum of U_f and U_0 in image plane. Assuming for simplicity that a_f and a_0 are uniform unit amplitudes, the irradiance can be written as

$$I(x, y) = 2\left\{1+\cos\left[\frac{2\pi}{\lambda}\Delta\Phi(x, y)\right]\right\}, \tag{2}$$

where

$$\Delta\Phi(x, y) = \int [n(x, y, z) - n_0]dz,$$

and the equation of a bright fringe is

$$\Delta\Phi(x, y) = N\lambda, \quad N = 0, 1, 2, \text{etc.} \tag{3}$$

N is called fringe order.

We are now in a position to study the relationship between the double exposure holographic interferometry and the distribution of refractive index of phase object. We can determine $\Delta\Phi$ directly in units of wavelength λ by assigning order numbers to the fringes and then inverting equation (3) to calculate $n(x, y, z)-n_0$.

Since a computer is utilized for post-processing, it is convenient to analyze a slice of a phase object in 2-D sampled form of $(M \times N)$ matrix.

As shown in Fig. 2, the phase field can be divided into discrete rectangular elements of dimensions Δx by Δy, where

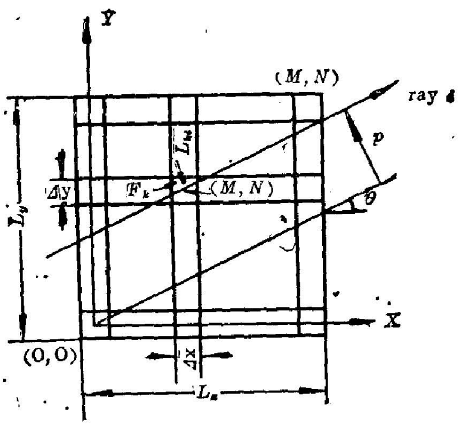


Fig. 2 Schematic diagram of coordinates of measured phase object field

$$\Delta x=\frac{L_x}{M+1}, \quad \Delta y=\frac{L_y}{N+1}.$$

Each element is considered to have a uniform refractive index. Let F_k denote the value of refractive index change of the element centered on the point $x=m\cdot\Delta x$, $y=n\cdot\Delta y$, where m and n are integers. The subscript k is related to m, n by

$$k=n(M+1)+m+1.$$

Let the fringe order number N_i be associated with the ith ray traversing the phase object. This ray is specified by the coordinates p, θ. If L_{ki} denotes the length of the segment of the ith ray which lies in the k-th element, the to tal optical pathlength for the i-th ray is given by

$$\Phi(x, y)=\sum_{k=1}^{K} L_{ki}F_k, \tag{4}$$

where $K=(M+1)(N+1)$ is the total number of elements.

Because during the double exposure the second medium is air, the optical pathlength difference for i-th ray can be written as

$$\Delta\Phi(x, y)=\sum_{k=1}^{K} L_{ki}F_k=\lambda N_i. \tag{5}$$

The coeffiecient L_{ki} can be determined geometrically as follows:

$$\begin{cases} 0 \quad \text{for} \quad |b| \geqslant (A\Delta x+\Delta y)/2, \\ 0 \quad \text{for} \quad |c| > \Delta x/2 \text{ and } |\theta|=\dfrac{\pi}{2}, \\ \Delta y \text{ for} \quad |c| < \Delta x/2 \text{ and } |\theta|=\dfrac{\pi}{2}, \\ \Delta x \sec\theta \quad \text{for} \quad |b| \leqslant (\Delta y-A\Delta x)/2 \text{ and } A \leqslant \Delta y/\Delta x, \\ \Delta y \sec\theta/A \quad \text{for} \quad |b| \leqslant (A\Delta x-\Delta y)/2 \text{ and } A > \Delta y/\Delta x, \\ \dfrac{\sec\theta}{A}\left(\dfrac{A\Delta x+\Delta y}{2}-|b|\right) \quad \text{for} \quad (|\Delta y-A\Delta x|)/2 < |b| \leqslant (|\Delta y+A\Delta x|)/2. \end{cases} \tag{6}$$

Where $A=|\tan\theta|$, $b=p\sec\theta+m\cdot\Delta x\tan\theta-n\Delta y$, $c=p+m\cdot\Delta x$.

In the experiment, the measured phase field is square and its center coincides with the center of rotation of object platform, as shown in Fih. 3.

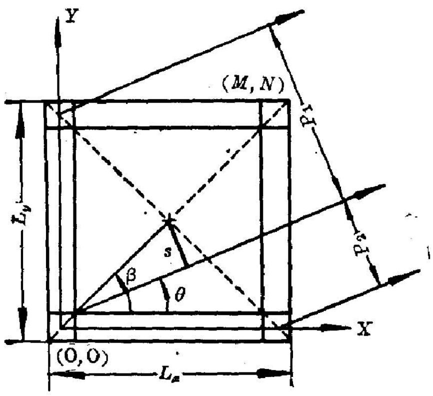


Fig. 3 Schematic diagram of geometrical relation between values p and θ

We call ray r reference ray and e and f edge rays. The ray r passes through the origin of coordinate. The position of reference ray can be defined by

$$s=\frac{1}{2}(L_y-\Delta y)\sin(\beta-\theta).$$

Here

$$\beta=45^\circ.$$

The positions of the two edge rays are

$$p_1=(L_y-\Delta y)\cos\theta,$$

$$p_2=(L_x-\Delta x)\sin\theta,$$

For square,

$$L_x=L_y,\quad \Delta x=\Delta y.$$

The value of p for each ray in the experiment can be assigned by the size of measured field with the values of p_1 and p_2.

It is apparent from equation (5), if the number of sampled rays is equal to the number of total elements K, a linear algebraic equations can be generated. The refractive index F thus can be obtained by solving these linear equations.

III. Experimentalprocedure

The experimental setup is shown schematically in Fig. 1. The object and reference beams were expanded and filtered using microscope objectves and pineholes. In order to make a high visibility hologram, it is important to adjust object and reference pathlength nearly equal and the ratio of irradiance of reference beam to object beam 1:1 to 1:5. Holograms were recorded on Kodak 649F films. A 50 mW He-Ne laser is used as light source. Two achromatic lenses L_1 and L_2 are used for collimation. To obtain multidirectional holograms, the object platform was rotated with the center of the measured phase field and the corresponding reference beam synchroneously changed orientation due to a rotatable mirror M4.

In the experiment, the size of measured phase field in the sample is 2×2 mm^2. The phase field was divided into 40×40 elements. The times of rotation of sample is 40 and a double exposure hologram is taken for each direction of ray. From each reconstructed interferogram, 40 sampling data were taken by the image sensing device. The accuracy of sample reture and relative position between the object and film in camera are less than 0.005 mm.

The reconstruction setup of the double exposure holograms is schematically shown in Fig. 4. In this system, the developed hologram can be illuminated from rear side by a conjugate reference beam at a certain angle, θ. The two reconstructed beams, i. e., U_f and U_0 in equation (1), would produce an interferogram. In order to perform a sequential reconstruction for each directional angle, a rotatable mirror was employed.

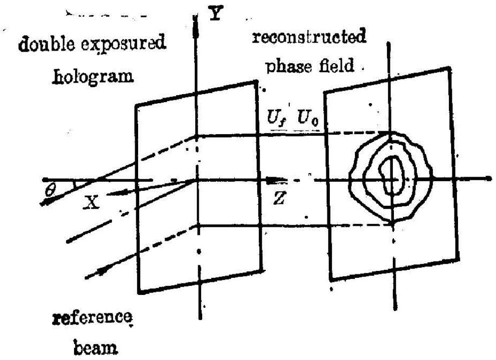


Fig. 4　Schematic diagram of the reconstruction setup of the double exposure hologram

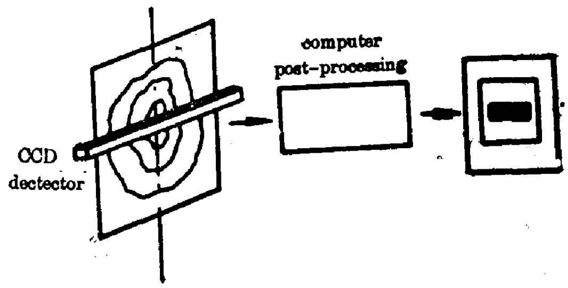


Fig. 5　Schematic diagram of the procedure of detecting and post-processing

In order to determine the fringe order of each ray, N_i, it is necessary to measure the position of each reconstructed fringe as indicated in equation (4). An image sensing device such as 1-D CCD array is used to detect reconstructed fringe as shown in Fig. 5. It is noted that any slice of an object field can be reconstructed by computer post-processing. Therefore, a 3-D reconstructed image of phase object field can be realized by applying a 1-D or 2-D image sensing device across the measured plane.

It is possible to measure $\Delta\phi$ directly in units of wavelength λ by assigning order numbers to the fringes. Various approaches can be used to define the fringe order and often the experimenter has sufficient knowledge of the field being examined. In the experiment, the number $N=0$ was assigned by reference fringe before the measurement stated. The subsequent bright fringes are assigned the order number $N=1, 2, 3, \cdots$ consecutively. The centers of all dark fringes are assigned numbers $N=0.5, 1.5, 2.5$, etc. A linear CCD array with 1024 pixels was employed. The size of each pixel is about 10 μ. The signals from CCD device were amplified by amplifying circuit and sent to a microcomputer by an interface system which acts as signal numeric conversion and keeping step with the microcomputer. The block diagram of computer post-processing is shown schematically in

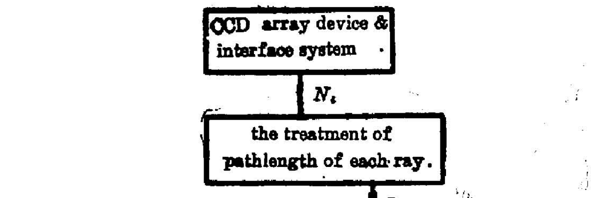
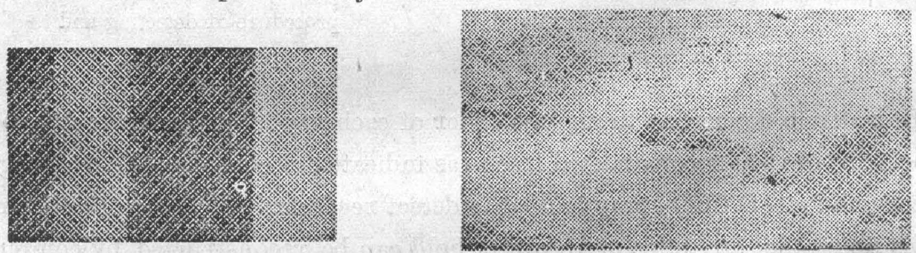


Fig. 6　Block diagram of computer postprocessing

Fig. 6. We only processed optical pathlength difference of each ray with the data from CCD device in the microcomputer, because its storage was usually not large enough to solve a $M\times N$ linear algebraic equations in the experiment and it could not output graph with greylevel. The last two steps of the procedure in Fig. 6 were done in a large IBM computer system.

As a final result of the post-processing, the matrix $\{F\}$ represents the refractive index of the object field. The refractive index of each element in the object field is marked by a proper grey level to display the value of refractive index. The matrix $\{F\}$ for ploting is 256×256. However, due to the limitation of the computer storage, the matrix dimension is reduced to be 40×40 for processing. The computer result is transfered back as a 256×256 matrix in output.

Two examples of result computed from reduced matrixes are presented in Fig. 7 which is the reconstruction of the in terferograms of two transparent samples. The greylevel shows the distribution of refractive index. The accuracy of index reconstruction is about 0.001 at the preliminary results.



(a) (b)

Fig. 7 Results of computer reconstruction

(a) A phase sample with three different uniform indexes: 1.103, 1.337, and 1.515;
(b) A phase sample with refractive indexes: 1.413, 1.586

IV. Discussion and conclusion

In this paper we have presented an al ternate method to determine refractive index distribution of 3-D or 2-D transparent phase objects. Double exposure multidirectional holographic technique is used to record the information of a phase object field. The multiplexed holograms are used to reconstruct different directional interferograms sequentially in an optical system. A series of path length differences of rays are detected by applying an image sensing device in the optical system. These data were then stored in a digital computer for post-processing. With both analog and digital reconstruction, the distribution of the refractive index of the phase field can be displayed with different grey levels.

As mentioned before, the measured phase field is square. It should be considered that when the object platform rotates an angle of α, i.e. the incident ray is at an angle of α to the surface of phase object, the ray will have a displacement $L_\alpha(\tan\alpha-\tan\theta)$ and the orientation of ray in measured phase object θ, equals arc $\sin(\sin\alpha/n)$. Here n is average index in the phase object.

It is no ted that the refraction errors can be neglected if the thickness of the measured object satisfies the inequality [5]:

$$\frac{\lambda L}{n' d_f^2} < 0.3, \tag{7}$$

where L——the length of measured object,

n'——the refractive index of a chosen point in the object,

d_f——the fringe spacing.

In the experiment, wavelength λ is 6328 Å and n' is often in arange of 1 to 1.7. However, the fringe spacing depends on the distribution of refractive index and the size of measured area in the phase object. It can satisfy the inequality (7) that the length of measured object is taken 1 mm to 7 mm, if the fringe spacing is 0.04 mm to 0.1 mm.

In order to reconstruct a measured phase field in detail a better resolution, a larger computer storage is needed. To determine the refractive indexes of a $M \times N$ matrix, a computer storage larger than $M^2 \times N^2$ is required.

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全息层析术在测定三维透明物体折射系数中的应用

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提 要

本文试就相干光的层面分析进行初步探索。介绍一种以全息术记录三维物体的位相信息，而以计算机作后处理输出，再现物体各层面位相的分布的方法。

关键词：层析术；全息术。

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