

# Nondestructive index profiling of a GRIN sphere: large aperture shearing harmonic interferometer

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## Abstract

The refractive index profile of a gradient index sphere can be measured rapidly, automatically and nondestructively by a large aperture shearing harmonic interferometer.

The apparatus, principle and technique of measurement are described and the data treatment and results are discussed. Computer simulation and experiments show that the curved ray analysis gives better accuracy than the straight ray analysis for large  $\Delta n$  in transverse interferometry.

## Introduction

It is no doubt that the refractive index profile of GRIN materials is a most important parameter. Profiling the refractive index of GRIN rod rapidly, nondestructively and accurately is still attracting the people in this field. Many nondestructive methods have been developed to measure the index profile of GRIN rods or fibers<sup>[1-6]</sup>.

There are two kinds of analytical methods in transverse or differential interference techniques, one is straight ray method (ST method<sup>[6]</sup>) and the other is real curved ray method (CT method)<sup>[5]</sup>.

The sample to be measured in the paper is a gradient index sphere, so the nondestructive technique must be chosen, because of its larger diameter and greater index difference between the center and the periphery, the differential interference system is preferable.

This paper describes a large aperture A. C. harmonic interferometer which was used to make the sphere index profile measurement rapidly, nondestructively and accurately. The further computer simulation was taken for large index difference  $\Delta n$

in order to compare the ST method and CT method, the simulation results show that the CT method gives better accuracy for large  $\Delta n$ .

### Set up (Harmonic interferometer)

In order to make the measurement of the index profile of GRIN materials rapidly, automatically and accurately, many techniques have been developed. After constant improvement, A. C. interferometry has become a popular interference technique meeting the above requirements<sup>[7~9]</sup>. Having high phase resolution, A. C. interferometer gets phase data easily and treats the data rapidly. Triangular and sinusoidal modulation were used in A. C. interferometry. Triangular modulation makes large motion of vibrated mirror and the system is difficult to get good linear response, the vibration modulation gets much better linear response, but the vibration is still a little bigger. The A. C. interference system described here, is called harmonic differential interferometer since the mirror  $M_2$  in Fig. 1 is vibrated harmonically (Equ. 2). This harmonic system just needs a  $\lambda/2$  peak-to-peak modulation and the modulation is nearly sinusoidal. The shearing plates  $SP_1$  and  $SP_2$  of two parallel plates are tilted to make the beams shearing a differential distance  $d_s$ . The  $B_1$ ,  $B_2$ ,  $M_1$  and  $M_2$  are large aperture beam splitters and mirrors, respectively,  $L_1$  and  $L_2$  are lens with large aperture and big  $f$ /number to meet the bigger bending of the light exited from the sample with large  $\Delta n$ . The gradient index sphere to be measured,  $S$  was immersed in index matching oil. The imaging system consisted in lens  $L_1$ ,  $L_2$  and  $L_3$ , images the observation plane of the sample on detector  $D_2$  (photodiode).

The mirror  $M_2$  scans such that each point passes the detector  $D_2$  and completes the data collecting automatically. The phase of signals from  $D_1$  (reference detector

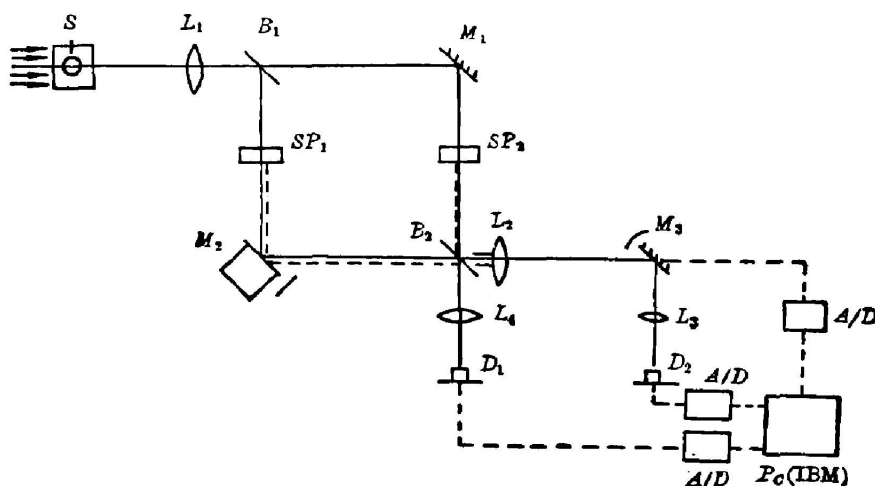


Fig. 1 Large aperture shearing harmonic interferometer

photodiode) is subtracted from the phase of signal from  $D_2$  to eliminate the phase noise produced at random from outside of the system. After amplifying and converting to a digital signal, the signal from  $D_1$  and  $D_2$  are sent to a microcomputer  $PC$ . The scan mirror  $M_3$  through  $A/D$  converter is also connected to the  $PC$ , so that the sample scanning is controlled by the  $PC$  automatically, and the phase map and refractive index profile can be obtained by the  $PC$  immediately.

The light intensity distribution of the field of the harmonic interferometer is

$$I(x, y, t) = I_1 + I_2 + 2\sqrt{I_1 I_2} \cos\{\theta(x, y) - \phi(t)\}, \quad (1)$$

where  $I_1, I_2$  is the intensity produced by the two branch beams independently,  $\theta(x, y)$  is the phase difference between the two beams which is dependent on the spatial coordinator,  $\phi(t)$  is the modulated phase varying with the time harmonically:

$$\phi(t) = A \sin \omega t + B \sin(2\omega t + \alpha). \quad (2)$$

Choosing the values of  $A, B$  and  $\alpha$  properly, the electrical phase can be equal the corresponded optical phase to be obtained at any time. It is easy to get electrical phase from the circuits. Let phase shift  $\alpha=0$ , the correct combination of the value of  $A$  with  $B$  leads the vibration of mirror  $M_2$  be modulated near sinusoidal wave. The less the value  $A$ , the more sinusoidal the modulated wave is, however too small value of  $A$  leads to a small signal to noise ratio. Generally, take  $A=\lambda/4$ ,  $\lambda$  is working wave length, for example  $A$  is 0.78 here, then  $B$  is 1.46. Fig. 2 shows the relationship of  $A$  and  $B$  such that "linear-solution" is obtained when  $\alpha=0$ . Fig. 3 is a one kind of wave shapes of the  $\phi(t)$ .

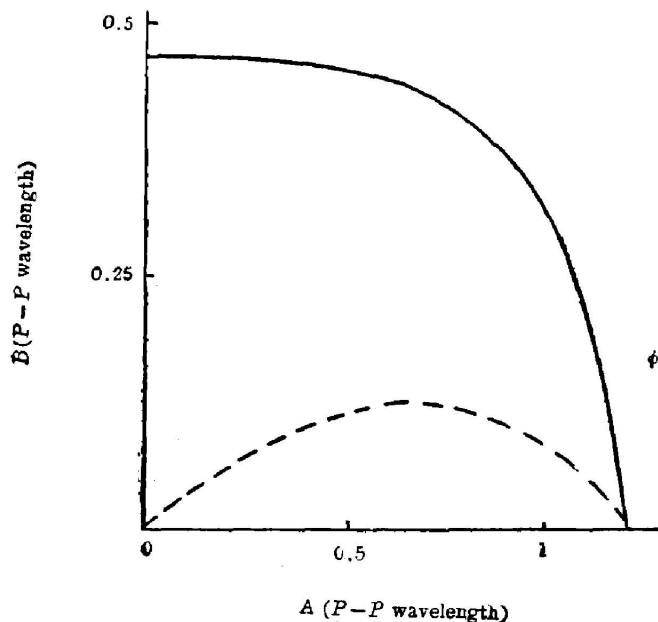


Fig. 2 Relationship  $A$  with  $B$  for linear solutions

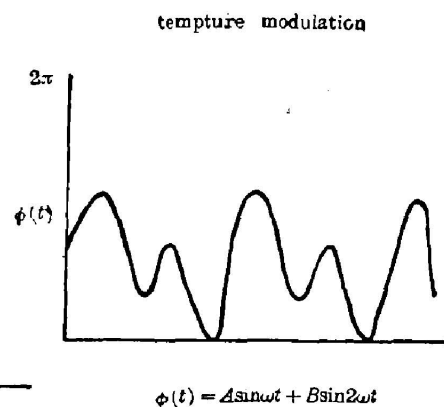


Fig. 3 An example of modulation wave shape

## Computer Simulation

There are mainly two kinds of theoretical analysis methods in calculating the index profile of a GRIN rod from the transverse interference pattern. One assumes that light beams traverse the gradient rod in a straight line, and the index profile is expressed with a polynomials, then the coefficients of the assumed polynomials are calculated from the interference pattern (ST). The other is based on the real curved ray trace of the light beams in a GRIN rod. It takes some approximation during Abel inversion of the index profile (CT). The further computer simulation was taken to compare the results of the two kinds of analytical techniques for larger  $\Delta n$ . Fig. 4 and Fig. 5 are results of the simulation for  $\Delta n=0.05$  and  $\Delta n=0.0015$  and different index matching oil. It is shown from the figures that the two methods are compatible when the  $\Delta n$  is small, the errors of the ST method, however, increased rapidly when  $\Delta n$  is larger, such as, when  $\Delta n=0.05$ , the error increases by an order of magnitude with respect to the  $\Delta n=0.015$ , but the error of the CT method is nearly constant.

It is also indicated that the effect of the index mis-matching between the periphery of the GRIN rod and the index oil in the CT method on the index profiling is almost uniform over the whole region of the GRIN rod. It is obvious that the CT method is more preferable when  $\Delta n$  is larger.

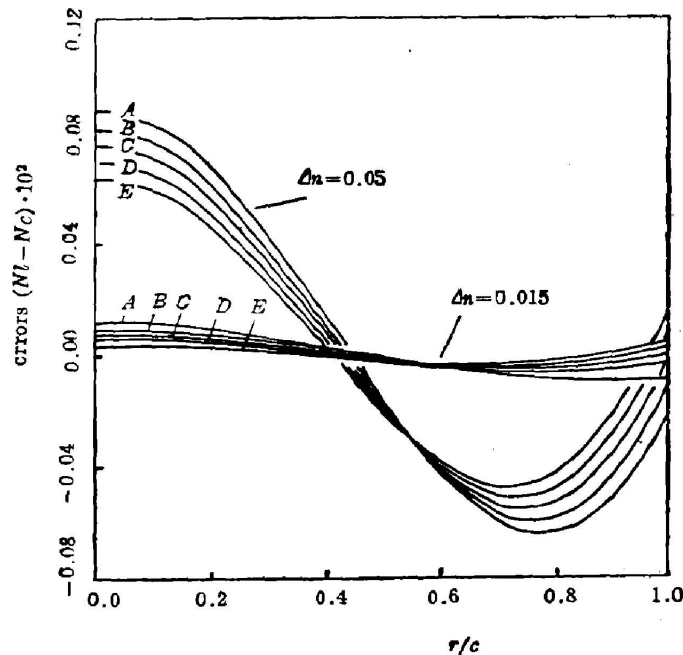


Fig. 4 Effect on  $\Delta n$  on the index discrepancies for straight ray method.

Index mismatching  $N_m - N_a$ : A—0.004; B—0.002; C—0.0; D—0.002; E—0.004

$N_m$ , index of matching oil;  $N_a$ , index of sample periphery;  $N_t$ , theoretical value;  $N_c$ , calculated value

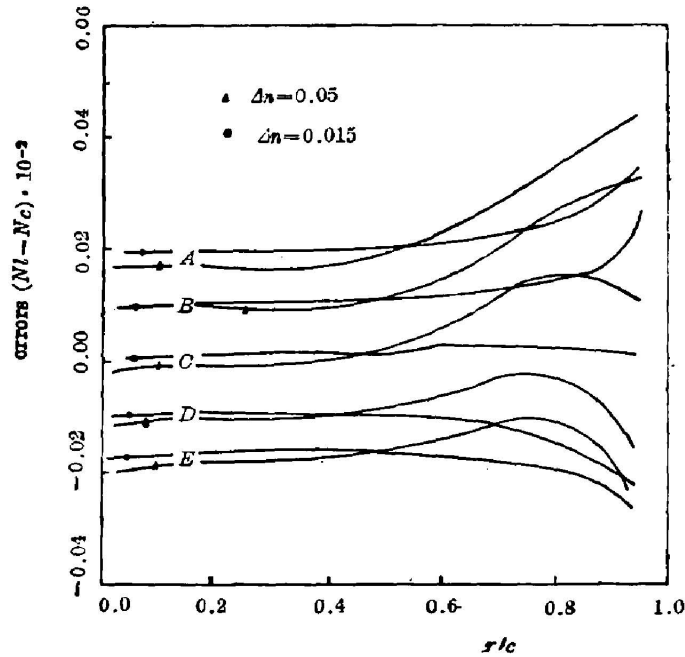


Fig. 5 Effect of  $\Delta n$  on the index discrepancies for curved ray method. Index mismatching  $N_m - N_a$ : A—0.004; B—0.002; C—0.0; D—0.002; E—0.004  
 $N_m$ , index of matching oil;  $N_a$ , index of sample periphery;  $N_t$ , theoretical value;  $N_c$ , calculated value

### Data Treatment and Results

Because of symmetry of the refractive index profile of the gradient index sphere, the light beams passing through the diameter cross section of the spheres have the same behaviors as the light beams traversed the gradient rod, the formulae used in the transverse method, therefore, can be applied to sphere with a slight change. On the other hand, the data obtained in harmonic interference system are the optical phase difference,  $\Delta\theta$ , between the probing beams and reference beams (in matching oil). The formula for calculating the refractive index profile of the sample in the first approximation<sup>[5]</sup> is

$$n(u) = N_m \exp \left\{ -\frac{1}{\pi} \int_u^\alpha \frac{\lambda}{2\pi} \frac{\Delta\theta(y)}{\Delta y} \frac{dy}{\sqrt{N_m^2 y^2 - u^2}} \right\}, \quad (3)$$

where  $u = n(r)r$ , is working wave length,  $\alpha$  is radius of the sphere,  $N_m$  is the refractive index of the matching oil. The gradient index sphere to be measured was made using iron diffusion method by Li Jinker's group, Xi'an Institute of Optics and Precision Mechanics, Academia Sinica, Xi'an, China, the diameter of the sphere is 9.182 mm, and the index of the periphery is 1.546.

The formula used to calculate the index profile from the data is equation (3). It is seen that the refractive index of every point of  $u$  is almost a result of whole phase

data measured. So the phase data was first fitted to a polynomial, then the polynomial is substituted in the equ. (3) to calculate the refractive index.

In order to eliminate the system errors as much as possible, after scanning the sample phase, the background phase also was scanned, then the background phase was subtracted from the sample phase.

Fig. 6 shows the phase map obtained from this method. The index profile of the sphere, shown on Fig. 7, was calculated from the phase data on the Fig. 6. Fig. 8 is the profiles of the sphere using common Mach-Zehnder interferometer. It is viewed from the figures that the experiment curve is in good agreement with the theoretical analysis of parabolic profile<sup>[40]</sup>. The index profile obtained from the harmonic interferometer as we see from the figures is in good agreement with the results achieved from the transverse method. It also shows that the index mis-matching effect of the index oil is approximately homogeneous over whole region of the sphere.

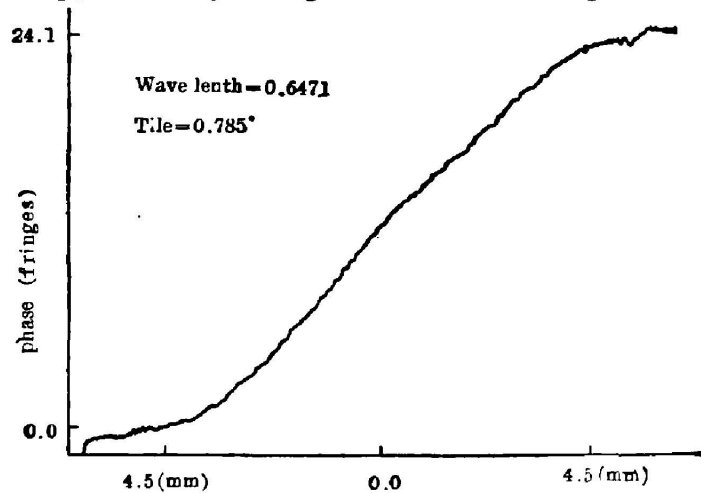


Fig. 6 Shearing phase map of the GRIN sphere

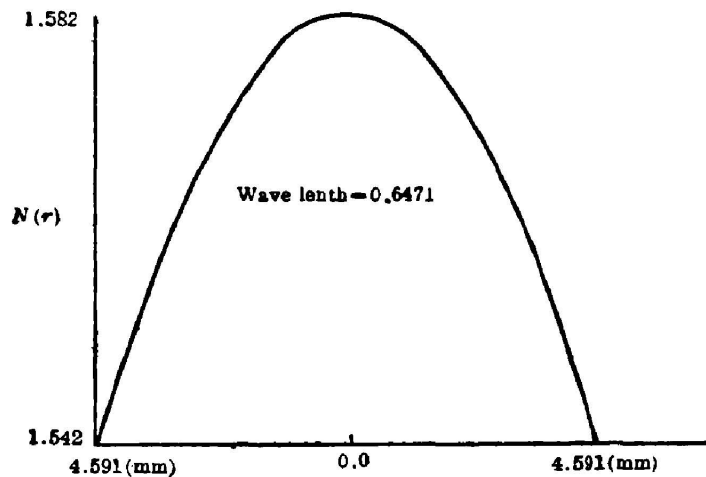


Fig. 7 Index profile of the GRIN sphere calculated from the phase map in Fig. 6

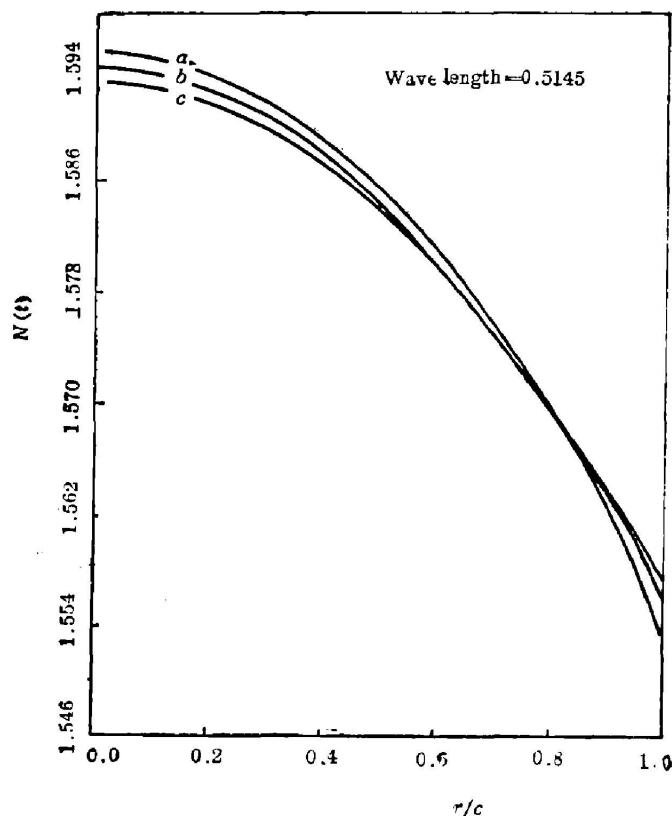


Fig. 8 Index profiles of the GRIN sphere as measured on Mach-Zehnder interferometer  
Matching oil: a, 1.550; b, 1.552; c, 1.554

### Conclusion

Computer simulation shows that the CT method is preferable to the ST method when the  $\Delta n$  of the sample is big.

Harmonic interferometer gets data easily and automatically, the amplitude of the modulation can be as small as possible.

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## 变折射率球的折射率分布无损测量: 大口径交流谐波剪切干涉仪

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### 提 要

本文提出用一种大口径交流谐波剪切干涉仪作变折射率球的折射率分布快速、自动、无损测量。文中叙述了实验装置、测量原理、实验技巧、数据处理及结果。计算机模拟和实验表明, 横向干涉法中的曲线轨迹分析精度优于直线轨迹分析。