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激光等离子体的回旋辐射

谭 维 翰
(中国科学院上海光学精密机械研究所)

提 要

本文研究的回旋辐射是由于在激光强场影响下的电子回旋运动产生的。对辐射能进行估算，激光等离子体回旋辐射的大小可与轫致辐射相比。这种情况要求更重视对激光等离子体回旋辐射的研究。

一、引 言

过去对激光等离子体的轫致辐射及逆轫致吸收分析较多[1~7]。对激光等离子体的回旋辐射，由于没有外加恒定磁场，则几乎没有探讨过。实际上虽未加恒定磁场，但在强的激光场作用下，如入射为圆偏振光，电子就作回旋运动。如为线偏振光，则总可分解为左、右圆偏振光，分别产生右、左旋的回旋运动。实际估算表明激光等离子体的回旋辐射与轫致辐射相比是不小的。下面先讨论等离子体密度起伏对辐射的贡献，然后计算回旋辐射的功率谱与电子有回旋运动情况下的介电张量，最后对回旋辐射的逆过程，即吸收作一些讨论。

二、电流密度起伏对等离子体辐射的贡献

将电流密度起伏$\delta\boldsymbol{J}(\boldsymbol{r},t)$展开为

$$\delta\boldsymbol{J}(\boldsymbol{r},t)=(1/V)\sum_{\boldsymbol{k}}\int(d\omega/2\pi)\delta\boldsymbol{J}(\boldsymbol{k},\omega)\exp[i(\boldsymbol{k}\cdot\boldsymbol{r}-\omega t)]。\tag{1}$$

设$\overline{OP}=\boldsymbol{R}_0=\hat{n}'R_0$, $\hat{n}'$为单位矢量，P点矢势$A(\hat{n}'R_0,t)=\boldsymbol{A}_{\hat{n}}(t)$的导数为$\dot{\boldsymbol{A}}_{\hat{n}}(t)$，当$R_0$很大时，$\dot{\boldsymbol{A}}_{\hat{n}}(t)$可写为[3]:

$$\begin{aligned}\dot{\boldsymbol{A}}_{\hat{n}}(t)&=(1/cR_0)\int d\boldsymbol{r}\delta\dot{\boldsymbol{J}}[\boldsymbol{r},t-(R_0/c)-(\hat{n}\cdot\boldsymbol{r}/v_p)]\\&=(1/cR_0)\sum_{\boldsymbol{k}}\int(d\omega/2\pi)(-i\omega)\delta\boldsymbol{J}(\boldsymbol{k},\omega)\delta(\boldsymbol{k},\hat{n}\omega/v_p)\\&\quad\times\exp[-i\omega(t-R_0/c)],\\\delta(\boldsymbol{k},\hat{n}\omega/v_p)&=(1/V)\int d\boldsymbol{r}\exp[-i(\boldsymbol{k}-\hat{n}\omega/v_p)\cdot\boldsymbol{r}],\end{aligned}\tag{2}$$

式中$\boldsymbol{r}=\overline{Q'O}$, $\hat{n}$为$\overline{QO}$方向的单位矢量，$\hat{n}'$为经等离子体界面OO'折射后的单位矢量(见图1所示)。v_p为电磁波在等离子体中的相速度，若等离子体为各向同性的、均匀的，则v_p为常数。图1中，如P与P'; Q与Q'处于同一波面上。$\overline{QO}$, $\overline{Q'O'}$; $\overline{OP}$, $\overline{O'P'}$为垂直于波面

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$\overline{Q'Q}$, $\overline{P'P}$ 的法线，波面 $\overline{Q'Q}$ 经折射后变为波面 $\overline{P'P}$。则

$$\overline{Q'O'}/v_p+\overline{O'P'}/c=\overline{QO}/v_p+\overline{OP}/c=\hat{n}\cdot\boldsymbol{r}/v_p+R_0/c。\tag{3}$$

将电场强度 $\boldsymbol{E}_{\hat{n}}(t)$，磁场强度 $\boldsymbol{H}_{\hat{n}}(t)$ 及坡印亭矢量 $\boldsymbol{S}_{\hat{n}}$ 等表示为

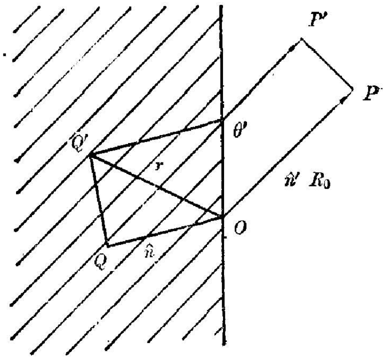

图 1

$$\boldsymbol{E}_{\hat{n}}(t)=(1/c)(\dot{\boldsymbol{A}}_{\hat{n}}(t)\times\hat{n})\times\hat{n},$$

$$\boldsymbol{H}_{\hat{n}}(t)=(1/c)\dot{\boldsymbol{A}}_{\hat{n}}(t)\times\hat{n},$$

$$\begin{aligned}\boldsymbol{S}_{\hat{n}}&=(c/4\pi)\langle\boldsymbol{E}_{\hat{n}}(t)\times\boldsymbol{H}_{\hat{n}}(t)\rangle\\&=\langle(c/4\pi)(1/R_0c^2)^2\sum_{\boldsymbol{k},\boldsymbol{k}'}\int(d\omega/2\pi)\\&\quad\times(-i\omega)\int(d\omega'/2\pi)(-i\omega')[(\delta\boldsymbol{J}(\boldsymbol{k},\omega)\\&\quad\times\hat{n})\times\hat{n}]\times[\delta\boldsymbol{J}(\boldsymbol{k}',\omega)\times\hat{n}]\\&\quad\times\delta(\boldsymbol{k},\hat{n}\omega/v_p)\delta(\boldsymbol{k}',\hat{n}\omega'/v_p)\exp[-\boldsymbol{i}(\omega+\omega')(t-R_0/c)\rangle\\&=(c/4\pi)(1/R_0c^2)\sum_{\boldsymbol{k}}\int(d\omega/2\pi)(-i\omega)\int(d\omega'/2\pi)(-i\omega')\\&\quad\times\langle[(\delta\boldsymbol{J}(\boldsymbol{k},\omega)\times\hat{n})\times\hat{n}]\times(\delta\boldsymbol{J}(\hat{n}\omega'/v_p,\omega')\times\hat{n})\rangle\\&\quad\times\delta(\boldsymbol{k},\hat{n}\omega/v_p)\exp[-\boldsymbol{i}(\omega+\omega')(t-R_0/c),\end{aligned}\tag{4}$$

式中

$$\begin{aligned}&[(\delta\boldsymbol{J}(\boldsymbol{k},\omega)\times\hat{n})\times\hat{n}]\times(\delta\boldsymbol{J}(\hat{n}\omega'/v_p,\omega')\times\hat{n})\\&\quad=[(\delta\boldsymbol{J}(\boldsymbol{k},\omega)\times\hat{n})\times(\delta\boldsymbol{J}(\hat{n}\omega'/v_p,\omega')\times\hat{n})]\hat{n}\langle(\delta\boldsymbol{J}(\boldsymbol{k},\omega)\times\hat{n})\\&\qquad\times(\delta\boldsymbol{J}(\hat{n}\omega'/v_p,\omega')\times\hat{n})\rangle\\&\quad=\langle JJ^*(\boldsymbol{k},\omega)\rangle_T\delta(\omega+\omega')\langle JJ^*(\boldsymbol{k},\omega)\rangle_T\\&\quad=N_e^2/2\epsilon_T^2(\boldsymbol{k},\omega)\int d\boldsymbol{v}f(\boldsymbol{v})[(\boldsymbol{k}\times\boldsymbol{v})^2/k^2]\delta(\omega-\boldsymbol{k}\cdot\boldsymbol{v})。\end{aligned}\tag{5}$$

$\langle JJ^*(\boldsymbol{k},\omega)\rangle_T$ 为电流密度起伏中横向部分[3]，$\epsilon_T(\boldsymbol{k},\omega)$ 表示电屏蔽[3,7]，

$$\begin{aligned}\epsilon_T(\boldsymbol{k},\omega)&=1-(\omega^2/k^2c^2)+(\omega_{pe}^2\omega/k^2c^2)\int(f_ed\boldsymbol{v}/\omega-\boldsymbol{k}\cdot\boldsymbol{v}-i\eta)\\&\quad+(\omega_{pi}^2\omega/k^2c^2)\int(f_id\boldsymbol{v}/\omega-\boldsymbol{k}\cdot\boldsymbol{v}-i\eta)。\end{aligned}\tag{6}$$

将(5)式代入(4)式便得 $\boldsymbol{S}_{\hat{n}}$ 及功率谱 $d^2P'/d\Omega\,d\omega$

$$\boldsymbol{S}_{\hat{n}}=(c\hat{n}/4\pi)(1/R_0c^2)^2\sum_{\boldsymbol{k}}\int d\omega\omega^2\langle JJ^*(\boldsymbol{k},\omega)\rangle_T\delta(\boldsymbol{k},\hat{n}\omega/v_p),\tag{7}$$

$$d^2P'/d\Omega\,d\omega=1/4\pi c^3\sum_{\boldsymbol{k}}[\omega^2\langle JJ^*(\boldsymbol{k},\omega)\rangle_T+\omega^2\langle JJ^*(\boldsymbol{k},-\omega)\rangle_T]_{k=\omega/v_p}。\tag{8}$$

设 $f(\boldsymbol{v})$ 为 Maxwell 分布，且 v_z 与 $\boldsymbol{k}$ 平行，则由(5)、(8)式得

$$\begin{aligned}d^2P'/d\Omega\,d\omega&=N_e^2\omega^2(\overline{v}_x^2+\overline{v}_y^2)/4\pi c^3\int dv_zf(v_z)[\delta(\omega-kv_z)/\epsilon_T^2(\boldsymbol{k},\omega)]\\&=[(N_e^2\omega^2KT/m)/4\pi c^3\epsilon_T^2(\boldsymbol{k},\omega)]\\&\quad\times(m/2\pi KT)^{1/2}(1/k)\exp[-\omega^2/2KTk^2]。\end{aligned}\tag{9}$$

由于 $\omega/k=v_p=c/\sqrt{1-\omega_p^2/\omega^2}$，上式又可写为

$$d^2P'/d\Omega\, d\omega=(N_e^2\omega/4\pi c^2)(KT/m)(1/\epsilon_T^2(\boldsymbol{k},\omega)\sqrt{1-\omega_p^2/\omega^2})$$
$$\times(m/2\pi KT)^{1/2}\exp[-mc^2/2KT(1-\omega_p^2/\omega^2)]。\tag{10}$$

对 Maxwell 分布，韧致辐射功率谱为[3]

$$d^2P/d\Omega\, d\omega=4c(Ze)^2r_e^2N_eN_i/3\pi(2\pi KT/m)^{1/2}V\{\ln[\sqrt{2}k_M(KT/m)^{1/2}/\omega]-c/2\}。\tag{11}$$

设 $\epsilon_T^2\simeq1$，于是有

$$(d^2P'/d\Omega\, d\omega)/(d^2P/d\Omega\, d\omega)$$
$$\simeq(3\pi k\lambda_D^2/4r_e)(1/\sqrt{1-\omega_p^2/\omega^2})\{\ln[\sqrt{2}\,k_M(KT/m)^{1/2}/\omega]$$
$$-c/2\}^{-1}\exp[-mc^2/2KT(1-\omega_p^2/\omega^2)],\tag{12}$$

(12)式为电流密度起伏(不考虑电子回旋辐射)横向部分的辐射功率谱与韧致辐射功率谱之比，式中经典电子半径 $r_e=2.83\times10^{-13}$ cm，设 $k=2\pi\times10^4\,\mathrm{cm}^{-1}$，因子 $3\pi/4\{\ \}^{-1}$ 的量级为 1，则

$$(1/\sqrt{1-\omega_p^2/\omega^2})(k\lambda_D^2/r_e)\exp[-(mc^2/2KT)(1/(1-\omega_p^2/\omega^2))]$$
$$\simeq1.23\times10^5\exp[-256/(1-\omega_p^2/\omega^2)]/\sqrt{1-\omega_p^2/\omega^2}。\tag{12$'$}$$

这表明电流密度起伏横向部分的辐射功率是可以忽略不计的，但考虑电子回旋辐射后，情况就不是这样。

三、电子回旋辐射功率谱

设圆偏振光沿 z 轴方向入射到等离子体上，使得电子除了无规运动($\boldsymbol{r}$, $\boldsymbol{v}$)外，还叠加一以无规运动($\boldsymbol{r}$, $\boldsymbol{v}$)为中心的回旋运动($\Delta\boldsymbol{r}$, $\Delta\boldsymbol{v}$)，

$$\Delta\boldsymbol{r}=eE/m\omega_0^2(\hat{i}\sin\omega_0t+\hat{j}\cos\omega_0t);$$
$$\Delta\boldsymbol{v}=eE/m\omega_0^2(\hat{i}\cos\omega_0t-\hat{j}\sin\omega_0t);\tag{13}$$
$$d\Delta\boldsymbol{v}/dt=\omega_0\Delta\boldsymbol{v}\times\hat{k}。$$

式中 $\hat{i}$, $\hat{j}$, $\hat{k}$ 分别为 x, y, z 方向的单位矢量。电流密度起伏中横向部分 $\langle JJ^*(\boldsymbol{k},\omega)\rangle_T$，在有电子回旋运动情形，可把(5)式表示为

$$\langle JJ^*(\boldsymbol{k},\omega)\rangle_T=(N_e^2/2\epsilon_T^2(\boldsymbol{k},\omega))(1/2\pi)\int_{-\infty}^{\infty}dt\int d\boldsymbol{v}\,d\Delta\boldsymbol{v}f(\boldsymbol{v})F(\Delta\boldsymbol{v})$$
$$\times((\boldsymbol{v}'+\Delta\boldsymbol{v}')\times\hat{n})\times((\boldsymbol{v}+\Delta\boldsymbol{v})\times\hat{n})\exp[-i\boldsymbol{k}\times(\boldsymbol{r}(t)$$
$$-\boldsymbol{r}(0))-i\boldsymbol{k}\times(\Delta\boldsymbol{r}(t)-\Delta\boldsymbol{r}(0))+i\omega t]。\tag{14}$$

设 $\hat{n}=\hat{i}\sin\theta+\hat{k}\cos\theta$，则有

$$(\Delta\boldsymbol{v}'\times\hat{n})\times(\Delta\boldsymbol{v}\times\hat{n})=(eE/m\omega_0)^2(\cos(\omega_0t+\varphi)\cos\varphi\cos^2\theta+\sin(\omega_0t+\varphi)\sin\varphi)。\tag{15}$$

电子回旋运动中心不受光场影响，故有 $\boldsymbol{v}'=\boldsymbol{v}$。

$$(\Delta\boldsymbol{v}\times\hat{n})\times\boldsymbol{v}+\boldsymbol{v}\times(\Delta\boldsymbol{v}'\times\hat{n})=(-\hat{i}\sin\varphi\cos\theta-\hat{j}\cos\varphi\sin\theta+\hat{k}\sin\varphi\sin\theta$$
$$-\hat{i}\sin(\omega_0t+\varphi)\cos\theta-\hat{j}\cos(\omega t+\varphi)\sin\theta+\hat{k}\sin(\omega t+\varphi)\sin\theta)\times\boldsymbol{v}。\tag{16}$$

又考虑到

$$\int_0^{2\pi} d\varphi \exp[-ix(\sin(\omega_0 t+\varphi)-\sin\varphi)]\begin{bmatrix}\cos\varphi\\ \cos(\omega_0 t+\varphi)\\ \cos(\omega_0 t+\varphi)\cos\varphi\\ 1\\ \sin(\omega_0 t+\varphi)\sin\varphi\\ \sin(\omega_0 t+\varphi)\\ \sin\varphi\end{bmatrix}$$

$$=2\pi\sum_{n=-\infty}^{\infty}\exp[-in\omega_0 t]\begin{bmatrix}n/xJ_n^2\\ n/xJ_n^2\\ (nJ_n/x)^2\\ J_n^2\\ (J'_n)^2\\ iJ_nJ'_n\\ -iJ_nJ'_n\end{bmatrix}$$

便得出对应关系

$$\cos\varphi\to nJ_n/x,\ \cos(\omega_0 t+\varphi)\to nJ_n/x,\ 1\to J_n,\ \sin\varphi\to -iJ'_n, \sin(\omega_0 t+\varphi)\to iJ'_n。\tag{17}$$

注意到

$$\int d\boldsymbol{v} f(\boldsymbol{v})v_y=0;\tag{18}$$

$$2\pi\int F(\Delta\boldsymbol{v})\Delta v_\perp d\Delta v_\perp d\Delta v_{/\!/}=1;\tag{19}$$

$$1/2\pi\int \exp[i(\omega-n\omega_0-\boldsymbol{k}\times\boldsymbol{v})t]dt=\delta(\omega-n\omega_0-\boldsymbol{k}\times\boldsymbol{v})。\tag{20}$$

由(16)、(17)式得

$$\int[(\Delta\boldsymbol{v}\times\hat{n})\times\boldsymbol{v}+\boldsymbol{v}\times(\Delta\boldsymbol{v}'\times\hat{n})]\exp[-ix(\sin(\omega_0 t+\varphi)-\sin\varphi)]d\varphi$$
$$=-\hat{j}2(nJ_n^2/x)\times\boldsymbol{v}=-2(nJ_n^2/x)v_y。\tag{21}$$

由于因子 $\exp[-i\boldsymbol{k}\times(\boldsymbol{r}(t)-\boldsymbol{r}(0))]=\exp[-i(k_x v_x+k_z v_z)t]$ 与 v_y 无关，故由(21)、(18)式得出(14)式中交叉项 $(\Delta\boldsymbol{v}\times\hat{n})\times\boldsymbol{v}+\boldsymbol{v}\times(\Delta\boldsymbol{v}'\times\hat{n})$ 的贡献为零，于是由(15)、(17)、(19)、(20)式得出(14)式为

$$\langle JJ^*(\boldsymbol{k},\ \omega)\rangle_T=N_n^2/2\epsilon_T^2(\boldsymbol{k},\ \omega)\int d\boldsymbol{v}f(\boldsymbol{v})\times\sum_{n=-\infty}^{\infty}\{J_n^2(v\times\hat{n})^2+(eE/m\omega_0)^2$$
$$\times[(nJ_n/x)^2\cos^2\theta+(J'_n)^2]\}\delta(\omega-n\omega_0-\boldsymbol{k}\cdot\boldsymbol{v})。\tag{22}$$

讨论(22)式的物理意义。注意到

$$x=(keE/m\omega_0^2)\sin\theta=(\omega/v_p)(eE/m\omega_0^2)\sin\theta=(\omega/\omega_0)(v_\perp/v_p)\sin\theta,$$
$$n/x=(n\omega_0/\omega)(v_\perp\sin\theta/v_p)^{-1}\simeq v_p/v_\perp\sin\theta=1/\beta_\perp\sin\theta。\tag{23}$$

当光场 $E\to 0$ 时，$x\to 0$，容易看出(22)式过渡到(5)式。若不考虑回旋中心的无规运动，可取 $f(\boldsymbol{v})=\delta(\boldsymbol{v})$，则有

$$\langle JJ^*(\boldsymbol{k},\ \omega)\rangle_T=[ve^2v_\perp^2/2\beta_\perp^2\epsilon_T^2(\boldsymbol{k},\ \omega)]\sum_{n=1}^{\infty}[(\cos\theta J_n/\sin\theta)^2+\beta_\perp^2(J_n')^2]\delta(\omega-n\omega_0)。\tag{24}$$

代入(8)式得

$$d^2P'/d\omega\, d\Omega=(N_e^2\omega v_p^2/4\pi c^3)(1/\epsilon_T^2(\boldsymbol{k},\ \omega))\times\sum_{n=1}^{\infty}[(\cos\theta J_n/\sin\theta)^2+\beta_\perp^2 J_n^2]\delta(\omega-n\omega_0)。\tag{25}$$

上式除因子 $1/\epsilon_T^2$ 外与文献[1](9.48)式相符。由此看出(22)式中 $J_n^2(\boldsymbol{v}\times\hat{n})^2$ 项表示集体效应,而后两项则表示粒子效应,是由单个粒子的回旋运动引起的。这三项都对辐射功率作出贡献,由(22)式代入(8)式给出总的辐射功率谱。

现按(22)式估算来表征集体效应与粒子效应的各项。当 $n=0$, $J_0^2(\boldsymbol{v}\times\hat{n})^2$ 项相当于不考虑电子回旋辐射情况下电流密度起伏对辐射的贡献,即(9)~(12)式; $n=1$ 各项即电子回旋辐射的贡献; $n>1$ 各项也是电子回旋辐射的贡献,但要小得多。现按(9)~(12)′式的方法估算 $n=0$, $n=1$ 各项的辐射功率谱 $d^2P_0'/d\Omega\, d\omega$, $d^2P_1'/d\Omega\, d\omega$, 并注意到

$$n=1,\quad \int dv_z f(v_z)\delta(\omega-\omega_0 kv_z)=(m/2\pi KT)^{1/2}(1/k)\exp[-(\omega-\omega_0)^2/2KTk^2]$$
$$\simeq(m/2\pi KT)^{1/2}k^{-1};\ n=0,\quad \int dv_z f(v_z)\delta(\omega-kv_z)=(m/2\pi KT)^{1/2}\times k^{-1}\exp[-\omega^2/2KTk^2]。$$

于是有(当 $x\ll1$ 时)

$$\begin{aligned}d^2P_1'/d\Omega\, d\omega)/(d^2P_0'/d\Omega\, d\omega)&\simeq\{[(J_1)^2(\boldsymbol{v}\times\hat{n})^2\\&+(eE/m\omega_0)^2((J_1/x)^2\cos^2\theta+(J_1')^2)]/J_0^2(\boldsymbol{v}\times\hat{n})^2\}\exp[\omega^2/2KTk^2]\\&\simeq(v_\perp/v_p)^2[1+(J_1')^2]\exp[256/(1-\omega_p^2/\omega^2)]。\end{aligned}\tag{25$'$}$$

由(12)、(12)′、(25)′各式,得

$$(d^2P_1'/d\Omega\, d\omega)/(d^2P/d\Omega\, d\omega)\simeq(v_\perp/v_p)^2(1+(J_1')^2)\times1.23\times10^5。$$

将激光功率密度 I 取为 $10^{14}\ \mathrm{W/cm^2}$, 激光波长 $\lambda_0=1\ \mu\mathrm{m}$, 则

$$v_\perp=eE/m\omega_0=2.5\times10^8\ \mathrm{cm/sec},$$

又由 $v_p=c/\sqrt{1-(\omega_p/\omega)^2}$ 得

$$(d^2P_1'/d\Omega\, d\omega)/(d^2P/d\Omega\, d\omega)\simeq8(1+(J_1')^2)(1-\omega_p^2/\omega^2)^{1/2}。\tag{25$''$}$$

四、介电张量计算

为计算(22)式的电屏蔽因子 $1/\epsilon_T(\boldsymbol{k},\ \omega)$, 就需要计算介电张量 $\epsilon(\boldsymbol{k},\ \omega)$。文献[2~4]虽给出有恒定磁场情况下的介电张量计算公式,但在激光场作用下,电子的回旋运动与恒定磁场下电子的回旋运动是有区别的。在激光场作用下,电子的回旋运动用 $(\boldsymbol{r}+\Delta\boldsymbol{r},\ \boldsymbol{v}+\Delta\boldsymbol{v})$ 来描述, $(\boldsymbol{r},\ \boldsymbol{v})$ 表示回旋中心的无规运动, $(\Delta\boldsymbol{r},\ \Delta\boldsymbol{v})$ 则表示相对于中心 $(\boldsymbol{r},\ \boldsymbol{v})$ 的回旋运动,且 $d\Delta\boldsymbol{v}/dt=\omega_0\Delta\boldsymbol{v}\times\boldsymbol{k}$。但电子运动的总速度并不满足此关系,即 $d(\boldsymbol{v}+\Delta\boldsymbol{v})/dt\neq\omega_0(\boldsymbol{v}+\Delta\boldsymbol{v})\times\boldsymbol{k}$, 在恒定磁场下电子的运动用 $(\boldsymbol{r},\ \boldsymbol{v})$ 来描述,电子运动的速度 $\boldsymbol{v}$ 满足 $d\boldsymbol{v}/dt=\Omega_0\boldsymbol{v}\times\boldsymbol{k}$, Ω_0 为回旋频率。由于这个区别,需要重新计算在强激光场作用下,并考虑电子回旋运动的

介电张量。为方便起见，先求出在外加恒定而均匀的磁场作用下的介电张量 $\epsilon_H(\boldsymbol{k},\ \omega)$，进一步求出在激光场作用下，并考虑电子回旋运动的 $\epsilon(\boldsymbol{k},\ \omega)$（参照文献[3, 4]）。

$$\epsilon_H(\boldsymbol{k},\ \omega)=\mathrm{II}+\sum_\sigma(\omega_\sigma^2/i\omega)\int_0^{2\pi}d\theta\int_0^\infty v_\perp dv_\perp\int_{-\infty}^{\infty}dv_{/\!/}\times\int_0^\infty d\tau\boldsymbol{v}[\partial f_\sigma(\boldsymbol{v}')/\partial\boldsymbol{v}']$$
$$\times[(1-\boldsymbol{k}\times\boldsymbol{v}'/\omega)\mathrm{II}+\boldsymbol{k}\times\boldsymbol{v}'/\omega]\exp[-i\phi(\tau)-\eta\tau], \tag{26}$$

其中

$$\partial f_\sigma(\boldsymbol{v}')/\partial\boldsymbol{v}')\times(\boldsymbol{k}\times\boldsymbol{v}'-\boldsymbol{k}\times\boldsymbol{v}'\mathrm{II})\times\delta\boldsymbol{E}$$
$$=[\partial f_\sigma(\boldsymbol{v}')/\partial\boldsymbol{v}'][\boldsymbol{v}'\times(\boldsymbol{k}\times\delta\boldsymbol{E})]$$
$$=(\boldsymbol{A}\times\boldsymbol{v}')\times\partial f_\sigma(\boldsymbol{v}')/\partial\boldsymbol{v}',\quad \boldsymbol{A}=\delta\boldsymbol{E}\times\boldsymbol{k}, \tag{27}$$
$$\phi(\tau)=Z_\sigma(\sin(\Omega_0\tau+\theta)-\sin\theta)+k_{/\!/}v_{/\!/}\tau-\omega\tau,\quad Z_\sigma=k_\perp v_\perp/\Omega_\sigma。$$

上式也可写成

$$[A_1(v_2'\partial/\partial v_3'-v_3'\partial/\partial v_2')+A_2(v_3'\partial/\partial v_1'-v_1'\partial/\partial v_3')$$
$$+A_3(v_1'\partial/\partial v_2'-v_2'\partial/\partial v_1')]f_\sigma(\boldsymbol{v}')。$$

对初始的各向同性的热平衡等离子体 $f_\sigma(\boldsymbol{v}')=\tilde{f}(v_x'^2+v_y'^2+v_z'^2)$，故有

$$(v_i'\partial/\partial v_j'-v_j'\partial/\partial v_i')f_\sigma(\boldsymbol{v}')=0, \tag{28}$$

即

$$(\partial f_\sigma(\boldsymbol{v}')/\partial\boldsymbol{v}')\times(\boldsymbol{k}\times\boldsymbol{v}'-\boldsymbol{k}\times\boldsymbol{v}'\mathrm{II})\times\delta\boldsymbol{E}=0。 \tag{29}$$

$$\epsilon_H(\boldsymbol{k},\ \omega)=\mathrm{II}+\sum_\sigma(\omega_\sigma^2/i\omega)\int_0^{2\pi}d\theta\int_0^\infty v_\perp dv_\perp\int_{-\infty}^{\infty}dv_{/\!/}\int_0^\infty d\tau\boldsymbol{v}(\partial f_\sigma(\boldsymbol{v}')/\partial\boldsymbol{v}')$$
$$\times\exp[-i\phi(\tau)-\eta\tau], \tag{30}$$

其中

$$\boldsymbol{v}\partial f_\sigma(\boldsymbol{v}')/\partial\boldsymbol{v}'=[v_\perp(\cos\theta\hat{i}+\sin\theta\hat{j})+v_{/\!/}\hat{k}][\partial f/\partial v_\perp)(\cos(\Omega_\sigma\tau+\theta)\hat{i}$$
$$+\sin(\Omega_\sigma\tau+\theta)\hat{j})+(\partial f/\partial v_{/\!/})\hat{k}]。 \tag{31}$$

又由 $Z_\sigma=k_\perp v_\perp/\Omega_\sigma$，将(31)式代入(30)式，并应用(17)式，便得

$$\epsilon_H(k,\ \omega)=\mathrm{II}-\sum(\omega_\sigma^2/\omega)\int_0^\infty v_\perp' dv_\perp'\int_{-\infty}^{\infty}dv_{/\!/}\sum_{n=-\infty}^{\infty}m_\sigma(v_\perp',\ v_{/\!/}';\ n)/(v\Omega_\sigma+k_{/\!/}v_{/\!/}'-\omega)。 \tag{32}$$

$$m_\sigma(v_\perp',\ v_{/\!/}';\ n)=[v_\perp((nJ_n/Z_\sigma)\hat{i}-iJ_n'\hat{j})+v_{/\!/}J_n\hat{k}][(\partial f/\partial v_\perp)$$
$$\times((nJ_n/Z_\sigma)\hat{i}+iJ_n'\hat{j})+(\partial f/\partial v_{/\!/})J_n\hat{k}]。$$

$$m_\sigma(v_\perp,\ v_{/\!/};\ n)$$
$$=\begin{pmatrix} v_\perp(\partial f/\partial v_\perp)(n^2J_n^2/Z_\sigma^2) & i(v_\perp nJ_nJ_n'/Z_\sigma)(\partial f/\partial v_\perp) & v_\perp(nJ_n^2/Z_\sigma)(\partial f/\partial v_{/\!/}) \\ -i(v_\perp nJ_nJ_n'/Z_\sigma)(\partial f/\partial v_\perp) & v_\perp J_n'^2(\partial f/\partial v_\perp) & -iv_\perp J_nJ_n'(\partial f/\partial v_{/\!/}) \\ v_{/\!/}(nJ_n^2/Z_\sigma)(\partial f/\partial v_\perp) & iv_{/\!/}J_nJ_n'(\partial f/\partial v_\perp) & v_{/\!/}J_n^2(\partial f/\partial v_{/\!/}) \end{pmatrix}。 \tag{33}$$

注意到　$-\omega/(n\Omega_\sigma+k_{/\!/}v_{/\!/}-\omega)=1-(n\Omega_\sigma+k_{/\!/}v_{/\!/}')/(n\Omega_\sigma+k_{/\!/}v_{/\!/}-\omega)$,

$$\sum nJ_n^2=\sum nJ_nJ_n'=0,\quad \sum J_nJ_n'=0,\quad \sum J_n^2=1,\quad \sum(J_n')^2Z^2=\sum n^2J_n^2=Z^2/2。 \tag{34}$$

便得

$$\epsilon_H(\boldsymbol{k},\ \omega)=\mathrm{II}+\sum(\omega_\sigma^2/\omega^2)\int_0^\infty v_\perp dv_\perp\int_{-\infty}^\infty dv_\parallel[(\hat{i}\hat{i}/2)v_\perp(\partial f/\partial v_\perp)$$
$$+(\hat{j}\hat{j}/2)v_\perp(\partial f/\partial v_\perp)+\hat{k}\hat{k}v_\parallel(\partial f/\partial v_\parallel)]$$
$$-\sum_\sigma(\omega_\sigma^2/\omega^2)\int_0^\infty v_\perp dv_\perp\int_{-\infty}^\infty dv_\parallel\sum_{n=-\infty}^\infty[(n\Omega_\sigma+k_\parallel v_\parallel)/(n\Omega_\sigma$$
$$+k_\parallel v_\parallel-\omega)]m_\sigma(v_\perp,\ v_\parallel;n)$$
$$=(1-\omega_p^2/\omega^2)\mathrm{II}-\sum_\sigma(\omega_\sigma^2/\omega^2)\sum_{n=-\infty}^\infty\int_0^\infty v_\perp dv_\perp\int_{-\infty}^\infty dv_\parallel$$
$$\times[(n\Omega_\sigma+k_\parallel v_\parallel)/(n\Omega_\sigma+k_\parallel v_\parallel-\omega)]m_\sigma(v_\perp,\ v_\parallel;n)。\tag{35}$$

在推导上式时，用到

$$\int(v_\perp/2)(\partial f/\partial v_\perp)v_\perp dv_\perp dv_\parallel=\int v_\parallel(\partial f/\partial v_\parallel)v_\perp dv_\perp dv_\parallel$$
$$=-\int fv_\perp dv_\perp dv_\parallel=-1;\quad \omega_p^2=\sum_\sigma\omega_p^2。$$

仍按前面假设 $f(\boldsymbol{v})=\tilde{f}(v_x^2+v_y^2+v_z^2)=\tilde{f}(v_\perp^2+v_z^2)$，故有 $v_\perp\partial f/\partial v_\parallel=v_\parallel\partial f/\partial v_\perp$，于是(35)式可写成

$$\epsilon_H(\boldsymbol{k},\ \omega)=(1-\omega_p^2/\omega^2)\mathrm{II}-\sum_\sigma\omega_\sigma^2/\omega^2\sum_{n=-\infty}^\infty\int_0^\infty v_\perp dv_\perp\int_{-\infty}^\infty dv_\parallel[(n\Omega_\sigma/v_\perp)(\partial f_\sigma/\partial v_\perp)$$
$$+k_\parallel\partial f_\sigma/\partial v_\parallel][\Pi_\sigma(v_\perp,\ v_\parallel;n)/(n\Omega_\sigma+k_\parallel v_\parallel-\omega-i\eta)],$$
$$\Pi_\sigma(v_\perp,\ v_\parallel;n)=\begin{pmatrix} n^2\Omega_\sigma^2J_n^2/k_\perp^2 & iv_\perp n\Omega_\sigma J_nJ_n'/k_\perp & v_\parallel n\Omega_\sigma J_n^2/k_\perp \\ -iv_\perp n\Omega_\sigma J_nJ_n'/k_\perp & v_\perp^2(J_n')^2 & -iv_\parallel v_\perp J_nJ_n' \\ v_\parallel n\Omega_\sigma J_n^2/k_\perp & iv_\parallel v_\perp J_nJ_n' & v_\parallel^2J_n^2\end{pmatrix}。\tag{36}$$

此即文献[2～3]的介电张量 $\epsilon_H(\boldsymbol{k},\ \omega)$ 表式。为求强激光作用下的介电张量 $\epsilon(\boldsymbol{k},\ \omega)$，可将(26)式作如下的推广

$$\boldsymbol{v}\to\boldsymbol{v}+\Delta\boldsymbol{v},\ \boldsymbol{v}'\to\boldsymbol{v}+\Delta\boldsymbol{v}',\ \Delta\boldsymbol{v}'=\Delta v_\perp[\hat{i}\cos(\omega_0t+\varphi)-\hat{j}\sin(\omega_0t+\varphi)],$$
$$f(\boldsymbol{v})\to f(\boldsymbol{v})F(\Delta\boldsymbol{v}),\ f(\boldsymbol{v}')\to f(\boldsymbol{v})F(\Delta\boldsymbol{v}')。\tag{37}$$

又设 $F(\Delta\boldsymbol{v}')=\delta(|\Delta\boldsymbol{v}'|-\Delta v_\perp)$ 与相角 $\omega_0t+\varphi$ 无关，而电子回旋运动保持 $|\Delta\boldsymbol{v}'|$ 不变，只是相角改变。故有

$$\partial f(\boldsymbol{v}')/\partial\boldsymbol{v}'\to(\partial f(\boldsymbol{v})/2\boldsymbol{v})F(\Delta\boldsymbol{v}')+f(\boldsymbol{v})(\partial F(\Delta\boldsymbol{v}')/\Delta v_\perp\partial(\omega_0t+\varphi))$$
$$=(\partial f(\boldsymbol{v})/\partial\boldsymbol{v})F(\Delta\boldsymbol{v}')。\tag{38}$$

于是有

$$\epsilon(\boldsymbol{k},\ \omega)=\mathrm{II}+\sum_\sigma(\omega_\sigma^2/i\omega)\int_0^{2\pi}d\theta\int_0^\infty\Delta v_\perp' d\Delta v_\perp'\int_{-\infty}^\infty d\Delta v_\parallel'\int d\boldsymbol{v}$$
$$\times\int_0^\infty d\tau(\boldsymbol{v}+\Delta\boldsymbol{v})(\partial f(\boldsymbol{v})/\partial\boldsymbol{v})F(\Delta\boldsymbol{v}')\times[(1-\boldsymbol{k}\cdot(\boldsymbol{v}+\Delta\boldsymbol{v}')/\omega)\mathrm{II}$$
$$+\boldsymbol{k}\cdot(\boldsymbol{v}+\Delta\boldsymbol{v}')/\omega]\exp[-i\phi(\tau)-\eta\tau]$$
$$=\mathrm{II}+\sum(\omega_\sigma^2/i\omega)(1/2\pi)\int_0^{2\pi}d\theta\int d\boldsymbol{v}\int_0^\infty d\tau(\boldsymbol{v}+\Delta\boldsymbol{v})(\partial f(\boldsymbol{v})/\partial\boldsymbol{v})$$
$$[(1-\boldsymbol{k}\times\Delta\boldsymbol{v}'/\omega)\mathrm{II}+(\boldsymbol{k}\times\Delta\boldsymbol{v}/\omega)]\exp[-i\phi(\tau)-\eta\tau]$$
$$=\mathrm{II}-\sum_\sigma(\omega_\sigma^2/\omega)\int d\boldsymbol{v}\sum_n(\Delta v_\perp nJ_n/Z_\sigma+v_xJ_n)\hat{i}+(\Delta v_\perp(-iJ_n')+v_yJ_n)\hat{j}$$

$$+v_z J_n \hat{k})(\partial f/\partial \boldsymbol{v}) \times [(J_n - (k_x \Delta v_\perp/\omega)(nJ_n/Z_\sigma) - (k_y \Delta v_\perp/\omega) iJ'_n)\mathrm{II} + \boldsymbol{k}\Delta v_\perp (nJ_n/Z_\sigma \hat{i} + iJ'_n \hat{j})/\omega](n\omega_0 + \boldsymbol{k}\cdot\boldsymbol{v} - \omega)^{-1}。 \tag{39}$$

引进矢量 $\Delta \boldsymbol{v}_\perp$, $\boldsymbol{J}_n$, 并矢 $\vec{J}_n$

$$\Delta \boldsymbol{v}_\perp = \Delta v_\perp(\hat{i}+\hat{j}); \quad \boldsymbol{J}_n = nJ_n Z_\sigma \hat{i} + iJ'_n \hat{j} + J_n \hat{k};$$
$$\vec{J}_n = nJ_n/Z_\sigma \hat{i}\hat{i} + iJ'_n \hat{j}\hat{j} + J_n \hat{k}\hat{k}。 \tag{40}$$

于是

$$\epsilon(\boldsymbol{k}, \omega) = \mathrm{II} - \sum_\sigma \omega_\sigma^2/\omega \sum_{-\infty}^{\infty}\int (J_n \boldsymbol{v} + \boldsymbol{v}_\perp \times \vec{J}_n)(\partial f/\partial \boldsymbol{v}) \times ((\hat{k} - k_x \Delta v_\perp/\omega \hat{i} - k_y \Delta v_\perp/\omega \hat{j}) \times \boldsymbol{J}_n \mathrm{II} + \boldsymbol{k}\Delta \boldsymbol{v}_\perp \times \vec{J}_n/\omega) d\boldsymbol{v}/(n\omega_0 + \boldsymbol{k}\times\boldsymbol{v} - \omega)。 \tag{41}$$

由介电张量 $\epsilon(\boldsymbol{k}, \omega)$ 求电屏蔽因子的公式如下[3]

$$1/\epsilon_T^2(\boldsymbol{k}, \omega) = 1/|[(T_r \epsilon(\boldsymbol{k}, \omega) - \boldsymbol{k}\times\epsilon(\boldsymbol{k}, \omega)\times \boldsymbol{k}/k^2)/2] - (kc/\omega)^2|^2。 \tag{42}$$

五、讨　论

用 V 除(11), (25)式的功率谱 $d^2P/d\Omega\, d\omega$, $d^2P'/d\Omega\, d\omega$, 便得出自发发射系数 $d^2P/d\Omega\, d\omega V$, $d^2P'/d\Omega\, d\omega V$。又用黑体辐射强度 $I(\nu, T) = (h\nu^3/c^2)(1/\{\exp[h\nu/KT]-1\})$ 除, 便得出相应的有效吸收系数 a_ω, a'_ω。当 $h\nu/KT \ll 1$ 时,

$$I(\nu, T) \simeq (\nu^2/c^2) KT; \tag{43}$$

$$a_\omega = (d^2P/V d\Omega\, d\omega)/(\nu^2 KT/c^2) = [4O(Ze)^2 r_e^2 n_e n_i / 3\pi (2\pi KT/m)^{1/2}] \times \{\ln(\sqrt{2}\, k_M (KT/m)^{1/2}/\omega) - c/2\} c^2/\nu^2 KT$$
$$= [16 e^6 Z^2 n_e n_i / 3c (2\pi m KT)^{1/2} \nu^2] \{\ln(\sqrt{2}\, k_M (KT/m)^{1/2}/\omega) - c/2\}; \tag{44}$$

$$a'_\omega = (d^2P'/V d\Omega\, d\omega)/(\nu^2 KT/c^2) = a_\omega (d^2P'/d\Omega\, d\omega)/(d^2P/d\Omega\, d\omega)$$
$$\simeq a_\omega (d^2P'_1/d\Omega\, d\omega)/(d^2P/d\Omega\, d\omega)。 \tag{45}$$

a_ω 的表达式 (44) 与文献 [5] 求得的相符。(45)式给出 a'_ω 与 a_ω 之比 $(d^2P'_1/d\Omega\, d\omega)/(d^2P/d\Omega\, d\omega)$, 由(25)″ 式给出, 这就表明通过电子回旋辐射的逆过程的吸收与逆韧致吸收相比也同样是不小的。当然这里的分析是就温度、密度均匀分布的等离子体进行的, 与实际的激光等离子体(温度、密度很不均匀且急剧变化, 还有各种不稳定和非线性问题)比, 是有很大距离的。

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Cyclotron radiation from laser plasma

TAN WEIHAN
(*Shanghai Institute of Optics and Fine Mechanics, Academia Sinica*)

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Abstract

The cyclotron radiation emitted from electrons in cyclotron motion under influences of the electromagnetic field is investigated. According to a moderate estimation of the emitted energy, the magnitude of cyclotron radiation from the laser plasma is comparative to that of Bremsstrahlung radiation. This result justifies a futher study on such phenomenon.