Squeezing-enhanced resolution of radio-frequency signals

We demonstrate a resolution enhancement scheme of radio-frequency signals by tailoring a phase-squeezed state. The echo radio-frequency signals collected by photonic radar give rise to displacements in the phase quadrature of a probe laser and are estimated by the balanced homodyne detector. In contrast to the conventional coherent state, the noise variances for radio-frequency estimation with a squeezed state are reduced by approximately 6.9 dB. According to the Rayleigh criterion that defines the resolution limit, the minimum resolvable displacement \( \Delta r \) with a squeezed state is reduced to 45% compared to that with a coherent state, demonstrating the quantum advantage. The squeezing-enhanced technique has extensive applications for multitarget recognition and tracking in contemporary photonic radar systems.

**Keywords:** phase-squeezed state; radio-frequency signal; resolution.

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1. Introduction

Squeezing-enhanced technique (SET), which is a momentous branch of quantum metrology\([1–3]\), enables measurement sensitivity beyond the standard quantum limit (SQL)\([4–7]\). Since proposed, massive efforts have been made to enhance the sensitivity of physical quantities including optical phase\([8,9]\), displacement\([10]\), rotation angle\([11]\), magnetic field\([12]\), and so on. Berni et al. demonstrated a quantum-enhanced optical phase estimation using squeezed vacuum states and real-time feedback control\([13]\). Yonezawa et al. achieved the unprecedented precision for optical-phase tracking using a phase-squeezed state\([14]\). Treps et al. and Sun et al. showcased subquantum noise level (sub-QNL) sensitivity in displacement measurements\([15,16]\). Liu et al. realized rotating-angle measurement beyond the quantum limit\([17]\). Li et al. obtained a 20% sensitivity improvement in an optomechanical magnetometer utilizing phase-squeezed light\([18]\). Yap et al. reduced the quantum backaction noise by 1.2 dB with the injection of amplitude-squeezed light\([19]\). Casacio et al. demonstrated a signal-to-noise ratio beyond the photodamage limit of conventional microscopy using bright quantum correlated illumination\([20]\). Xia et al. demonstrated the entanglement-enhanced radio-frequency (RF) photonic sensing with a squeezing source and variable beam splitters\([21]\).

Recently, squeezed vacuum states have been tailored for advanced LIGO\([22]\) and advanced Virgo detectors\([23]\), enabling a 3 dB sensitivity enhancement.

Precise spatial resolution is of paramount significance to multitarget recognition, positioning, and tracking for military or civil radars\([24,25]\). In order to achieve high spatial resolution, multiple radar systems have been proposed: for instance, the phased array antenna\([26]\), synthetic aperture radar\([27]\), microwave photonic radar\([28]\), millimeter-wave radar\([29]\), and so on. However, the existing photonic radars are not accurate enough to identify closely spaced targets. Therefore, it is always beneficial to improve the spatial resolution of photonic radar.

In this paper, we propose an innovative squeezing-enhanced photonic radar system, where the echo signals from closely spaced targets are simulated by RF signals, and the spatial resolution is enhanced by utilization of squeezed light. According to the Rayleigh criterion\([30–32]\), it offers a 55% resolution enhancement in contrast to that with coherent light.

2. Theoretical Analysis and Experimental Setup

Figure 1(a) illustrates the principle of squeezing-enhanced photonic radar. It commences with the introduction of a squeezed operator \( \hat{S}(r, \theta) \), which generates a squeezed vacuum state by acting upon the vacuum state \( \hat{0} \). Notably, the squeezed vacuum state has no coherent amplitude, and its noise fluctuation in phase quadrature is lower than the quantum noise limit (QNL)\([33–35]\). At the signal acquisition module, the echo RF signal from the target is expressed as \( E = A \sin(\Omega t + \phi) \), where \( A \),
\(\Omega\) and \(\phi\) are the amplitude, frequency, and phase, respectively. The squeezing vacuum state is modulated by an electro-optical transducer (EOT) driven by the echo RF signal, introducing a displacement \(\alpha_{RF} = A\kappa \sin(\phi)\) along the orthogonal phase axis in the phase space. Apparently, there is no displacement when the phase \(\phi = 0\), while the maximum (minimum) displacement occurs at \(\phi = \frac{\pi}{2} (\frac{-\pi}{2})\). At the signal processing terminal, the displacement \(\alpha_{RF}\) is measured by a balanced homodyne detector (BHD), from which we can acquire the amplitude and phase information of echo RF signals.

The experimental setup is depicted in Fig. 1(b). A continuous-wave single-frequency fiber laser at 1550 nm is injected into the mode cleaner1 (MC1), suppressing the intensity noise and phase noise for fundamental frequency while improving its beam quality. The output of MC1 is split into two beams, one of which traverses another mode cleaner2 (MC2), ulteriorly suppressing the noise. The other beam is subsequently resonant in the second-harmonic generation (SHG) resonator, where photons undergo parametric upconversion through the second-order nonlinear effect of the crystal, yielding a frequency-doubled light at 775 nm. For both 1550 and 775 nm, the front end of the crystal is highly reflective and the back end is antireflective. The reflectivity of a concave mirror is 84.3% for 1550 nm and 97.8% for 775 nm, respectively. With a 21 mm air gap between the PPKTP crystal and the output mirror, the finesse of OPA is 35 for the 1550 nm and 200 for the 775 nm laser. In virtue of the doubly resonant characteristic of OPA, the cavity length is stabilized by frequency-doubled laser with meticulously designed proportional integral derivative (PID) feedback loop. The co-resonance condition for fundamental laser and frequency-doubled laser is satisfied by fine-tuning the temperature of the PPKTP crystal. By controlling the relative phase between seed beam and frequency-doubled laser to 0, a phase-squeezed light is generated with a maximum squeezing degree of \(-7.1\) dB±0.2 dB, while the antisqueezing degree is 10.0 dB±0.2 dB.

The RF signal acquired by photonic radar is transformed to a probe laser at the fundamental frequency by an EOT, introducing a displacement \(\alpha_{RF}\) along the quadrature phase in phase space[36,37]. In the experimental scenario, the unknown RF signal is simulated by a sinusoidal function whose frequency is 10 MHz. Subsequently, the probe laser is interfered with a strong local oscillator at a 50:50 beam splitter and injected towards the BHD. In order to access the displacement in phase quadrature, the relative phase between probe laser and local oscillator needs to be controlled at \(\pi/2\). The desired information in the time domain is acquired by demodulating the AC output of BHD.
with another sinusoidal function at 10 MHz and collected by a digital oscilloscope.

3. Experimental Results

Figure 2 contrasts the phase-squeezed state with the coherent state aiming at the estimation of RF phase. We initially estimated the RF phase utilizing the coherent state, which is generated by blocking the frequency-doubled laser for OPA. By automatically scanning the phase across 0 deg to 360 deg range with a step size of 10 deg, the averaged displacements are represented by the purple spheres, while the noise variances are displayed by the lavender shaded area. Subsequently, the phase-squeezed state is employed to estimate the RF phase. Sweeping the phase with the same parameters and collecting data automatically, the noise variances are illustrated by the dark purple shaded area. The black curve is a fitting for the experimental data, showing a sinusoidal pattern. Obviously, the noise variances for estimation of all RF phases are reduced by approximately 6.9 dB compared to that with the coherent state. Moreover, a gradient algorithm and a regional optimization algorithm are employed to locate the extremum of displacement measurements. As a result, the maximum and minimum displacements are represented by two red stars, corresponding to RF phases $\phi_1$ and $\phi_2$, respectively.

Suppose two RF signals, $E_1 = A \sin (\Omega t + \phi_1)$ and $E_2 = A \sin (\Omega t + \phi_2)$, are echo signals from closely spaced targets, respectively. The noise variances for estimation of two signals are interlacing. Moreover, when the phases of two RF signals are 0 or $\pi$, the first derivative of the sine function is the largest, corresponding to the point where phase estimation is most sensitive.

Figure 3(a) illustrates the estimated noise variances with the coherent state by fixing the RF amplitude and automatically scanning the RF phase. The red region showcases the RF amplitudes are distinguishable with the squeezed state, while the dark blue region exhibits the distinguishable RF amplitude with the squeezed state. It is apparent that the distinguishable RF amplitude is diminished by harnessing the phase-squeezed state.

The resolution enhancement becomes notably evident upon reconstructing the Wigner function. To this end, the relative phase between the probe laser and local oscillator is scanned via a phase shifter. By setting the RF phases to $\phi_1$, 0, and $\phi_2$, respectively, the time-domain information for any quadrature is recorded by the oscilloscope. After extracting one complete cycle from the data, we derived the density matrix and Wigner function and projection in phase space while employing the maximum likelihood (MaxLik) technique. Figure 4(a) showcases the reconstructed Wigner function and projection in phase space while employing the coherent state to estimate the RF signal. The red region represents the most densely concentrated area within the phase space. Apparently, three RF signals are superposing together and indistinguishable when the coherent state is acting as a probe.
Figure 4(b) displays the Wigner function and projection in phase space arising from the estimation results with the squeezed state. In contrast, three RF signals are discernible clearly by utilizing the phase-squeezed state, underscoring a 55% enhancement in spatial resolution compared to that with the coherent state.

4. Conclusion

We have demonstrated a 55% enhancement in the resolution of RF signals by tailoring a phase-squeezed state with a squeezing degree of $-7.1 \, \text{dB}$. The echo RF signals from the target lead to displacements in the phase quadrature of the probe laser and are estimated by a BHD. In contrast to traditional coherent light sources, the noise variances for RF estimation with the squeezed state are reduced by approximately 6.9 dB. According to the Rayleigh criterion, the minimum resolvable displacement $\Delta a$ with the squeezed state is reduced to 0.45 when $\Delta a$ with the coherent state is normalized to unity, demonstrating the quantum advantage. Automated scanning techniques accelerate the experimental process and improve the effectiveness of data acquisition. The proposed methodology holds significant promise for multitarget recognition and tracking in current photonic radar systems.

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