Ghost imaging—its physics and application [Invited]

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Since its first experimental demonstration in 1995, “ghost imaging” has attracted much attention, perhaps not only because of its interesting physics but also because of its attractive application. This review article discusses the physics and application of ghost imaging: (1) emphasizes the nonlocal two-photon interference nature of ghost imaging, including detailed analysis and calculations; (2) introduces three types of applications with unique advantages of ghost imaging, including a light detection and ranging device with imaging ability, namely, an Imaging Lidar or ILidar system; a turbulence-resistant, or turbulence-free, imaging technology; and a vibration-resistant X-ray microscope of high resolving capability. This article is prepared for a Special Issue of Chinese Optics Letters, intended for general audiences, especially young researchers and students who are interested in ghost imaging.

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1. Introduction

Since its first experimental demonstration in 1995, “ghost imaging” has attracted great attention from the science community. The physics community even named it a “ghost.” The important physics demonstrated in that experiment, however, may not be the so-called “ghost.” Indeed, the original purpose of the experiment was to study the nonlocal Einstein–Podolsky–Rosen (EPR) correlation in position and in momentum for an entangled biphoton system. The ghost imaging experiment confirmed a point-to-point EPR-type nonlocal correlation from the distant joint-photodetection event of an entangled signal-idler photon pair created from spontaneous parametric down-conversion (SPDC); although the signal photon and the idler photon can be observed at any point on the object plane and ghost image plane, respectively, if the signal photon is observed at a certain point on the object plane, the idler must be observed at a unique corresponding point on the ghost imaging plane, jointly. It is this EPR correlation playing the role of an image-forming function that produced the ghost image. In fact, a classic imaging device, such as a camera, is able to produce a point-to-point image-forming function between the object plane and image plane with the help of an image lens. In Dirac’s opinion, namely the theory of quantum mechanics, this classic point-to-point image-forming function is the result of a photon interfering with the photon itself. Following Dirac, the nonlocal point-to-point ghost image-forming function must be the result of two-photon interference: a pair of entangled photons interferes with the pair itself. [Notes: (1) To avoid any possible confusion with the quantum interference picture of ghost imaging, this article will not review or discuss its classical simulations, for example, the “speckle-to-speckle” image-forming function. The review and discussion on the physics and applications of classical simulations of ghost imaging can be found in other contributions to this special issue. The author sincerely apologizes for this. (2) To avoid any misunderstanding, we would like to emphasize that (i) two-photon interference is not the interference between two photons. In Dirac’s opinion: “. . . photon . . . only interferes with itself.” (ii) Two-photon interference is not limited by “two-term” superposition. The superposition of a large number of two-photon amplitudes is a typical quantum interference phenomenon.]

This interference is indeed “nonlocal”; the superposition of the two-photon amplitudes happens at two different space-time points, and the photodetection events of the signal photon and the idler photon can be easily arranged into space-like separation.

Ten years after the experimental demonstration of ghost imaging with entangled photon pairs, we found that the Fresnel near-field, natural, non-factorizable, point-to-point ghost image-forming function is not only the property of entangled photon pairs. Two-photon interference of randomly created and randomly paired photons in the thermal state can produce a similar point-to-point image-forming correlation without the use of any imaging lens. This type of ghost imaging is often referred to as “lensless ghost imaging,” although it does not prevent the use of lenses for secondary imaging. Perhaps, due to the thermal state nature of the jointly measured photons, a debate about the classical or quantum nature of ghost imaging started to confuse the community. At the same time,
Among these waves, the image produced by a speckle-to-speckle correlation is more like a "projection shadow" rather than an image. More importantly, it can never achieve the "peculiar" benefits of ghost imaging that we have come to expect.

Ghost imaging has received a great deal of attention, perhaps not only because of its interesting physics but also because of its attractive applications. This article emphasizes the nonlocal two-photon interference nature of ghost imaging and discusses three types of applications of ghost imaging. This article starts with classical imaging. The detailed calculation and analysis of the point-to-point correspondence between the object plane and the imaging plane conclude that the classical image-forming function is the result of first-order interference, i.e., a photon interfering with the photon itself. We then introduce biphoton ghost imaging and thermal light ghost imaging. The detailed calculations and analysis conclude that the ghost image-forming function is the result of a two-photon interference: a pair of photons, either entangled or randomly created and randomly paired, interferences with the pair itself. In the application sections, we first discuss the working mechanism of a light detection and ranging device with imaging ability, namely, an Imaging Lidar or ILidar. We then follow with a discussion about turbulence-resistant, or turbulence-free, ghost imaging technology for nonlocal measurements. The last application we analyze is an X-ray microscope, which takes the unique advantages of the lensless and vibration-resistant property of ghost imaging.

2. Classical Imaging

The concept of optical imaging was well-developed in optics prior to the electromagnetic wave theory of light. The early theories of imaging provided quite a few phenomenological solutions for the point-to-point relationship between an object plane and its image plane. In these theories, especially the theory of geometric optics, radiation is treated as "ray of light" (this "ray of light" is not Einstein's "bundle of rays"), and the image is explained as the result of the peculiar way of their propagation. A later theory of classical imaging, namely the theory of physical optics, is based on the concept of waves. Light is treated as waves that propagate to and interfere at a space-time point. The image is considered the result of constructive-destructive interferences among these waves.

Figure 1 schematically illustrates a classic imaging setup, an object that is illuminated by an incoherent light source. Spatially incoherent radiations are reflected or scattered from the object plane. An imaging lens is used to image the randomly radiated or scattered radiations from the object onto an image plane, which is defined by the "Gaussian thin-lens equation\\(^{[3]}\),

\[
\frac{1}{s_i} + \frac{1}{s_o} = \frac{1}{f},
\]

where \( s_o \) is the distance between the object and the imaging lens, \( s_i \) is the distance between the imaging lens and the image plane, and \( f \) is the focal length of the imaging lens. The Gaussian thin-lens equation defines an image plane that has a point-to-point relationship with the object plane: any radiation starting from a point on the object plane must collapse or stop at a unique point on the image plane due to the constructive-destructive interferences.

A perfect point-to-point correspondence between the object plane and the image plane produces a perfect image. Illuminated by a radiation, the image produced by an imaging system is a reproduction, either magnified or demagnified, of the illuminated object, mathematically corresponding to a convolution between the intensity distribution function of the object plane, \( |A(\mathbf{\rho}_o)|^2 \), and a \( \delta \)-function like image-forming function, which characterizes the point-to-point relationship between the object plane and the image plane:
\[ \langle I(\vec{p}_i) \rangle = \left\langle \left| \int_{\text{obj}} d\vec{p}_o A(\vec{p}_o) \delta \left( \vec{p}_o + \frac{\vec{p}_i}{\mu} \right) \right|^2 \right\rangle \]

\[ = \int_{\text{obj}} d\vec{p}_o |A(\vec{p}_o)|^2 \delta \left( \vec{p}_o + \frac{\vec{p}_i}{\mu} \right) \]

\[ = |A(\vec{p}_o)|^2 \otimes \delta \left( \vec{p}_o + \frac{\vec{p}_i}{\mu} \right) \]

\[ \approx |A(\vec{p}_i/\mu)|^2, \quad (2) \]

where \( \langle I(\vec{p}_i) \rangle = |A(\vec{p}_i/\mu)|^2 \) is the mean intensity distribution function, i.e., the reproduction of the aperture function in the image plane, \( A(\vec{p}_o) \) is the complex aperture function of the object plane, \( \vec{p}_o \) and \( \vec{p}_i \) are 2-D vectors of the transverse coordinates in the object plane and the image plane, respectively, \( \mu = s_i/s_o \) is the image magnification factor (\( \mu \) may take a positive value or a negative value), and the mathematical symbol \( \otimes \) represents convolution. In Eq. (2), we have assumed an incoherent illumination, namely an incoherent imaging process in which \( A(\vec{p}_o) \) contains a random phase.

In reality, limited by the finite size of the imaging system, we may never obtain a perfect point-to-point correspondence. The incomplete constructive-destructive interference turns the point-to-point correspondence into a point-to-“spot” relationship. The \( \delta \)-function in the convolution of Eq. (2) has to be replaced by a point-spread function:

\[ \langle I(\vec{p}_i) \rangle = \left\langle \left| \int_{\text{obj}} d\vec{p}_o A(\vec{p}_o) \text{somb} \left( \frac{R \omega}{s_o c} \vec{p}_o + \frac{\vec{p}_i}{\mu} \right) \right|^2 \right\rangle \]

\[ = \left| A(\vec{p}_o) \otimes \text{somb} \left( \frac{R \omega}{s_o c} \vec{p}_o + \frac{\vec{p}_i}{\mu} \right) \right|^2 \]

\[ = |A(\vec{p}_o)|^2 \otimes \text{somb}^2 \left( \frac{R \omega}{s_o c} \vec{p}_o + \frac{\vec{p}_i}{\mu} \right), \quad (3) \]

where the sombrero-like function, or the Airy disk, is defined as

\[ \text{somb}(x) = \frac{2f_1(x)}{x}, \]

where \( f_1(x) \) is the first-order Bessel function, \( R \) is the radius of the imaging lens, and \( R/s_o \) is known as the numerical aperture of the imaging system. The sombrero-like point-spread function, or the Airy disk, defines the observed spot size on the image plane that is produced by the radiation field coming from point \( \vec{p}_o \). It is clear from Eq. (3) that a larger imaging lens and shorter wavelength will result in a narrower point-spread function, and thus a higher spatial resolution of the image. The finite size of the observed spot on the image plane determines the spatial resolution of the imaging system.

It should be emphasized that we must not confuse a trivial “projection shadow” with an image. Similar to an X-ray photograph, projection makes a shadow of an object, instead of an image of the object. Figure 2 distinguishes a projection shadow from an image. The object-shadow correspondence is essentially defined by the propagation direction of the light rays, and there is no unique imaging plane. The shadow can be found in any plane behind the object. A projection shadow is very different from an image, both from a fundamental and a practical viewpoint. There is no spatial resolution defined in terms of Rayleigh’s criterion for a projection shadow.

Figure 3 illustrates a simplified schematic experimental setup of an imaging system. Based on this simplified setup, we calculate and analyze the imaging process. The following calculation is divided into two parts: (1) we calculate the point-to-point “propagator” from the object plane to the image plane, the radiation starting (emitted, scattered, reflected, or transmitted) from a point in the object plane must collapse or stop at a unique point in the image plane; (2) we calculate the classic image as a convolution between the aperture function and the point-spread image-forming function. To simplify the mathematics, we assume the object is illuminated by a spatially incoherent radiation of single frequency. By assuming \( \omega_i \) to be a constant, the following analysis and calculation will focus on the spatial behavior of the field and ignore the temporal coherence discussion.

(1) The point-to-point “propagator” from the object plane to the image plane.

Selecting a radiation field \( E(\vec{p}_o, z_o = 0, t_o = 0) \) from an arbitrary point of the object plane, propagating \( E(\vec{p}_o, 0, 0) \) to the image plane by means of an appropriate propagator \( \tilde{g}_{\vec{p}_i}(\vec{p}_i, z_i, t_i) \),

\[ E(\vec{p}_i, s_o + s_i, t_i) = E(\vec{p}_o) \tilde{g}_{\vec{p}_i}(\vec{p}_i, z_i, t_i) \]

\[ = \int_{\text{lens}} d\vec{p}_o E(\vec{p}_o) \left[ \tilde{g}_{\vec{p}_i}(\vec{p}_i, z_i, t_i) \right] \left[ g_{\text{lens}}(\vec{p}_o) \right], \quad (4) \]
where we have taken \((z_o = 0, t_o = 0)\) and ignored the zeros from \(E(\vec{p}_o, z_o, t_o)\); \(g_{\vec{p}_2}(\vec{p}_1, z_i, t_i)\) and \(g_{\vec{p}_2}(\vec{p}_1, z_i, t_i)\) are Green’s functions that propagate the field \(E(\vec{p}_o)\) from the object plane to the imaging lens plane, and from the imaging lens plane to the image plane, respectively; and \(g_{\text{lens}}\) is the Gaussian function introduced by the imaging lens. Due to the single frequency, \(\omega = \text{constant}\) assumption, we have dropped \(\omega\) from Green’s functions, or propagators.

Assuming Fresnel near-field propagations from the object plane to the imaging lens plane, and from the imaging lens plane to the image plane, the following propagator \(g_{\vec{p}_2}(\vec{p}_1, z_i, t_i)\) will propagate field \(E(\vec{p}_o)\) from the object plane to the image plane,

\[
g_{\vec{p}_2}(\vec{p}_1, z_i, t_i) = \int_{\text{lens}} d\vec{p}_1 \frac{-i\omega e^{i\varphi}}{2\pi c \cdot s_o} \mathcal{G}(|\vec{p}_1 - \vec{p}_o|, \frac{\omega}{cs_o}) \times \mathcal{G}(|\vec{p}_1 - \vec{p}_o|, \frac{\omega}{cs_o}),
\]

where \(\vec{p}_o, \vec{p}_1,\) and \(\vec{p}_2\) are 2-D vectors defined, respectively, in the object plane, imaging lens plane, and image plane. The term in the first bracket pair is the free-space Fresnel propagator that propagates the field from the source/object plane to the imaging lens plane; the term in the second bracket pair is the Gaussian function introduced by the imaging lens; the term in the third is the free-space Fresnel propagator that propagates the field from the imaging lens plane to the observation plane, i.e., the image plane. The Fresnel propagator includes a spherical wave function \(\exp[i\omega(z_j - z_k)/c(z_j - z_k)]\) and a Fresnel phase factor \(G(|\vec{p}_1|, \beta) = \exp[i(\beta/2)|\vec{p}_1|^2] = \exp[i\omega|\vec{p}_o - \vec{p}_k|^2/2c(z_j - z_k)]\).

The term in the third bracket pair adds a phase factor, \(G(|\vec{p}_1|, \beta)\), which is introduced by the imaging lens.

Applying the properties of the Gaussian function, Eq. (5) can be simplified into the following form:

\[
g_{\vec{p}_2}(\vec{p}_1, z_i, t_i) = \frac{-i\omega^2}{(2\pi c)^2 s_o s_i} e^{i\varphi} \mathcal{G}(\vec{p}_1, \frac{\omega}{cs_o}) \mathcal{G}(\vec{p}_o, \frac{\omega}{cs_o}) \times \int_{\text{lens}} d\vec{p}_1 \mathcal{G}(|\vec{p}_1|, \frac{\omega}{c} \left(\frac{1}{s_o} + \frac{1}{s_i} - \frac{1}{R}\right) e^{-i\varphi/2}),
\]

where \(s_o, s_i,\) and \(R\) are the magnification of the imaging system. Replacing \(s_o, s_i,\) and \(R\) by the imaging system,

\[
\mathcal{G}(\vec{p}_1, \frac{\omega}{cs_o}) \mathcal{G}(\vec{p}_o, \frac{\omega}{cs_o}) \times \int_{\text{lens}} d\vec{p}_1 \mathcal{G}(|\vec{p}_1|, \frac{\omega}{c} \left(\frac{1}{s_o} + \frac{1}{s_i} - \frac{1}{R}\right) e^{-i\varphi/2}),
\]

The image plane is defined by the Gaussian thin-lens equation of Eq. (1); hence, the integral in Eq. (6) simplifies and gives, for a finite sized lens of radius \(R\), the so called point-spread function of the imaging system:

\[
\text{somb}(x) = \frac{2J_1(x)}{x} \quad \text{with } x = \frac{R \omega}{s_o c} \left|\vec{p}_o + \frac{\vec{p}_i}{\mu}\right|.
\]

Here, again, \(J_1(x)\) is the first-order Bessel function, and \(\mu = s_i/s_o\) is the magnification of the imaging system. Replacing the somb-function with the integral, we have

\[
g_{\vec{p}_2}(\vec{p}_1, z_i, t_i) = \frac{-i\omega^2}{(2\pi c)^2 s_o s_i} e^{i\varphi} \mathcal{G}(\vec{p}_1, \frac{\omega}{cs_o}) G(\vec{p}_o, \frac{\omega}{cs_o}) \times \text{somb} \left(\frac{R \omega}{s_o c} \left|\vec{p}_o + \frac{\vec{p}_i}{\mu}\right|\right).
\]

For a large sized imaging lens, the somb-function can be approximated as \(\delta(\vec{p}_o + \vec{p}_i/\mu)\),

\[
g_{\vec{p}_2}(\vec{p}_1, z_i, t_i) = \frac{-i\omega^2}{(2\pi c)^2 s_o s_i} \delta \left(\vec{p}_o + \frac{\vec{p}_i}{\mu}\right),
\]

indicating a point-to-point propagator: the field \(E(\vec{p}_o, z_o, t_o)\) that is emitted from a point \((\vec{p}_o, z_o)\) in the object plane must collapse or stop at a unique point in the image plane \((\vec{p}_i, z_i)\). In Eq. (9), we have absorbed all phases into \(\varphi_s\).

After the above point-to-spot propagation of Eq. (8) or the point-to-point propagation of Eq. (9), the intensity distribution of the radiation field \(E(\vec{p}_o, s_o + s_i, t_i) = E(\vec{p}_o)g_{\vec{p}_2}(\vec{p}_1, z_i, t_i)\) in the image plane becomes

\[
\langle|E(\vec{p}_o)g_{\vec{p}_2}(\vec{p}_1, z_i, t_i)|^2\rangle \propto \text{somb}^2 \left(\frac{R \omega}{s_o c} \left|\vec{p}_o + \frac{\vec{p}_i}{\mu}\right|\right),
\]

for an imaging lens with finite diameter \(D\), and

\[
\langle|E(\vec{p}_o)g_{\vec{p}_2}(\vec{p}_1, z_i, t_i)|^2\rangle \propto \delta \left(\vec{p}_o + \frac{\vec{p}_i}{\mu}\right),
\]

for a large sized imaging lens as an approximation. It is interesting to find that, although the field \(E(\vec{p}_o)\) may propagate to all possible directions and may take all possible optical paths with different optical delays to arrive at the image plane, the point-to-point propagator forces it to collapse or stop at a unique spot around \(\vec{p}_i\) of the image plane.

This point-to-spot propagator is the result of a superposition that sums all subfields originating from point \(\vec{p}_o\) and passing through each and all points \(\vec{p}_i\) of the imaging lens along all possible optical paths with different optical delays. These subfields interfere constructively at a unique spot (approximated as a point) on the image plane while their optical path lengths are equal, and they interfere destructively at all other points on the image plane while their optical path lengths are unequal. The imaging lens makes this constructive-destructive interference possible. In Dirac’s language: a photon coming from \(\vec{p}_o\) may pass the imaging lens to all possible directions and may take all possible optical paths with different optical delays to arrive at the image plane; these quantum amplitudes are superposed constructively at a unique spot (approximated as a point) of the image plane while their optical path lengths are equal, and they are superposed destructively at all other points of the image plane while their optical path lengths are unequal. The constructive-destructive superposition of these quantum probability
amplitudes forces the photon to stop at a unique spot around point \( \vec{\rho} \) on the image plane.

(II) The classical image: a convolution between the aperture function and the point-spread image-forming function.

Examining the simplified experimental setup of Fig. 3, again, the measured intensity \((I(\vec{\rho}_i, z_i, t_i))\) in the image plane is calculated as follows:

\[
I(\vec{\rho}_i, z_i, t_i) = |\langle E(\vec{\rho}_i, s_o + s_i) \rangle |^2
= \left| \int_{\text{obj}} d\vec{\rho}_o E(\vec{\rho}_o) g(\vec{\rho}_i, \vec{\rho}_o, z_i, t_i) \right|^2
= \left| \int_{\text{obj}} d\vec{\rho}_o A(\vec{\rho}_o) g(\vec{\rho}_i, \vec{\rho}_o, z_i, t_i) g_{\text{lens}}(\vec{\rho}_i, z_i, t_i) \right|^2
= \left| \int_{\text{obj}} d\vec{\rho}_o A(\vec{\rho}_o) \int_{\text{lens}} d\vec{\rho}_i g(\vec{\rho}_i, \vec{\rho}_o, z_i, t_i) \right|^2. \tag{12}
\]

Assuming Fresnel near-field propagation, the following Gaussian propagators will be able to propagate each and all fields \( E(\vec{\rho}_o) \) from the object plane to the image plane and superpose at each point, \( (\vec{\rho}_i, s_o + s_i) \), of the image plane,

\[
E(\vec{\rho}_i, s_o + s_i) = \int_{\text{obj}} d\vec{\rho}_o \int_{\text{lens}} d\vec{\rho}_i A(\vec{\rho}_o) \left[ -\frac{i \omega}{2 \pi c} \right] \frac{e^{i \vec{\rho}_o \cdot \vec{\rho}_i}}{s_o} \frac{\omega}{c s_o} G\left( \frac{\vec{\rho}_i - \vec{\rho}_o}{s_o}, \frac{\omega}{c s_o} \right) \times \left[ G\left( \frac{\vec{\rho}_i}{s_i} - \frac{\omega}{cf} \right) \right] \left[ -\frac{i \omega}{2 \pi c} \right] \frac{e^{i \vec{\rho}_i \cdot \vec{\rho}_o}}{s_i} \frac{\omega}{c s_i} G\left( \frac{\vec{\rho}_i - \vec{\rho}_o}{s_i}, \frac{\omega}{c s_i} \right), \tag{13}
\]

which is the same as Eq. (5), except the integral of \( \vec{\rho}_o \).

Applying the properties of the Gaussian function, Eq. (13) can be simplified into the following form:

\[
E(\vec{\rho}_i, z = s_o + s_i) = -\frac{\omega^2}{(2\pi c)^2 s_o s_i} G\left( \frac{\vec{\rho}_i}{s_o}, \frac{\omega}{c s_o} \right) \int_{\text{obj}} d\vec{\rho}_o A(\vec{\rho}_o) G\left( \frac{\vec{\rho}_o}{s_o}, \frac{\omega}{c s_o} \right) \times \int_{\text{lens}} d\vec{\rho}_i G\left[ \frac{\vec{\rho}_i}{s_i}, \frac{\omega}{c} \left( \frac{1}{s_o} + \frac{1}{s_i} \right) \right] e^{-\frac{1}{2} \left( \frac{\vec{\rho}_i^2}{s_o^2} + \frac{\vec{\rho}_i^2}{s_i^2} \right)} \vec{\rho}_i. \tag{14}
\]

The image plane is defined by the Gaussian thin-lens equation of Eq. (1); hence, the second integral simplifies and gives, for a finite sized lens of radius \( R \), the so-called point-spread function of the imaging system:

\[
E(\vec{\rho}_i, z = s_o + s_i) = -\frac{\omega^2}{(2\pi c)^2 s_o s_i} G\left( \frac{\vec{\rho}_i}{s_o}, \frac{\omega}{c s_o} \right) \int_{\text{obj}} d\vec{\rho}_o A(\vec{\rho}_o) G\left( \frac{\vec{\rho}_o}{s_o}, \frac{\omega}{c s_o} \right) \times \text{somb} \left( \frac{R \omega}{s_o c}, \frac{\vec{\rho}_o}{\mu} \right), \tag{15}
\]

where, again, \( \text{somb}(x) = 2J_1(x)/x \) and \( J_1(x) \) is the first-order Bessel function. The somb-function is the result of an integral that sums all subfields emitted from \( \vec{\rho}_o \) and passing through each and all points, \( \vec{\rho}_i \), of the lens along all possible optical paths with different optical delays. These subfields interfere constructively when their optical path lengths are equal and interfere destructively when their optical path lengths are unequal. The image observed from the measurement of \((I(\vec{\rho}_i, z_i))\) is thus

\[
(I(\vec{\rho}_i)) \propto \left| \int_{\text{obj}} d\vec{\rho}_o A(\vec{\rho}_o) \text{somb} \left( \frac{R \omega}{s_o c}, \frac{\vec{\rho}_o + \vec{\rho}_i}{\mu} \right) \right|^2
= \int_{\text{obj}} d\vec{\rho}_o |A(\vec{\rho}_o)|^2 \text{somb}^2 \left( \frac{R \omega}{s_o c}, \frac{\vec{\rho}_o + \vec{\rho}_i}{\mu} \right)
\approx |A(\vec{\rho}_o)|^2 \otimes \delta(\vec{\rho}_o + \vec{\rho}_i/\mu) = |A(\vec{\rho}_i/\mu)|^2, \tag{16}
\]
indicating a reproduction of the aperture function, i.e., a magnified or demagnified image of the object. An image, magnified or demagnified by a factor of \( \mu \), is thus reproduced by an incoherent imaging process, which is mathematically represented by the convolution between the squared modulus of the aperture function of the object and the point-spread function, or the image-forming function. In Eq. (16), we have assumed a large sized imaging lens and approximated the point-spread function \( \delta \)-function, \( \delta(\vec{\rho}_o + \vec{\rho}_i/\mu) \); we have also absorbed the Fresnel phase \( G(\vec{\rho}_o, \frac{\omega}{c \mu}) \) into the complex aperture function, which is reasonable for incoherent imaging.

### 3. Biphonon Ghost Imaging

In this section, we review the physics of a biphonon ghost image, i.e., ghost image produced by a pair of entangled photons. We prove that biphonon ghost imaging is the result of two-photon interference: a pair of entangled photons interferes with the pair itself.

The schematic setup of the 1995 ghost imaging experiment is shown in Fig. 4 [11]. The entangled photon source is a continuous wave (CW) laser beam pumped nonlinear crystal, usually called SPDC. Roughly speaking, the process of SPDC involves sending a pump laser beam in to a nonlinear material, such as a non-centrosymmetric crystal. Occasionally, the nonlinear interaction inside the crystal leads to the annihilation of a high-frequency pump photon and the creation of a pair of entangled lower frequency photons as signal and idler, namely a "biphonon." In this experiment, the nonlinear crystal BBO is cut for degenerate type-II phase matching to produce a pair of orthogonally polarized signal (e-ray of the crystal) and idler (o-ray of the crystal) photons. The pair emerges from the crystal as collinear, with \( \omega_s \approx \omega_i \approx \omega_p/2 \). The pump is then separated from the signal—idler pair by a dispersion prism, and the remaining signal and idler beams are sent in different directions by polarization beam splitting (Thompson prism). The signal beam passes through a convex lens with a 400 mm focal length and illuminates a chosen aperture (mask). As an example, one of the demonstrations used
Fig. 4. Schematic setup of the first ghost imaging experiment of 1995. letters “UMBC” for the object mask. Behind the aperture is the “bucket” detector package \(D_1\), which consists of a short focal length collection lens with an avalanche photodiode in its focal spot. \(D_1\) is mounted in a fixed position during the experiment. The idler beam is met by detector package \(D_2\), which consists of an optical fiber whose output is mated with another avalanche photodiode. The input tip of the fiber is scannable in the transverse plane by two step motors. The output pulses of each detector, which operate in photon-counting mode, are sent to a coincidence circuit for counting the joint-detection event of the signal–idler pair.

By recording the coincidence counts as a function of the fiber tip’s transverse plane coordinates, the image of the chosen aperture (for example, “UMBC”) is observed, as reported in Fig. 5, if the following experimental condition is satisfied: the focal length of the imaging lens \(f\), the aperture’s optical distance from the lens \(s_o\), and the image’s optical distance from the lens \(s_i\) (which is from the imaging lens going backward along the signal photon path to the biphoton source of SPDC crystal then going forward along the path of the idler photon to the image) satisfy the Gaussian thin-lens equation. When choosing \(s_o/s_i = 2\), it is interesting to find that the observed image measures 7 mm × 14 mm while the size of the “UMBC” aperture inserted in the signal beam is only about 3.5 mm × 7 mm. The observed image in the joint detection of \(D_1\) and \(D_2\) is magnified exactly by a factor of \(m = s_o/s_i = 2\). In the measurement of Fig. 5, \(s_o\) was chosen to be \(s_o = 600\) mm, and the twice magnified clear image was found when the fiber tip was on the plane of \(s_i = 1200\) mm. While \(D_2\) was scanned on other transverse planes other than that of \(s_i = 1200\) mm, the images blurred out.

The first reaction of many people may be negative: “Absolutely impossible!” (Note: this experiment was rejected by the referees of Physical Review Letters.) Examining the two observers \(D_1\) and \(D_3\) \(D_1\) is in the signal beam behind the object; however, it is a “bucket” detector, so it does not have any ability to image the object. What \(D_1\) can do is simply counting the signal photons; whenever a photoelectron is created by a signal photon, \(D_1\) sends an electronic pulse to open the “gate” of the coincidence circuit for \(D_2\). \(D_2\), however, is placed in the idler beam, so how could \(D_2\) “see” the object that is in the signal beam? The measurement of the signal and the idler subsystem may further strengthen the negative reaction. In fact, the single photon-counting rate of \(D_1\) and \(D_2\) was recorded during the scanning of the image and was found fairly constant in the entire region of the image. The counting rate of neither \(D_1\) nor \(D_2\) is a function of the fiber tip transverse coordinates.

The EPR-correlation \(\delta\)-functions, \(\delta(\vec{r}_s - \vec{r}_i)\) and \(\delta(\vec{k}_s + \vec{k}_i)\) of SPDC in transverse dimension, are the key to understand this interesting phenomenon. In a degenerate SPDC, although the signal–idler photon pair has equal probability to be emitted from any points on the output surface of the nonlinear crystal, the transverse position \(\delta\)-function indicates that, if one of them is observed at one position, the other one must be found at the same position. In other words, the pair is always emitted from the same point on the output plane of the biphoton source. The transverse momentum \(\delta\)-function defines the angular correlation of the signal–idler pair: the transverse momenta of a signal–idler amplitude are equal but pointed in opposite directions, \(\vec{k}_s = -\vec{k}_i\), i.e., the signal–idler pair is always existing at roughly equal yet opposite angles relative to the pump. This then allows for a simple explanation of the experiment in terms of “usual” geometrical optics in the following manner: we envision the nonlinear crystal as a “hinge point” and “unfold” the schematic of Fig. 4 into that shown in Fig. 6. The signal–idler biphoton amplitudes can then be represented by straight lines (but keep in mind the different propagation directions); therefore, the image is well-produced in coincidences when the aperture, lens, and fiber tip are located according to the Gaussian thin-lens equation. The image is exactly the same as one would observe on a screen placed at the fiber tip if detector \(D_1\) was replaced by a point-like light source and the nonlinear crystal by a reflecting mirror.

Fig. 5. (a) A reproduction of the actual aperture “UMBC” placed in the signal beam. (b) The image of “UMBC”: coincidence counts as a function of the fiber tip’s transverse plane coordinates. The step size is 0.25 mm. The image shown is a “slice” at the half-maximum value.
observing a joint-photodetection event at space-time points According to the quantum theory of light, the probability of and the ghost image plane. Constructively at this unique pair of points on the object plane of these biphoton amplitudes. It is the imaging lens that made the image thin-lens equation:

$$1 \over s_o + 1 \over s_i = 1 \over f.$$ 

Although the placement of the lens, the object, and the detector $D_2$ obeys the Gaussian thin-lens equation, it is important to remember that the geometric rays in the figure actually represent the biphoton amplitudes of an entangled signal-idler pair. The point-to-point correspondence is the result of the superposition of these biphoton amplitudes.

Following a similar analysis in geometric optics, it is not difficult to find that any geometrical “light spot” on the subject plane, which is the intersection point of all possible signal–idler amplitudes coming from the entangled biphoton source, corresponds to a unique geometrical “light spot” on the image plane, which is another intersection point of all the possible signal–idler amplitudes. This point-to-point correspondence made the “ghost” image of the subject-aperture possible. Despite the completely different physics from classical geometrical optics, the remarkable feature is that the relationship of the focal length $f$; the aperture’s optical distance from the lens $s_o$, and the image’s optical distance from the lens $s_i$ satisfies the Gaussian thin-lens equation:

where $\omega_j, k_j (j = s, i, p)$ are the frequency and wavevector of the signal ($s$), idler ($i$), and pump ($p$), respectively, $a_s^\dagger$ and $a_i^\dagger$ are creation operators for the signal and the idler photon, respectively, and $\Psi_0$ is a normalization constant. In general, the field operator $\hat{E}^+(\mathbf{r}, t)$ at space-time point $(\mathbf{r}, t)$ can be written in terms of the Green’s function, which propagates each Fourier mode from space-time point $(\mathbf{r}_s, t_s)$ of the source to $(\mathbf{r}, t)$ of the photodetection:

$$\hat{E}^+(\mathbf{r}, t) = \sum_k \hat{E}^+(\mathbf{k}, \mathbf{r}, t_s)g(\mathbf{k}, \mathbf{r} - \mathbf{r}_s, t - t_s).$$

To calculate the field operators, we follow the unfolded experimental setup shown in Fig. 7 to establish Green’s functions $g(\mathbf{k}_s, \omega_s, \mathbf{p}_s, z_s)$ and $g(\mathbf{k}_o, \omega_o, \mathbf{p}_o, z_o)$. In arm-1, the signal propagates freely over a distance $d_1$ from the output plane of the source to the imaging lens, then passes an object aperture at distance $s_o$, and then is focused onto a photon-counting detector $D_1$ by a collection lens. We will evaluate $g(\mathbf{k}_s, \omega_s, \mathbf{p}_s, z_s)$ by propagating the field from the output plane of the biphoton source to

$$G^{(2)}(\mathbf{r}_1, t_1; \mathbf{r}_2, t_2)$$

$$= \langle \Psi | \hat{E}^-(\mathbf{r}_1, t_1) \hat{E}^-(\mathbf{r}_2, t_2) \hat{E}^+(\mathbf{r}_2, t_2) \times \hat{E}^+(\mathbf{r}_1, t_1) | \Psi \rangle,$$

(17)

where $\hat{E}^-(\mathbf{r}_j, t_j)$ and $\hat{E}^+(\mathbf{r}_j, t_j), j = 1, 2$, are the negative-frequency and the positive-frequency field operators of the detection event at space-time point $(\mathbf{r}_j, t_j)$; $| \Psi \rangle$ is the quantum state of the signal–idler photon pair,

$$| \Psi \rangle = \Psi_0 \sum_{s, i} \delta(\omega_s + \omega_i - \omega_p)\delta(k_s + k_i - k_p)a_s^\dagger(a_i^\dagger a_p^\dagger | 0 \rangle,$$

(18)

Fig. 7. In arm-1, the signal propagates freely over a distance $d_1$ from the output plane of the source to the imaging lens, then passes an object aperture at distance $s_o$, and then is focused onto a photon-counting detector $D_1$ by a collection lens. In arm-2, the idler propagates freely over distance $d_2$ from the output plane of the source to a point-like photon-counting detector $D_2$. 

$\text{Fig. 6.}$ An unfolded setup of the “ghost” imaging experiment, which is helpful for understanding the physics. Since the biphoton “light” propagates along “straight-lines,” it is not difficult to find that any geometrical light point on the subject plane corresponds to a unique geometrical light point on the image plane. Thus, a “ghost” image of the subject is made nonlocally in the image plane. Although the placement of the lens, object, and detector $D_2$ obeys the Gaussian thin-lens equation, it is important to remember that the geometric rays in the figure actually represent the biphoton amplitudes of an entangled signal-idler pair. The point-to-point correspondence is the result of the superposition of these biphoton amplitudes.
Here, we treat it as a thin lens:

\[ g(\vec{k}_0, \omega; \vec{p}_o, z_o) = d_1 + \frac{s_o}{\alpha} \]

where \( \vec{p}_S \) and \( \vec{p}_I \) are the transverse vectors defined, respectively, on the output plane of the source and on the plane of the imaging lens. The terms in the first and third bracket pairs in Eq. (20) describe free-space propagation from the output plane of the source to the imaging lens and from the imaging lens to the object plane, respectively. The function \( G(\vec{p}_l, \frac{\alpha}{c \delta}) \) in the second bracket pair is the transformation function of the imaging lens. Here, we treat it as a thin lens: \( G(\vec{p}_l, \frac{\alpha}{c \delta}) \cong \exp(-i\omega|\vec{p}_l|^2/2c\delta). \)

(II) Arm-2 (from source to ghost image).

In arm-2, the idler propagates freely from the source to the plane of \( D_2 \), which is also the plane of the image. Green’s function is thus

\[ g(\vec{k}_0, \omega; \vec{p}_2, z_2 = d_2) \]

\[ = \frac{-i\omega}{2\pi c d_2} e^{\frac{i\omega}{c \delta}} \int_{\text{source}} d\vec{p}_S G\left( |\vec{p}_S - \vec{p}_2|, \frac{\alpha}{c \delta} \right) e^{i\vec{k}_0 \cdot \vec{p}_S}, \]

where \( \vec{p}_S \) and \( \vec{p}_2 \) are the transverse vectors defined, respectively, on the output plane of the source and on the plane of photodetector \( D_2 \).

(III) \( \Psi(\vec{p}_o, \vec{p}_i) \) (object plane–ghost image plane).

Now we introduce the concept of the biphoton effective wavefunction\(^1\) that is defined from the second-order coherence function \( G^{(2)}(\vec{r}_1, t_1; \vec{r}_2, t_2) \),

\[ G^{(2)}(\vec{r}_1, t_1; \vec{r}_2, t_2) = |\langle 0| \hat{E}^{(+)}(\vec{r}_2, t_2) \hat{E}^{(+)}(\vec{r}_1, t_1) |\Psi\rangle|^2 \]

\[ = |\Psi(\vec{r}_1, t_1; \vec{r}_2, t_2)|^2, \]

where \( \Psi(\vec{r}_1, t_1; \vec{r}_2, t_2) \) is the biphoton effective wavefunction. To evaluate \( \Psi(\vec{r}_1, t_1; \vec{r}_2, t_2) \), we need to propagate the field operators from the source to the space-time coordinates \( (\vec{r}_1, t_1) \) and \( (\vec{r}_2, t_2) \). The two-photon effective wavefunction \( \Psi(\vec{p}_1, z_1, t_1; \vec{p}_2, z_2, t_2) \) is thus calculated as

\[ \Psi(\vec{p}_1, z_1, t_1; \vec{p}_2, z_2, t_2) \]

\[ = \int_0^1 d\omega' d\vec{k}' g(\vec{k}_0', \omega'; \vec{p}_2, z_2) e^{-i\omega' t_1; \hat{a}(\omega', \vec{k}')} \]

\[ \times \int d\omega'' d\vec{k}'' g(\vec{k}_0'', \omega''; \vec{p}_1, z_1) e^{-i\omega'' t_2; \hat{a}(\omega'', \vec{k}'')} \]

\[ \times \hat{a}_1^\dagger(\omega'' - \omega', \vec{k}_s + \vec{k}_i) a_2(\omega'', \vec{k}_s) \]

\[ \times \hat{a}_1(\omega - \omega', \vec{k}_s) \]
\[ \Psi(\vec{p}_o, \vec{p}_i) \propto \int_{\text{lens}} d\vec{p}_i G(|\vec{p}_i - \vec{p}_o|, \frac{\omega}{c s_i}) G(|\vec{p}_i|, \frac{\omega}{c f}) \times G(|\vec{p}_i - \vec{p}_a|, \frac{\omega}{c s_o}). \] (27)

Although the signal and idler may propagate to any different directions along two optical arms, interestingly, Eq. (27) looks similar to that of a classical imaging setup, if we imagine that the fields start propagating from a point \( \vec{p}_o \) on the object plane to the lens and then stop at point \( \vec{p}_o \) on the imaging plane. The mathematics is consistent with our previous qualitative analysis of the experiment.

The integral on \( d\vec{p}_i \) yields a point-to-point relationship between the object plane and the image plane that is defined by the Gaussian thin-lens equation:

\[ \int_{\text{lens}} d\vec{p}_i G(|\vec{p}_i|, \frac{\omega}{c s_i + \frac{1}{s_i} - \frac{1}{f}}) e^{-i\frac{\pi}{c} \vec{p}_i \cdot \vec{p}_o} \propto \delta\left(\vec{p}_o + \frac{\vec{p}_i}{m}\right). \] (28)

where the integral is approximated to infinity and the Gaussian thin-lens equation of \( 1/s_i + 1/s_o = 1/f \) is applied. We have also defined \( m = s_i/s_o \) as the magnification factor of the imaging system. The function \( \delta(\vec{p}_o + \vec{p}_i/m) \) indicates that a point of \( \vec{p}_o \) on the object plane corresponds to a unique point of \( \vec{p}_i \) on the image plane. The two vectors point to opposite directions, and the magnitudes of the two vectors hold a ratio of \( m = |\vec{p}_i|/|\vec{p}_o| \).

If the finite size of the imaging lens has to be taken into account (finite diameter \( D \)), the integral yields a point-spread function of \( \text{somb}(x) \):

\[ \int_{\text{lens}} d\vec{p}_i e^{-i\frac{\pi}{c} \vec{p}_i \cdot \vec{p}_o} \propto \text{somb}\left[\frac{R}{s_o c} \left(\vec{p}_o + \frac{\vec{p}_i}{m}\right)\right]. \] (29)

where, again, \( \text{somb}(x) = 2j_1(x)/x, j_1(x) \) is the first-order Bessel function, and \( R/s_o \) is named as the numerical aperture. The point-spread function turns the point-to-point correspondence between the object plane and the image plane into a point-to-“spot” relationship and thus limits the spatial resolution. This point is discussed in detail in the last section.

Therefore, by imposing the condition of the Gaussian thin-lens equation, the transverse biphoton effective wavefunction is approximated as a \( \delta \) function,

\[ \Psi(\vec{p}_o, \vec{p}_i) \propto \delta\left(\vec{p}_o + \frac{\vec{p}_i}{m}\right). \] (30)

where \( \vec{p}_o \) and \( \vec{p}_i \), again, are the transverse coordinates on the object plane and the image plane, respectively, defined by the Gaussian thin-lens equation. Thus, the second-order spatial coherence function, or correlation function, \( G^{(2)}(\vec{p}_o, \vec{p}_i) \) turns to be

\[ G^{(2)}(\vec{p}_o, \vec{p}_i) = |\Psi(\vec{p}_o, \vec{p}_i)|^2 \propto \delta\left(\vec{p}_o + \frac{\vec{p}_i}{m}\right)^2. \] (31)

Equation (31) indicates a point-to-point EPR correlation between the object plane and the image plane, i.e., if one observes the signal photon at a position of \( \vec{p}_o \) on the object plane, the idler photon can only be found at a unique position of \( \vec{p}_i \) on the image plane satisfying \( \delta(\vec{p}_o + \vec{p}_i/m) \) with \( m = s_i/s_o \).

We now include an object-aperture function, a collection lens, and a photon-counting detector \( D_1 \) into the optical transfer function of arm-1 as shown in Fig. 4.

First, we treat the collection-lens-\( D_1 \) package as a “bucket” detector. The “bucket” detector integrates all \( \Psi(\vec{p}_o, \vec{p}_i) \) that pass the object aperture \( A(\vec{p}_o) \) as a joint-photodetection event. This process is equivalent to the following convolution:

\[ R_{1,2} \propto \left| \int_{\text{obj}} d\vec{p}_i A(\vec{p}_o) \Psi(\vec{p}_o, \vec{p}_i) \right|^2 \propto \left| A(\vec{p}_o) \otimes \delta\left(\vec{p}_o + \frac{\vec{p}_i}{m}\right) \right|^2 = \left| A\left(-\frac{\vec{p}_i}{m}\right) \right|^2, \] (32)

where \( R_{1,2} \) is the coincidence counting rate of \( D_1 \) and \( D_2 \) \( \otimes \) means convolution, and again, \( D_2 \) is scanning in the image plane, \( \vec{p}_2 = \vec{p}_i \). Equation (32) indicates a magnified (or demagnified) image of the object-aperture function by means of the joint-detection events between distant photodetectors \( D_1 \) and \( D_2 \).

The “-” sign in \( A(-\vec{p}_i/m) \) indicates opposite orientation of the image. The model of the “bucket” detector is a good and realistic approximation.

Second, we calculate Green’s function from the source to \( D_1 \) in detail by including the object-aperture function, the collection lens, and the photon-counting detector \( D_1 \) into arm-1. Green’s function of Eq. (20) becomes

\[ g(\vec{x}_o, \omega_o, \vec{p}_1, \vec{z}_1 = 1 + s_o + f_{\text{coll}}) \]

\[ = e^{i\omega z_1} \int_{\text{obj}} d\vec{p}_i \int_{\text{lens}} d\vec{p}_1 \int_{\text{source}} d\vec{p}_s \left[ -i\frac{\omega_s}{2\pi c d_1} \right] e^{i\vec{p}_s \cdot \vec{p}_1} \times G\left(|\vec{p}_s - \vec{p}_i|, \frac{\omega}{c f_1}\right) \times G\left(|\vec{p}_1|, \frac{\omega}{c f_{\text{coll}}}\right) \times G\left(|\vec{p}_s|, \frac{\omega}{c f_{\text{coll}}}\right) \]

\[ = \left| A(\vec{p}_o) \otimes \delta\left(\vec{p}_o + \frac{\vec{p}_i}{m}\right) \right|^2, \] (33)

where \( f_{\text{coll}} \) is the focal length of the collection lens and \( D_1 \) is placed at the focal point of the collection lens. Repeating the previous calculation, we obtain the transverse biphoton effective wavefunction:

\[ \Psi(\vec{p}_1, \vec{p}_2) \propto \int_{\text{obj}} d\vec{p}_o A(\vec{p}_o) \delta\left(\vec{p}_o + \frac{\vec{p}_i}{m}\right) \]

\[ = A(\vec{p}_o) \otimes \delta\left(\vec{p}_o + \frac{\vec{p}_i}{m}\right), \] (34)
which is the same as Eq. (32). Notice, in Eq. (34) we have ignored the phase factors, which have no contribution to the formation of the image. The joint-detection counting rate, $R_{1,2}$, between photon-counting detectors $D_1$ and $D_2$ is thus

$$R_{1,2} \propto G^{(2)}(\vec{\rho}_1, \vec{\rho}_2) \propto |A(\vec{\rho}_o) \otimes \delta(\vec{\rho}_o + \vec{\rho}_2) m|^2 = |A(\vec{\rho}_2)/m|^2,$$

(35)

where, again, $\vec{\rho}_2 = \vec{\rho}_1$.

As we discussed earlier, the point-to-point EPR correlation is the result of the coherent superposition of the biphoton probability amplitudes. In principle, one signal–idler pair contains all the necessary biphoton probability amplitudes that produce the ghost image. We name this kind of image the two-photon coherent image to distinguish the two-photon incoherent image of thermal light.

4. Thermal Light Ghost Imaging

In this section we discuss the physics of thermal light ghost imaging. (Note: To avoid any possible confusion with the quantum interference picture of ghost imaging, again, this article will not review or discuss the classical simulations of ghost imaging, such as the "speckle-to-speckle" image-forming function. The review and discussion on the physics and applications of classical simulations of ghost imaging can be found in other contributions of this special issue. The author sincerely apologizes for this again.) Similar to biphoton ghost imaging, thermal light ghost imaging is also produced by two-photon interference. Different from biphoton ghost imaging, the two photons are in the thermal state. We will prove that the thermal light ghost image-forming function is the result of a pair of randomly created and randomly paired photons interfering with the pair itself.

Ten years after the experimental demonstration of ghost imaging with entangled photon pairs, Valencia et al. found that the near-field, natural, non-factorizable, point-to-point image-forming correlation is not only the property of entangled photon pairs\[8\]. Two-photon interference of randomly created and randomly paired photons in the thermal state can produce a similar point-to-point image-forming correlation without the use of any imaging lens. This type of ghost imaging technique is commonly named lensless ghost imaging. With no lens requirement, in addition to visible light, the two-photon interference produced image-forming correlation is well-suited for these radiations for which no effective imaging lenses are available.

Lensless Fresnel near-field ghost imaging with pseudo-thermal radiation was demonstrated by Wang et al.[10], Zhu et al.[9], Valencia et al.[5], D’Angolo et al.[16], and Scarcelli et al.[11] in the years from 2004 to 2006. (Note: Fresnel near-field is not “near-surface-field.” The Sun has an angular size of $\sim 0.5^\circ$ relative to Earth and is in the Fresnel near-field.) Figure 8 is the schematic experimental setup of Scarcelli’s 2006 demonstration. A pseudo-thermal radiation with a narrow spectral bandwidth, $\Delta \omega$, and a fairly large bandwidth of spatial frequency, $\Delta k_y$, resulting from a large angular sized disk-like source, is divided into two by an optical beam splitter. The reflected light is propagated and focused onto a point-like photodetector $D_2$ (bucket detector) after passing through an object mask, which is a simple double-slit in Fig. 8. $D_2$ is fixed in the focal plane of the focusing convex lens. It is clear that $D_2$, known as a bucket detector, cannot retrieve any information about the spatial distribution or the aperture function of the object mask. The transmitted light is freely propagated to the plane of $x_1$ to be detected by the scanning point-like photodetector $D_1$. The joint detection between $D_1$ and $D_2$ is realized either by a photon-counting-coincidence counter or by a standard Hanbury Brown and Twiss (HBT)-type

![Fig. 8. Lensless Fresnel near-field ghost imaging with pseudo-thermal light demonstrated in 2006 by Scarcelli et al.[11]. D1 is a point-like photodetector that is scanable along the x1-axis. The joint detection between D1 and D2 is realized either by a photon-counting coincidence counter or by a standard HBT linear multiplier (RF mixer). In this measurement, D2 is fixed in the focal point of a convex lens, playing the role of a bucket detector. The counting rates or the photocurrents of D1 and D2, respectively, are measured to be constants. Surprisingly, an image of the 1-D object is observed in the joint detection between D1 and D2 by scanning D1 in the plane of z1 = z2 along the x1-axis. The image is blurred out when z1 $\neq$ z2. There is no doubt that thermal radiations propagate to any transverse plane in a random and chaotic manner. There is no lens applied to force the thermal radiation “collapsing” to a point or speckle either. What is the physical cause of the point-to-point image-forming correlation in coincidences?](image)

![Fig. 9. The experimentally observed ghost image of a double-slit. The lensless ghost image is observed to have equal size to that of the object. The ghost image has 50% contrast if measured by either a photon-counting coincidence circuit or by a standard HBT-type analog correlation circuit. If the measurement is for photon number fluctuation correlation or intensity fluctuation correlation, the visibility of the ghost image is close to 100%.](image)
sured image of the double-slit in photon number fluctuation of the original near-field lensless ghost imaging experiment [12]. Figure 11 shows their observed ghost image. This experiment Figure 10 schematically illustrates their experimental setup.

number fluctuation correlation or intensity fluctuation correlation or intensity correlation measurements. However, if the measurement is for photon number correlation or intensity correlation of a universal constant \(z\) that is observed in the joint detection when \(z_1 = z_2\) along the \(x_1\)-axis. The image contrast measures almost 50%, which is the maximum contrast we can expect from photon number fluctuation correlation or intensity correlation measurements. However, if the measurement is for photon number fluctuation correlation or intensity fluctuation correlation, the image contrast can be \(\sim 100\%\). Figure 9 reports the measured image of the double-slit in photon number fluctuation correlation.

In 2008, Meyers et al. published a modified version of the original near-field lensless ghost imaging experiment [12]. Figure 10 schematically illustrates their experimental setup. Figure 11 shows their observed ghost image. This experiment improves the ghost imaging experiment of Scarcelli et al. [11] in two aspects: (1) the bucket detector is not triggered by the transmitted light that passes through a mask but instead is triggered by the randomly scattered and reflected photons from the surface of a toy soldier; (2) no scanning point photodetector, but instead a CCD array operated at photon-counting regime, is used for the joint detection with the bucket detector \(D_2\). A ghost image of the toy soldier is captured by the gated CCD when taking \(z_1 = z_2\). The spatial resolution of the ghost image is determined by the angular size of the thermal source.

The above ghost images are either observed from the photon number fluctuation correlation \(\langle \Delta n(r_1, t_1) \Delta n(r_2, t_2) \rangle\) by using photon-counting-coincidence circuits, or from the intensity fluctuation correlation \(\langle I(r_1, t_1)I(r_2, t_2) \rangle\) by using standard HBT-type analog correlation circuit. Photon number fluctuation correlation is part of the photon number correlation \(\langle n(r_1, t_1)n(r_2, t_2) \rangle\). Intensity fluctuation correlation is part of the intensity correlation \(\langle I(r_1, t_1)I(r_2, t_2) \rangle\):

\[
\langle n(r_1, t_1)n(r_2, t_2) \rangle = \langle n(r_1, t_1) \rangle \langle n(r_2, t_2) \rangle + \langle \Delta n(r_1, t_1) \Delta n(r_2, t_2) \rangle,
\]

\[
\langle I(r_1, t_1)I(r_2, t_2) \rangle = \langle I(r_1, t_1) \rangle \langle I(r_2, t_2) \rangle + \langle \Delta I(r_1, t_1) \Delta I(r_2, t_2) \rangle,
\]

(36)

where \(n(r_1, t_1)\) and \(n(r_2, t_2)\), \([I(r_1, t_1)\) and \(I(r_2, t_2)\)] are the instantaneous photon numbers (intensities) measured at space-time coordinates \((r_1, t_1)\) and \((r_2, t_2)\), respectively. Photon number correlation (intensity correlation) is a measure of the second-order coherence of the field \(G_2(r_1, t_1; r_2, t_2)\). To calculate the photon number fluctuation (intensity fluctuation) correlation of the thermal field, we introduce Einstein’s picture of light.

In 1905, Einstein introduced a granularity picture to radiation, abandoning the continuum interpretation of Maxwell [13]. This led to a microscopic picture of radiation and a statistical view of light. Although Einstein did not name his “bundle of ray,” which is an English translation from German “strahlbündel,” a photon, at the heart of Einstein’s theory is the particle picture of radiation. Einstein assumed that the radiation energy is quantized into localized bundles as light quanta. The energy of the bundle or light quantum is initially localized in a small volume of space and remains localized as it moves away from the radiation source with velocity \(c\). The energy of the bundle or light quantum is related to its frequency by multiplication of a universal constant \(h\). In the photoelectric process, one bundle of energy \(E = h\nu\), or one photon, is completely absorbed by one electron originally bound with the metal. In Einstein’s picture, a natural light source, such as the Sun or a distant star, consists of many point-like sub-sources, each of which emits their own subfields that carry energy \(E = h\nu\) in a random manner: the \(m\)th subfield (photon) that is emitted from the \(m\)th point-like sub-source (an atomic transition in which the atom changes its state from higher energy level \(E_2\) to lower energy level \(E_1\) and releases a photon with energy \(h\nu = E_2 - E_1\) may propagate in all possible directions with all possible random phases. Einstein’s concept of “bundle of ray” or “subfield” refers a quantized microscopic realistic substance of the electromagnetic field, corresponding to the quantum mechanical concept of photon. In Einstein’s picture, the radiation measured at coordinate \((r, t)\) is the result of a superposition among a large number of subfields:
where $E_m(r,t)$ is the observed $m$th subfield at space-time $(r,t)$, the subscript index $m$, $m = 1, 2, \ldots, M$, labels the $m$th subfield that is created from the $m$th point-like sub-source (atomic transition) at space-time coordinate $(r_m, t_m)$ of the $m$th sub-source with energy $\hbar \omega_m = E_m - E_1$. $M$ is the total number of quantized subfields contributed to the measurement. In the following discussion, we will simplify the expression of sum from $\sum_{m=1}^{M}$ to $\sum_{m}$, and calculate for one polarization of the field.

Assuming a light source of finite size that contains a large number of independent and randomly radiating point-like sub-sources, such as spontaneous atomic transitions, the radiation fields $E(r_1, t_1)$ and $E(r_2, t_2)$, measured at photodetectors $D_1$ and $D_2$, respectively, are the result of superposition among a large number of subfields, labeled $E_m(r_m, t_m)$, originating from each of these independent sub-sources. The measured photon number correlation (intensity correlation) is thus

\[
\langle n(r_1, t_1) n(r_2, t_2) \rangle \propto \langle I(r_1, t_1) I(r_2, t_2) \rangle
\]

\[
= \left( \sum_{m,n,p,q} E_m^*(r_1, t_1) E_n(r_1, t_1) E_p(r_2, t_2) E_q(r_2, t_2) \right)
\]

\[
= \left( \sum_{m \neq n} E_m^*(r_1, t_1) E_n(r_1, t_1) \sum_{n} E_n^*(r_2, t_2) E_n(r_2, t_2) \right)
\]

\[
+ \sum_{m = n} E_m^*(r_1, t_1) E_n(r_1, t_1) E_n(r_2, t_2) E_m(r_2, t_2)
\]

\[
= \sum_{m,n} \frac{1}{\sqrt{2}} \left[ E_m^*(r_1, t_1) E_n(r_2, t_2) + E_n^*(r_1, t_1) E_m(r_2, t_2) \right]^2,
\]  

where $m, n, p, q$ label the randomly created and randomly distributed subfields and their corresponding point-like sub-sources. Considering the random phases of the subfields, as a result of an interference cancellation when taking into account all possible random phases of the subfields, the only surviving terms from the ensemble sum are the following: (1) $m = q$ and $n = p$; (2) $m = p$ and $n = q$. Comparing Eq. (36) with Eq. (38), we recognize immediately that

\[
\langle n(r_1, t_1) \rangle \langle n(r_2, t_2) \rangle \propto \langle I(r_1, t_1) I(r_2, t_2) \rangle
\]

\[
= \sum_{m} E_m^*(r_1, t_1) E_m(r_1, t_1) \sum_{n} E_n^*(r_2, t_2) E_n(r_2, t_2)
\]

\[
= \sum_{m \neq n} E_m^*(r_1, t_1) E_n(r_1, t_1) E_n(r_2, t_2) E_m(r_2, t_2).
\]

\[
\langle \Delta n(r_1, t_1) \Delta n(r_2, t_2) \rangle \propto \langle \Delta I(r_1, t_1) \Delta I(r_2, t_2) \rangle
\]

\[
= \sum_{m \neq n} E_m^*(r_1, t_1) E_n(r_1, t_1) E_n(r_2, t_2) E_m(r_2, t_2).
\]

It is interesting to find that the photon number fluctuation (intensity fluctuation correlation) is the sum of the cross terms of the following two-photon superposition:

\[
\frac{1}{\sqrt{2}} \left[ E_m(r_1, t_1) E_n(r_2, t_2) + E_n(r_1, t_1) E_m(r_2, t_2) \right]^2,
\]

for two different yet indistinguishable alternatives for a pair of randomly created and randomly paired subfields, or photons, to produce a joint-photodetection event: (1) the $m$th subfield, or photon, produces a photodetection event at $D_1$, and the $n$th subfield, or photon, produces a photodetection event at $D_2$; (2) the $n$th subfield, or photon, produces a photodetection event at $D_1$, and the $n$th subfield, or photon, produces a photodetection event at $D_2$, namely, two-photon interference, randomly created and randomly paired subfields (photons) interfering with the mirror itself. Examining the “unfolded” lensless ghost imaging experiment schematic setup of Fig. 12, the two-photon interference induced intensity fluctuation correlation measured by the scannable photodetector $D_1$ and the bucket photodetector $D_2$ can be easily calculated from Einstein’s picture.

\[
\langle \Delta n(\vec{r}_1, z_1) \Delta n(\vec{r}_2, z_2) \rangle
\]

\[
= \sum_{m,n} \left[ E_m(\vec{r}_1, z_1) E_m(\vec{r}_2, z_2) \right] \left[ E_n(\vec{r}_1, z_1) E_n(\vec{r}_2, z_2) \right]
\]

\[
\times \left[ \int d\vec{p}_{\vec{r}_1} E_m(\vec{p}_{\vec{r}_1}) g_m^{*}(\vec{p}_{\vec{r}_1}, z_1) \right]
\]

\[
\times \left[ \int d\vec{p}_{\vec{r}_2} E_m(\vec{p}_{\vec{r}_2}) g_m^{*}(\vec{p}_{\vec{r}_2}, z_2) \right]
\]

\[
= \sum_{m} \left[ E_m(\vec{r}_1, z_1) \right] \left[ \int d\vec{p}_{\vec{r}_1} E_m(\vec{p}_{\vec{r}_1}) g_m^{*}(\vec{p}_{\vec{r}_1}, z_1) \right]^2
\]

\[
\times \left[ \int d\vec{p}_{\vec{r}_2} E_m(\vec{p}_{\vec{r}_2}) g_m^{*}(\vec{p}_{\vec{r}_2}, z_2) \right]^2,
\]

where $m$ and $n$ label the $m$th and the $n$th subfield emitted from the $m$th and the $n$th sub-source. In Eq. (41), we have assumed a
perfect second-order temporal coherence as usual. We apply the Fresnel near-field approximation to propagate the field from each sub-source to the photodetectors by means of the following Green’s function:

$$g_m(\omega; \rho_j, z_j) = \frac{c_0}{z_j} e^{i\omega \rho_j} e^{i\omega \rho_j / c z_j^2},$$

(42)

where $c_0$ is a normalization coefficient and $\tau_j = t_j - z_j/c, j = 1, 0$. Assuming a disk-like source and randomly distributed and randomly radiated point-like sub-sources, we can approximate the sum of $m$ into an integral of $\rho_j$ on the source plane of $z_s = 0$. The photon number fluctuation, or intensity fluctuation, correlation measurement between $D_1$ and $D_2$ yields a diffraction limited correlation between the planes of $z_1 = d$ and $z_o = d$,

$$\left| \langle \Delta n(\rho_1, z_1) \Delta n_2 \rangle \big|_{z_1 = z_o} \right|^2 \approx \left| \int \int d^2 \rho \delta(\omega; \rho_1, z_1) \left[ \int d^2 \rho_o A(\rho_o) g_m(\omega; \rho_1, z_1) \right]^2 \right|^2 = \left| A(\rho_o) \otimes \text{somb} \left( \frac{\pi \Delta \theta}{\lambda} |\rho_1 - \rho_o| \right) \right|^2,$$

(43)

where the somb-function is defined as $\text{Somb}(x) = \frac{1}{2} \left( 1 + \frac{x \lambda}{8} ight)^{\frac{1}{2}}$ and $\Delta \theta \approx 2R/d$ is the angular diameter of the radiation source viewed at the photodetectors. It is clear from Eq. (43) that we may conclude the following. (1) Similar to the ghost image reproduced by entangled photon pairs, the observed lensless ghost image is the result of a convolution between the aperture function of the object, $A(\rho_0)$, and the two-photon diffraction limited point-to-point image-forming function, $\text{somb} \left( \frac{\pi \Delta \theta}{\lambda} |\rho_1 - \rho_o| \right)$. (2) The spatial resolution of the lensless ghost image is determined by the wavelength of the thermal field and the angular diameter of the thermal source: the shorter the wavelength and the larger the angular size of the light source, the higher the spatial resolution of the achievable lensless ghost image.

For a large value of $\Delta \theta$, the point-to-“spot” sombrero function can be approximated as a delta-function, $\delta(|\rho_1 - \rho_o|)$,

$$\left| \langle \Delta n(\rho_1, z_1) \Delta n_2 \rangle \big|_{z_1 = z_o} \right|^2 \approx \left| \int d^2 \rho_o A(\rho_o) \delta(|\rho_1 - \rho_o|) \right|^2 = \left| A(\rho_o) \right|^2 \left| \delta(|\rho_1 - \rho_o|) \right|^2.$$

(44)

To confirm that the observation is an image but not a “projection shadow,” in fact, the first lensless ghost imaging demonstration of Valencia et al. in 2005 was observed from a secondary imaging plane of the lensless ghost image\(^{[69]}\). Their unfolded experimental setup is illustrated in the top part of Fig. 13. By using a convex imaging lens of focal length $f$, the primary lensless ghost image is mapped onto a secondary image plane, where the scanning photodetector $D_1$ is placed. The secondary image of the primary lensless ghost image is recorded in the joint photodetection between $D_1$ and $D_2$ by means of either photon-counting coincidences or photocurrent–photocurrent correlation. The bottom part of Fig. 13 shows a secondary image of the primary lensless ghost image of “UMBC” with a magnification factor of $m = |s_i/s_o| \sim 2.9$. The secondary imaging system is useful in certain experimental conditions, especially when the size of the 1:1 lensless ghost image is either too big or too small to be captured by a CCD (scanning photodetector $D_1$). A magnified or demagnified secondary ghost image would be helpful for certain applications, such as satellite imaging (field is too large) and X-ray microscopes (field is too small).

The following calculation confirms that the magnified or demagnified secondary ghost image is reproduced by a two-photon interference induced diffraction limited point-to-point image-forming correlation. We first calculate the point-to-point photon number fluctuation correlation between the object plane, $z_o = d_A$, and the secondary ghost image plane, $z_1 = d_A + s_o + s_i$, where $s_o$ is the distance from the primary ghost image to the lens and $s_i$ is the distance from the lens to the secondary ghost image, satisfying the Gaussian thin-lens equation $1/s_o + 1/s_i = 1/f$ with $f$ the focal length of the lens. Following Eq. (41), we have
\[
(\Delta n(\rho_o, z_o) \Delta n(\rho_1, z_1)) \\
\propto \left| \int d\rho_o g_{\rho_o} (\rho_o, \rho_1, z_1) \left[ \int d\rho_o A(\rho_o) g_{\rho_o}^* (\rho_o, \rho_o, z_o) \right] \right|^2 \\
= \left| \int d\rho_o \left\{ g_{\rho_o} (\rho_o, \rho_1, z_1) \left[ \int d\rho_o g_{\rho_o}^* (\rho_o, \rho_1, z_1) \right] g_{\rho_o} (\rho_o, \rho_o, z_o) \right\} \right|^2 \\
\times \left[ \int d\rho_o A(\rho_o) \left\{ g_{\rho_o} (\rho_o, \rho_1, z_1) \right\} \right]^2 \\
\approx \int d\rho_o A(\rho_o) \left| \int d\rho_o \delta(\rho_o - \rho_1) \right|^2 \\
\times \text{somb} \frac{\pi D}{s_o} |\rho_o - \rho_1/m|^2.
\] 

(45)

where \( g_{\rho_o} (\rho_o, \rho_1, z_1) \) and \( g_{\rho_o} (\rho_o, \rho_1, z_1) \) are Green’s functions along the ghost image paths that propagate the field from the source plane to the one-to-one primary ghost image plane, from the primary ghost image plane to the lens plane, from the input plane of the lens to the output plane of the lens, and from the lens plane to the secondary ghost image plane, respectively; \( g_{\rho_o} (\rho_o, \rho_o, z_o) \) is Green’s function along the bucket detector path that propagates the field from the source plane to the object plane. It is interesting to conclude a diffraction limited nonlocal second-order correlation between the object plane \( z_o = d_A \) and the secondary ghost image plane \( z_1 = d_A + s_o + s_i \) in the presence of a lens: the sombrero-like function indicates a distant correlation between the two propagating arms of the thermal fields.

Now, we calculate the magnified or demagnified lensless ghost imaging. Examining the “unfolded” schematic setup of Fig. 13, following Eqs. (41) and (45), the two-photon interference induced intensity fluctuation correlation measured by the scannable point-like photodetector, \( D_1 \), and the “bucket” photodetector, \( D_2 \), is calculated as follows:

\[
(\Delta I(\rho_1, z_1) \Delta I_2) \\
\propto \left| \int d\rho_o A(\rho_o) \delta(\rho_o - \rho_1) \right|^2 \\
= |A(\rho_o) \otimes \delta(\rho_o - \rho_1) / m|^2 = |A(\rho_1) / m|^2, 
\] 

(47)

indicating a magnified or demagnified secondary ghost image by means of a point-to-point mapping of the aperture function. It is necessary to emphasize that the mathematics of the convolution between the aperture function and the point-to-point image-forming function has no difference in any optical imaging systems, including traditional classical imaging, ghost imaging of entangled states, and ghost imaging of thermal fields. The differences between different imaging systems come from different mechanisms that produce the point-to-point, or realistically point-to-“spot,” image-forming function in that particular imaging system. In a classical imaging system, it is the first-order constructive-destructive interference that causes the point-to-point correspondence between the object and image planes, i.e., any radiation that is radiated (or reflected) from a point on the object plane will arrive at a unique point on the image plane. In the lensless ghost imaging system of the thermal field, it is the two-photon interference that causes the nonlocal second-order correlation between the object plane and the image plane. Analogous to the ghost image of entangled photon pairs, this natural, non-factorizable, point-to-point image-forming correlation represents a nonlocal interference of randomly created and randomly paired photons in thermal state: neither photon-one nor photon-two “knows” precisely where to go when it is created at each independent sub-source; however, if one is observed at a point on the object plane, the other one has twice the probability of arriving at a unique corresponding point on the image plane. (Note: similar to the far-field HBT correlation, the contrast of the Fresnel near-field point-to-point image-forming function in the measurement of photon number correlation or intensity correlation is 50%, i.e., a 2:1 ratio between the maximum value and the constant background. This Fresnel near-field point-to-point image-forming function turns to be 100% in the measurement of photon number fluctuation correlation or the intensity fluctuation correlation.)

5. Classical Simulation of Ghost Imaging

It is always possible to replace the two-photon interference produced natural, non-factorizable, point-to-point correlation of thermal light or entangled photon pairs by an artificial correlation made from a radiation source in which the “light knows where to go” when it is prepared at the source. There have been quite a few attempts to simulate two-photon interference produced ghost image-forming correlation. For instance, one may prepare two identical copies of intensity “speckles” on the object plane and on the image plane, respectively.
The speckle-to-speckle correlation plays the role of a spot-to-spot image-forming function. The object function \( A(\vec{\rho}_{\text{obj}}) \) is thus reproduced in the coincidence measurement of identical speckles. The following should be emphasized. (1) The resolution and the quality of the classically simulated ghost image may not be able to achieve that of the true ghost image produced from the two-photon interference produced natural, non-factorizable, point-to-point correlation of randomly created and randomly paired photons in the thermal state or photon pairs in the entangled state. (2) For a known artificial speckle distribution, it is unnecessary to use two photodetectors for joint detection. One bucket detector is sufficient to reproduce the “image” of the object. (3) The “image” is more likely a projection, instead of an image, of the object.

We briefly discuss three classical simulations in the following.

(I) Correlated laser beams.

In 2002, Bennink et al. simulated ghost imaging by two correlated laser beams\[14,15\]. The authors intended to show that two correlated co-rotating laser beams can simulate similar physical effects of entangled photon pairs. Figure 14 is a schematic picture of their experiment. Different from ghost imaging, here the point-to-point correspondence between the object plane and the “image plane” is made artificially by two co-rotating laser beams “shot by shot.” The laser beams propagate in opposite directions and are focused on the object and image planes, respectively. If laser beam-1 is blocked by the object mask, there would be no joint detection between \( D_1 \) and \( D_2 \) for that “shot,” while if laser beam-1 is unblocked, a coincidence count will be recorded against that angular position of the co-rotating laser beams. A shadow of the object mask is then reconstructed in coincidences by the blocking-partial blocking-unblocking of laser beam-1.

The point-to-point correlation of Bennink et al. is made shot by shot between “correlated” laser beams, which is not only different from that of ghost imaging but also different from the standard statistical intensity fluctuation correlations. Nevertheless, the experiment of Bennink et al. obtained a ghost shadow, which may be useful for certain purposes. In fact, this experiment can be considered as a good example to distinguish a man-made factorizable classical intensity–intensity correlation from a natural, non-factorizable second-order correlation that is caused by nonlocal two-photon interference.

(II) Correlated speckles.

Following a similar philosophy, Gatti et al. proposed a classical correlation between “speckles”\[16,17\]. The experimental setup of Gatti et al. is depicted in Fig. 15. Their experiments use either entangled photon pairs of SPDC or pseudo-thermal light for simulating ghost images in coincidences. The “ghost image” observed in coincidences comes from a man-made classical speckle-to-speckle correlation. The speckles observed on the object and image planes are the classical images of the speckles of the radiation source, reconstructed by the lenses shown in the figure (the lens may be part of a CCD camera used for the joint measurement). Each speckle on the source, such as the \( j \)th speckle near the top of the source, has two identical images on the object plane and on the image plane. Mathematically, the speckle-to-speckle correlation is factorizable into a product of two classical images,

\[
\gamma(\vec{\rho}_1, \vec{\rho}_2) \approx \delta(\vec{\rho}_1 - \vec{\rho}_1/m)\delta(\vec{\rho}_2 - \vec{\rho}_2/m),
\]

where \( \vec{\rho}_i \) is the transverse coordinate of the light source plane, \( m = s_1/s_0 \) is the classical imaging magnification factor, and \( m = 2f/2f = 1 \) and \( 1/2f + 1/2f = 1/f \) are defined as the optical distance between the plane of the light source and the planes of the object and the ghost image. (Note: the original publications of Gatti et al. choose \( z_1 \) and \( z_2 \) to image the speckles of the source onto the object plane and the ghost image plane.) The choices of \( z_1 \) and \( z_2 \) must satisfy the Gaussian thin-lens equation, respectively (see Fig. 15). It is easy to see from Fig. 15 that \( D_1 \) and \( D_2 \) will have more chance to be triggered jointly when they are in the position within the two identical speckles, such as the two \( j \)th speckles near the bottom of the object plane and the image plane. It is also easy to see that the size of the identical speckles determines the spatial resolution of the ghost shadow. This observation has been confirmed by quite a few experimental demonstrations. The classical simulation of Gatti et al. might be useful for certain applications. However, the man-made
speckle-to-speckle correlation of Gatti et al. is fundamentally different from the natural, non-factorizable point-to-point image-forming correlation observed in the ghost imaging experiment of Pittman et al. with entangled photon pairs and the lensless ghost imaging experiment of Scarcelli et al. with pseudo-thermal light.

(III) Computational ghost imaging.

Shapiro proposed a computational ghost imaging (CGI) experiment, which consists of a controllable (deterministic) light source, an object for imaging, and a bucket photodetector\[^18\]. For a nondeterministic light source, such as an entangled biphoton system or a thermal radiation source with randomly created and randomly paired photons in the thermal state, a photon or subfield may be observed randomly at any arbitrary position in space, or say a photon does not “know” where to go when it is created at the light source, and joint measurements between a reference detector (the CCD array) and a probe detector (the bucket detector) are always necessary. The reference detector defines the spatial coordinate of \( \vec{\rho}_o \) of the observation, and the probe detector measures the value of the aperture function \( |A(\vec{\rho}_p)|^2 \). To reproduce a ghost image exactly and accurately, the joint measurement must be able to distinguish the values of the aperture function, such as \( |A(\vec{\rho}_p)|^2 \) and \( |A(\vec{\rho}_o)|^2 \), in terms of each reference coordinate of the ghost image, such as \( \vec{\rho}_i \) and \( \vec{\rho}_o \). However, for a deterministic light source, in which the light “knows” where to go in each shot of its operation, it is unnecessary to use two photodetectors at all. One bucket detector is good enough to reproduce the image of the object. The working principle is very simple: in each shot of the operation, the light beam propagates to a chosen “spot” of the object, or say the light knows where to go when it is created at the source. The coordinate \( \vec{\rho}_o \), which is chosen by the light source, is recorded against the counting rate of the bucket detector at that coordinate, which is proportional to \( |A(\vec{\rho}_o)|^2 \). The object function \( |A(\vec{\rho}_p)|^2 \) is thus reproduced or calculated after a large number of such records. If the purpose of the imaging is for recognizing the “shape” of the object only, the light source may not be necessary to prepare a precise spot on the target object in each of its shot-to-shot operations. Instead, the light source may prepare a known “function” of intensity speckles on the target object plane, which is randomly determined by the source from shot to shot. The measured counting rate of the bucket detector will be added to these coordinates in each shot of its operation. After a large number of accumulations, the shape of the object, or the shape of the mask, can be estimated statistically.

It should be emphasized that we cannot expect a high-quality and high-resolution image from classically simulated ghost “imaging.”

6. Application - I

In the following three sections, we discuss a few practically useful applications of ghost imaging. Compared with classical imaging, ghost imaging technology has its peculiar features, uniqueness, and advantages. For instance, (1) it is nonlocal, (2) it has the property of resisting turbulence and environmental vibration and suppressing noise, and (3) it can be lensless.

One obviously interesting application is a light detection and ranging (Lidar) system with imaging ability. We name it Imaging Lidar (ILidar). A conceptual schematic of an ILidar is given in Fig. 16. This particular ILidar uses an entangled photon pair to reproduce a ghost image of a distant target and a set of joint-photodetection histograms of each pixel of the CCD, \( D_2 \), and the point-like photodetector \( D_1 \). The observed ghost image and the histograms provide both transverse (image) and longitudinal (ranging) space-time information of the target. A 3-D ghost image is finally observable with the help of a massive

![Fig. 16. Conceptual schematic of an Imaging Lidar.](image-url)
computation device. It should be emphasized that the two-photon source is not limited to entangled photons; for instance, it can be a light source that creates randomly distributed and randomly paired photons, or groups of identical photons, to produce ghost images of a distant target with 50% contrast.

First, this ILidar is an imaging device. Compared with the setup of the first ghost imaging experiment of 1995 in Fig. 4, it is not too difficult to figure out the imaging function of this ILidar and understand its imaging working mechanism. The only difference between Figs. 16 and 4 is that the imaging lens is moved from one arm of the setup to the other arm of the setup. With the help of a cleverly designed optical system, including the telescope coupling, a demagnified ghost image of the distant target is observable in the local laboratory.

Second, this ILidar is a ranging device. Ranging is the main goal of Lidar anyway. More interestingly, the ranging function of this ILidar is “turbulence-resistant,” or “turbulence-free,” i.e., any rapid phase variations along the optical path due to random changes in composition, density, length, index of refraction, or medium vibration, as well as any interference noise, do not affect its precise timing and positioning. How does a ghost imaging device implement the ranging function? Why is it turbulence-resistant? These two questions will be addressed as follows.

To understand its ranging function and turbulence-resistant property, let us simplify the measurement to 1-D by considering one pixel of the CCD as a point-like photodetector $D_2$ and focus on the second-order temporal correlation measurement of $D_1$ and $D_2$, namely the measurement of two-photon coherence function $G^{(2)}(t_1 - t_2)$. To simplify the calculation, we assume an entangled biphoton source similar to that of the 1995 ghost imaging experiment in which pairs of entangled signal–idler photons are created from a CW laser beam pumped SPDC. In a CW laser pumped SPDC, both the signal field and the idler field are in the form of continuous waves. Nevertheless the coincident measurement of $D_1$ and $D_2$, a measure of the second-order temporal coherence of the biphoton field, at distance is able to achieve a $\delta$-function-like correlation. The actual width of the $\delta$-function like $G^{(2)}(t_1 - t_2)$ is primarily determined by the response time of the measurement electronics.

Examining the ILidar in Fig. 16, the signal photon is sent to a distant target at position $r_1$ through a telescope while the idler is directly sent to a local photodetector $D_2$ at $r_2 \approx 0$. The reflected signal photon is received by the telescope and is annihilated at photodetector $D_1$, which is placed at the focal point of the telescope. The registration times of the signal photon and idler photon, $t_1$ and $t_2$, are recorded by two “event timers” whose time bases are synchronized by a clock. The individual time history records can be analyzed by software to obtain the second-order temporal correlation function $G^{(2)}(t_1 - t_2)$. The joint-detection counting rate, which is proportional to $G^{(2)}(t_1 - t_2)$ will show a maximum value when $t_1 - t_2 \equiv (|r_1 - r_2| / c)$, while the joint-detection events are produced by each biphoton, respectively. Determining the value of $t_1 - t_2$ at maximum value of $G^{(2)}(t_1 - t_2)$, the position of the target is then easily estimated.

The higher accuracy of determination of $t_1 - t_2$, the higher accuracy of the positioning measurement.

To calculate the temporal coherence function $G^{(2)}(t_1 - t_2)$, we approximate the state of the signal–idler photon pair as

$$|\Psi\rangle \approx \Psi_0 \sum_{\omega, \omega'} \delta(\omega + \omega_i - \omega_e) a_\omega^d a_{\omega'}^\dagger |\omega\rangle |0\rangle.$$  

Here, the subscripts $s$, $i$, and $p$ denote the signal, the idler, and the pump, respectively. The two-photon effective wavefunction of the biphoton $\Psi(t_1, t_2)$ is calculated as

$$\Psi(t_1, t_2) = \langle 0 | E_{t_1}^{(+)}(r_2, t_2) E_{t_2}^{(+)}(r_1, t_1) | \Psi \rangle \approx \Psi_0 \sum_{\omega, \omega'} \delta(\omega + \omega_i - \omega_e) e^{i\omega t_1} e^{-i\omega t_2}$$  

$$= \Psi_0 \sum_{\omega, \omega'} \Psi_{\omega, \omega'}(\omega_i, \omega_e),$$  

where $t_j = t_j - r_j / c$. It is interesting to see that the effective wavefunction represents a coherent superposition of a large number of two-photon amplitudes. This superposition indicates a nonlocal two-photon interference: a pair of entangled photons at a distance interfering with the pair itself. Approximating the sum (superposition) as a Fourier transform,

$$\Psi(t_1; r_2) \approx \Psi_0 e^{-i(\omega_0 t_1 + \omega_0 t_2)} \int \int d\omega d\omega' e^{-i(\omega t_1 - \omega' t_2)}$$  

$$= |\Psi_0 e^{-i(\omega_0^s + \omega_0^i)(t_1 + t_2)}| \left\{ F_{t_1 - t_2}[f(\nu)] e^{i(\omega_0^i - \omega_0^s)(t_1 - t_2)} \right\},$$  

where $\omega_0^s$ and $\omega_0^i$ are the central frequencies of the signal and idler, respectively; $\nu \equiv \omega_0^s - \omega_0^i$ is the detuning of the signal field; and $F_{t_1 - t_2}[f(\nu)]$ is the Fourier transform of the spectral function of the signal–idler field. The temporal (longitudinal) correlation function $G^{(2)}(t_1 - t_2)$ is thus

$$G^{(2)}(t_1 - t_2) \propto |F_{t_1 - t_2}[f(\nu)]|^2.$$  

The Fourier transform is usually a $\delta$-function-like function of $t_1 - t_2$ in the case of SPDC due to its wide spectrum. Assuming a constant distribution of $\nu$ within a certain spectrum $\Delta \nu$, the Fourier transform can be approximated as a sinc-function,

$$G^{(2)}(t_1 - t_2) \propto \text{sinc}^2 \frac{\Delta \nu |t_1 - t_2|}{2\pi}.$$  

The sinc-function ($\sin x / x$) becomes a $\delta$-function-like narrow and sharp function of $t_1 - t_2$ when $\Delta \nu \sim \infty$. At $t_1 - t_2 = 0$, i.e., $t_1 - t_2 = (r_1 - r_2) / c$, the sinc-function takes its maximum value of one. Indeed, it is the nonlocal two-photon interference resulting in the narrow, sharp histogram from the measurement of CW waves.

Based on this two-photon superposition, or two-photon interference, it is not too difficult to find that, if all two-photon
amplitudes $\Psi_s(\omega_t, \omega_i)$ of the signal–idler experience the same turbulence along their optical paths in space-time with the same phase variations,

$$\Psi^T(\tau_1, \tau_2) \simeq \Psi_0 \sum_{i,j} \delta(\omega_i + \omega_j - \omega_p)[e^{-i\delta\phi(\tau_i)}]e^{-i\omega_i \tau_1}e^{-i\omega_i \tau_2}$$

$$= [\Psi_0 e^{-i\delta\phi(\tau_i)}] \Psi(\tau_1, \tau_2), \quad \text{(54)}$$

the nonlocal two-photon correlation function

$$G^{(2)}(\tau_1 - \tau_2) = |\Psi^T(\tau_1, \tau_2)|^2 = |\Psi(\tau_1, \tau_2)|^2 \quad \text{(55)}$$

will be invariant. The measured joint detection histogram is therefore turbulence-resistant. (Note: experiencing the same turbulence for all superposed two-photon amplitudes, not limited by “two-term” superposition, is the only experimental requirements of turbulence-resistant or turbulence-free ghost imaging.)

The turbulence-resistant nature of the ranging function has been experimentally demonstrated\(^{[19]}\). Figure 17 shows a typical measurement of the two-photon temporal coherence function: number of coincidence counts versus $t_1 - t_2$, which is proportional to $G^{(2)}(t_1 - t_2)$. In this measurement, significant atmospheric turbulence and strong background light noise were introduced. The histogram shows undisputedly that the target is 750.14 ± 0.03 m away. Figure 18 illustrates two enlarged $G^{(2)}(t_1 - t_2)$ functions around their maximum values. The left-side histogram is a measurement without turbulence and background noise, and the right-side histogram is a measurement with significant turbulence and background noise. Comparing the two histograms, it is hard to find any change in either the shifting of the peak (accuracy) or the function width (resolution) of the histogram, except for slightly increased background coincidences and a slightly decreased peak count due to the scattering loss caused by the applied atmospheric turbulence. In these measurements, the introduced atmospheric turbulence is strong enough to blur out the interference pattern of a classic Young’s double-slit interferometer. The background noise is 20 times stronger than the signal.

It is worth noting that the calculated width of the two-photon coherence function is much narrower than the measured width. This is mainly due to the slow response time of the photodetectors:

$$G^{(2)}(\tilde{t}_1 - \tilde{t}_2) = \int_{t_c} dt_1 dt_2 G^{(2)}(t_1 - t_2) D(t_1 - \tilde{t}_1)D(t_2 - \tilde{t}_2), \quad \text{(56)}$$

where $t_c$ is the response time, or the characteristic time of the photodetector. Any meaningful physics we learn from the measurement cannot go beyond that timescale. $\tilde{t}_j$, $j = 1, 2$, is the measured “mean” time of the photodetection event of the $j$th photodetector. The mean time $\tilde{t}_j$ has a minimum basic timescale of $t_c$. $D(t_1 - t_2)$ is the normalized response function of the $j$th photodetector. For a sub-nanosecond photodetector, the femtosecond $G^{(2)}(t_1 - t_2)$ is basically playing the role of a $\delta$-function in the above convolution.

We have recognized that a ghost imaging device is able to achieve turbulence-resistant ranging. Either the imaging function or the turbulence-resistant ranging function of this ILidar is the result of two-photon interference. The accuracy and resolution of the imaging and ranging function of this ILidar rely on the second-order spatial coherence (imaging) and temporal coherence (ranging) of the two-photon field. Combining all histograms, pixel by pixel, with the ghost image, a 3-D picture of the target is observable from this ILidar. A powerful computation element is the key to the success of this 3-D ILidar.

### 7. Application - II

The multi-photon interference nature of ghost imaging determines its peculiar features. In this section, we are focusing on its “turbulence-resistant” applications.

The turbulence-resistant, or turbulence-free, ghost imaging was demonstrated by Meyers \textit{et al.} in 2011\(^{[20]}\). The schematic setup of their experiment is shown in Fig. 19. It is a typical thermal light ghost imaging setup, which captures the secondary ghost image of the primary lensless ghost image. This
experiment, however, added a set of powerful heating elements underneath the optical paths to produce laboratory atmospheric turbulence. Figure 19 illustrates the most serious situation in which turbulence occurs in all optical paths of the setup. The heating elements can be isolated to produce turbulence for any individual optical path too. Heating of the air causes temporal and spatial fluctuations on its index of refraction that cause the classic image of the object to jitter about randomly on the image plane of a classic camera.

Similar to their earlier demonstration of ghost imaging, pseudo-thermal light is generated from a fairly large angular sized pseudo-thermal source and is split into two by a 50%-50% beamsplitter. One of the beams illuminates an object located at $z_1$, as shown with the letters "ARL" in Fig. 19. The scattered and reflected photons from the object are collected and counted by a bucket detector, which is simulated by the right-half of the photon-counting CCD in Fig. 19. The other beam propagates to the ghost image plane of $z_1 = z_2$. We have learned that, from early analysis of thermal light ghost imaging experiments, by placing a CCD array on the ghost image plane, the CCD array will capture the ghost image of the object if its exposure is gated by the bucket detector. In this experiment, the CCD array is replaced by a piece of glossy white paper. The scattered and reflected light from the glossy white paper, which contains the information of the ghost image, is then captured by the left-half of the high-resolution and high-sensitivity photon CCD camera, which operates at the photon-counting regime. The CCD camera is focused onto the ghost image plane and is gated by the bucket detector for the observation of the secondary ghost image. The secondary ghost image captured by the left-half CCD camera is the image of the primary lensless ghost image located at $z_1 = z_2$. In this special setup, the left-half and the right-half of the CCDs individually, as two independent classic cameras, may play the roles of two independent classic cameras in their “normal” ungated operation and simultaneously capture the secondary ghost image in their gated joint-detection operation. The hardware circuit and the software program are designed to monitor the outputs of the left-half and the right-half of the CCDs, individually, as two independent classic cameras, and simultaneously to monitor the gated output of the left-half

Fig. 18. Typical measured histograms: number of coincidence counts versus temporal delay, $t_1 - t_2$ in nanoseconds. Upper left, without turbulence and background-noise; upper right, with significant turbulence and background-noise; bottom, two histograms, without and with significant turbulence, in one plot for comparison. The introduced atmospheric turbulence is strong enough to blur out the interference pattern of a classic Young’s double-slit interferometer. The background noise is 20 times stronger than the signal.

Fig. 19. Schematic setup of a typical thermal light ghost imaging experiment that captures the secondary image of the primary lensless ghost image. This experiment, however, added a set of powerful heating elements underneath the optical paths to produce laboratory atmospheric turbulence. The dashed line and arrows indicate the optical path of the "bucket" detector. The solid line and arrows indicate the optical path of the ghost image arm.
CCDs as a ghost camera. In the measurement, the classic image and the secondary ghost image of the object were captured and monitored simultaneously when the turbulence is introduced to each or to all optical paths.

The effect of turbulence on a classic image can be easily seen from the blurring of the images. Technically the turbulence is characterized by the refractive index structure parameter $C_n^2$. This experiment achieved $C_n^2 = 1.2^{-12}$ for the CCD arm and $C_n^2 = 1.5{-12}$ for the bucket detector arm. These values correspond to extremely high levels of atmospheric turbulence, causing significant temporal and spatial fluctuations on the index of refraction, as well as the blurring of classic images. Under the same turbulence, however, the ghost images behave differently, as neither its spatial resolution nor its contrast was affected by the turbulence.

The turbulence-resistant ghost imaging is the result of turbulence-resistant non-factorizable point-to-point image-forming correlation, which is caused by two-photon interference: superposition between two different yet indistinguishable alternative ways for randomly created and randomly paired photons to lead a joint-photodetection event. We give a simple analysis in the following, starting from

$$G^{(2)}(\tilde{p}_1, z_1; \tilde{p}_2, z_2) = \int d\tilde{x}_1d\tilde{x}_2 \frac{1}{\sqrt{2}} [g_2(\tilde{p}_2, z_2; \tilde{x}_2)g_4(\tilde{p}_1, z_1; \tilde{x}_1) + g_4(\tilde{p}_2, z_2; \tilde{x}_2)g_2(\tilde{p}_1, z_1; \tilde{x}_1)]^2,$$

which indicates an interference between two quantum amplitudes, corresponding to two alternatives, different yet indistinguishable, which leads to a joint-photodetection event. This interference involve both arms of the optical setup as well as two distant photodetection events at $(\tilde{p}_1, z_1)$ and $(\tilde{p}_2, z_2)$, respectively.

Optical turbulence is defined as a random change in index of refraction due to changes in the composition or density of the propagation medium. These changes are related to the index of refraction along the propagation path, such that $n_i(r) = n_0 + \delta n_i(r), j = 1, 2$, where $r$ is the coordinate along the optical path. To more efficiently model this change in the index of refraction, we introduce a random phase shift, which is dependent on the change in index of refraction along the optical path,

$$\delta \phi_j = \frac{2\pi}{\lambda} \int dr \delta n_i(r).$$

We thus introduce an arbitrary phase disturbance $e^{i\delta \phi_2}$ into the ghost image arm and another phase disturbance $e^{i\delta \phi_1}$ into the bucket detector arm, respectively, to simulate the turbulences.

The spatial part of the second-order coherence with turbulences turns to be

$$G^{(2)}_{\text{turb}}(\tilde{p}_1, z_1; \tilde{p}_2, z_2) = \int d\tilde{x}_1d\tilde{x}_2 \frac{1}{\sqrt{2}} [g_4(\tilde{p}_2, z_2; \tilde{x}_2)g_4(\tilde{p}_1, z_1; \tilde{x}_1) + g_4(\tilde{p}_2, z_2; \tilde{x}_2)g_4(\tilde{p}_1, z_1; \tilde{x}_1)]^2,$$

It is easy to see that the phase turbulence has a null effect on the second-order correlation function $G^{(2)}(\tilde{p}_1, z_1; \tilde{p}_2, z_2)$. The normalized non-factorizable point-to-point image-forming correlation $G^{(2)}_{\text{turb}}(\tilde{p}_1, z_1; \tilde{p}_2, z_2)$ of thermal field is thus turbulence-resistant. The joint-photodetection counting rate between the bucket detector and the CCD array will therefore reproduce the aperture function as a turbulence-resistance ghost image,

$$\langle n(\tilde{p}_1, z_1)n(\tilde{p}_2, z_2) \rangle = \langle n(\tilde{p}_1, z_1)\rangle \langle n(\tilde{p}_2, z_2) \rangle + \langle \Delta n(\tilde{p}_1, z_1) \rangle \langle \Delta n(\tilde{p}_2, z_2) \rangle \propto G^{(2)}_{\text{turb}}(\tilde{p}_1, z_1; \tilde{p}_2, z_2) \approx n_{\alpha} + |A(\tilde{p}_1)|^2,$$

where $n_{\alpha}$ is a constant and $A(\tilde{p}_1)$ is the aperture function of the object.

In a realistic application, such as sunlight satellite imaging, it is not easy to place a beamsplitter to produce a ghost image. In fact, the real applications of the turbulence-resistant ghost imaging of Meyers et al. do not have a beamsplitter (BS in Fig. 19). In that case, the lensless ghost image is right on top of the object. We name it “ghost camera.” It should be emphasized that, in a real application in which natural light sources are used to produce ghost image, the second-order temporal coherence has to be taken into account. Short-pulse gated fast CCD is a solution.

Applying the same mechanism, a turbulence-resistant camera has been demonstrated. The turbulence-resistant camera is schematically illustrated in Fig. 20. It looks like a classic CCD (CMOS) camera, except for the following: (1) The image is divided into two paths, path-1 and path-2, in the camera by an optical beamsplitter. A photon-counting CCD (CMOS) array, $D_1$, and an integrated bucket photodetector, $D_2$, respectively, are placed on the image plane and on the focal plane of the camera lens. (2) $D_1$ and $D_2$ measure the photon number fluctuations, $\Delta n(\tilde{p}_1)$ and $\Delta n_2 = \int d\tilde{p}_2 \Delta n(\tilde{p}_2)$, where $\tilde{p}_1$ and $\tilde{p}_2$ label the transverse coordinates of the image planes of path-1 and path-2 of the camera, respectively. The photon number fluctuations $\Delta n(\tilde{p}_1)$ and $\Delta n_2$ are counted, respectively, and calculated, jointly, by a novel PNFC circuit to obtain the photon number fluctuation correlation $\langle \Delta n(\tilde{p}_1) \Delta n_2 \rangle$. This circuit is especially useful for satellite imaging in which the complicated lengthy statistical calculation can be performed on the ground. The hardware located in the satellite records two sets of data only: the registration time of each photodetection event for each photo-element (pixels) of $D_1$ (located at $\tilde{p}_1$), and for the bucket detector of $D_2$.

Figure 21 reports a set of preliminary results of turbulence-resistant camera demo-unit. The camera is imaged at the group 0 of a 1951 USAF Resolution Testing Gauge and is
placed 2 m away from the 0.5 mm lines (0 group of the gauge). Figure 21(a) shows the clear classical image in the measurement of \( n(\hat{\rho}_1) \) without atmospheric turbulence. Figure 21(b) shows the “blurred” first-order classic image in the presence of atmospheric turbulence. Figure 21(c) shows the image produced by the turbulence-resistant camera. In Figs. 21(b) and 21(c), row (i) shows weak turbulence, row (ii) shows medium turbulence, and row (iii) shows strong turbulence. It is clear that, although the first-order classic images in \( n(\hat{\rho}_1) \) are “blurred” out in the presence of significant turbulence, the image in \( \Delta n(\hat{\rho}_1)\Delta n_2 \) is always an improvement.

In fact, the setup of the turbulence-resistant camera in Fig. 20 is the same as the ghost imaging setup of Fig. 13, except that the primary lensless ghost image plane is right on top of the object plane, i.e., on top of the building of Fig. 20. The secondary ghost image of the primary lensless ghost image, which is observed from \( \langle n(\hat{\rho}_3) \rangle \), is completely “blurred” due to the influence of the atmospheric turbulence. However, a turbulence-resistant image is observed from the measurement of \( \langle \Delta n(\hat{\rho}_3) \Delta n_2 \rangle \), i.e., any atmospheric density, refractive index, or phase variations do not have any influence on this image. In this setup, the turbulence may appear either in the optical paths between the camera and the object or in the optical paths between the object and the light source, or appear in both.

8. Application - III

In this section, we discuss another useful application of ghost imaging: X-ray microscope.

In classical imaging setups, focusing optics such as imaging lenses play a critical role in producing a diffraction-limited point-to-point relation between the object plane and the image plane, forming a magnified or demagnified image of the object. If desired, an additional lens system is then able to map the primary image onto a secondary image plane for further magnification or demagnification, notably making an optical microscope possible. This type of optical microscope is commonly used to obtain high-resolution images of detailed surface structures of objects. Unlike visible light, an X-ray can pass through many materials allowing X-ray imaging devices to image the
internal structure of an object. If our expectation is to obtain high-resolution images of the detailed internal structure of an object or material, an X-ray microscope is a necessary but difficult goal. The first difficulty faced was that traditional lenses are not practical to use for high-energy X-rays because the refractive index is close to $n = 1$ for high energy X-rays in all known materials. To overcome this, some alternative focusing X-ray optics have been developed in recent years such as compound refractive lenses, focusing mirrors, and zone plates. While zone plates and focusing mirrors are typically limited to soft X-rays ($< 10$ keV), compound refractive lenses can be designed for $> 10$ keV. Unlike projectional radiography, which is a projection, or “shadow,” of the X-ray that passes through the object with different materials and thicknesses of materials causing more absorption in some areas compared to others, focusing X-ray optics take full advantage of the short wavelength of soft X-rays with significantly higher resolution when compared with current projectional radiography technology. Unfortunately, there are still many barriers in producing effective focusing devices for hard X-rays. Due to this, the X-ray imaging technique most commonly used in practice is still projectional radiography. Even as focusing X-ray optics are made more readily available, it is difficult for a classic imaging device to obtain images of deeper interior structures of an object due to their limited angular resolution. To take advantage of the resolving power of short-wavelength X-rays, the object plane should be near the focal point. Often the focal length of a traditional lens and focusing X-ray optics is restricted to a certain value, meaning any structure deeper than this value into the object would be inaccessible at the desired resolution.

Obviously, lensless ghost imaging is rightly suitable for X-ray imaging. (1) Lensless imaging does not need any lens. (2) Due to the “ghost” nature of the ghost image, an additional imaging device with limited angular resolution can be placed as close as possible to the selected ghost image plane, which corresponds to a “sliced” internal structure of the object, to “force” the angular separation of close neighboring points large enough to be resolvable. In addition, by scanning different lensless ghost image planes along the optical axis, which correspond to different internal cross sections, or “slices,” of the object, a set of slices of the internal structure of the object can be grouped together to form a magnified secondary 3-D ghost image of the object.

In this section, we apply the mechanism of two-photon ghost imaging to produce a sub-nanometer resolution, lensless, image-forming correlation between the image plane and object plane, for which X-rays allow imaging of the internal structure of the object in 3-D. Through the help of a secondary imaging device, either focusing X-ray optics or a scintillator paired with a visible-light lens system, the primary lensless ghost image in sub-nanometer resolution can be mapped onto a secondary image plane with significant magnification to be resolvable by a standard CCD or CMOS, namely, an X-ray ghost microscope. In principle, once some experimental barriers are overcome, this X-ray “ghost microscope” may achieve sub-nanometer resolution and open up new capabilities that would be of interest to the fields of physics, material science, and medical imaging.

Similar to lensless ghost imaging in the visible spectrum, the X-ray lensless image-forming correlation is the result of two-photon interference: two randomly created and randomly paired X-ray photons interfering with the pair itself. It has been proved that two-photon interference phenomena can be set up to achieve turbulence-resistant measurements. This is achieved when the superposed two-photon amplitudes experience the same turbulence and medium vibrations along their optical paths, meaning any composition, density, length, refractive index, or medium vibration induced random phase variations along the optical paths do not have any effect on each individual two-photon interference. The X-ray ghost microscope analyzed in this section is a typical turbulence-resistant imaging device. The turbulence-resistant nature is especially important for the extremely high resolution imaging obtainable with the X-ray ghost microscope, as we knew that vibrations would typically cause a blurred classic image.

(I) Primary lensless ghost imaging of X-ray.

To better visualize the X-ray ghost microscope, it is best to start with the working mechanism of X-ray lensless ghost imaging. This simple experimental setup, schematically illustrated in Fig. 22, consists of an X-ray beamsplitter that divides the X-ray beam from a disk-like source into two beams. (Note: this is trivial when using visible light but becomes more difficult with high-energy X-rays. So far, Laue diffraction of crystals has been used with success, but advancement in kinoform X-ray beam splitters may prove useful in the future.) A 2-D array of X-ray detectors, $D_1$, is placed in beam-one (transmitted in Fig. 22) at a selected plane of $z_1$ in the Fresnel near-field. Following the output of
beam-two (reflected in Fig. 22), we place an object followed by a bucket X-ray photodetector, $D_2$, which collects all X-rays transmitted from the object. This is a typical lensless ghost imaging setup. To calculate the X-ray ghost image-forming correlation, we apply the Fresnel near-field propagator, or Green’s function, to propagate the field from one space-time location at $(\rho_m, z_m)$ to another space-time location at $(\rho_j, z_j)$,

$$
g_m(\rho_j, z_j) = \frac{\epsilon_0}{|z_j - z_m|} e^{i\text{imag} \beta(z_m-z_j)},
$$

(58)

where $\epsilon_0$ is a normalization constant. (Note: due to the use of short wavelength X-rays, higher-order approximation, or numerical computation, may be necessary instead of a simple Fresnel propagator. However, it is a mathematical conclusion that higher-order approximations will produce “narrower” correlation than that of the Fresnel. To simplify the mathematics, we thus keep the Fresnel approximation in the following “prove-principle discussion.”) For the following discussion, we will assume a monochromatic source to simplify the calculation as usual. Assuming a disk-like source and randomly distributed and randomly radiated point-like sub-sources, we can approximate the sum of $m$ into an integral of $\rho_j$ on the source plane of $z_j = z_m = 0$. Before considering the detectors, we can look at the object plane on beam-two (reflected), $z_o$, and the corresponding plane on beam-one (transmitted), which we will label $z_{gi}$. The result of X-ray intensity fluctuation correlation is

$$\langle \Delta I(\rho_{gi}, z_{gi})\Delta I(\rho_o, z_o) \rangle |_{z_{gi}=z_o} \propto \left( \int d\rho_o \int d\rho_{gi} [g_o(\rho_o, z_o)g_{gi}(\rho_{gi}, z_{gi})] \right)^2 \propto \text{somb}^2 \frac{\pi \Delta \theta}{\lambda} |\rho_{gi} - \rho_o|, \quad (59)$$

where the somb-function is defined as $2f(x)/x$ and $\Delta \theta \approx 2R/d$ is the angular diameter of the radiation source. [Note: again, due to the use of short wavelength X-rays, higher-order approximation or numerical computation may be necessary. We will have a much narrower somb-function instead of $2f(x)/x$.] This point-to-spot correlation is similar to that of the original HBT experiment that started the practice of optical correlation measurements, except that it is measured in the Fresnel near-field instead of the Fraunhofer far-field. This point-to-spot correlation has also been demonstrated at X-ray synchrotron sources. Due to the high-energy (short-wavelength) nature of X-rays, this point-to-spot correlation is much more narrow than that of the visible-light. For instance, a $\sim 0.5 \times 10^{-10}$ m wavelength ($\sim 25$ keV) X-ray source with angular diameter of $> 10^{-1}$ rad may achieve sub-nanometer correlation. We can approximate this point-to-spot correlation as a point-to-point correlation between $\rho_{gi}$ and $\rho_o$. This point-to-point correlation, or lensless ghost image-forming function, identifies a unique internal plane of the object, $z_{gi} = z_o$, with the X-ray ghost image observed from the X-ray intensity fluctuation, or photon number fluctuation, correlation. Together with $z_{gi} = z_o$, we may approximate the X-ray intensity fluctuation correlation as

$$\langle \Delta I(\rho_{gi}, z_{gi})\Delta I(\rho_o, z_o) \rangle \propto \delta(z_{gi} - z_o)\delta(|\rho_{gi} - \rho_o|). \quad (60)$$

We now calculate the X-ray lensless ghost image by including a 2-D photodetection array on the $z_1 = z_{gi}$ plane and a bucket detector, $D_2$, which integrates all possible X-rays transmitted through each transverse coordinate point $\rho_o$ of the object. The internal “aperture function” of an object cannot be simply represented as a 2-D function $A(\rho_o, z_o)$ as typically done for visible-light imaging. In visible-light imaging, $A(\rho_o)$ is usually used to represent the surface plane of an object; however, for X-ray ghost imaging, it is reasonable to model a 3-D aperture function representing the internal structure of an object,

$$A(\rho_o) \approx \int dz_o A(\rho_o, z_o). \quad (61)$$

Assuming perfect temporal correlation is satisfied experimentally, the intensity fluctuation correlation results in

$$\langle \Delta I(\rho_1, z_1)\Delta I(\rho_o, z_o) \rangle = \int d\rho_o \int dz_o A(\rho_o, z_o) \times \left( \delta(z_1 - z_o) \text{somb} \frac{\pi \Delta \theta}{\lambda} |\rho_1 - \rho_o| \right)^2 \approx \int d\rho_o \int dz_o A(\rho_o, z_o) \times \delta(z_1 - z_o) \delta(|\rho_1 - \rho_o|))^2 \approx |A(\rho_1 = \rho_o, z_1 = z_o)|^2, \quad (62)$$

indicating the reproduction of a 2-D X-ray ghost image of the internal transverse cross section of $z_o = z_1$, i.e., $z_{gi}$. In other words, the longitudinal position of the 2-D X-ray detector array, $z_1$, selected an object plane, $z_o = z_1$, of the internal structure of the object to image. When the 2-D X-ray detector array is scanned from $z_0$ to $z_0'$ along the optical axis, the selected plane will be changed from $z_o = z_1$ to $z_0' = z_1'$ and $z_o'' = z_1''$, respectively, as illustrated in Fig. 12. By scanning the position of the 2-D X-ray detector array, a set of slices of the internal structure of the object can be grouped together to form a 3-D ghost image of the object with sub-nanometer resolution. This differs from traditional X-ray computerized tomography (CT) imaging and ghost tomography (GT) demonstrated by Kingston et al., which rely on rotating the object or revolving the detectors around the object 360°.

Unfortunately, the sub-nanometer resolution lensless ghost image is unresolvable by any state-of-the-art 2-D photodetector array, such as CCD or CMOS sensors, unless the photodetector array technology ever advances to sub-nanometer-sized pixels. In order to resolve the two-photon X-ray ghost image with a standard 2-D photodetector array, which may have micrometer sized pixels, significant magnification is necessary.

(II) Secondary ghost image reproduced by an X-ray lens system.
One solution would be to magnify the primary X-ray ghost image to a secondary ghost image plane with an X-ray lens system, such as X-ray zone plates, compound refractive lenses, or other types of X-ray lenses. Note that X-ray optics may limit the usable energy levels of the X-ray source (soft X-ray) unless the technology advances to accommodate higher energy levels. The schematic design of this type of X-ray ghost microscope is shown in Fig. 23. A magnified 3-D secondary ghost image is expected from the measurement of the X-ray intensity fluctuation correlation between the bucket X-ray detector, which is placed behind the object, and the 2-D array of X-ray detectors, which is placed in the secondary ghost image plane.

To confirm the nonlocal point-to-point correlation between the object plane, \( z_o \), and the secondary image plane, \( z_i \) (where \( D_1 \) will be placed, \( z_i = z_j \)), we adjust Eq. (59) to include the lens system,

\[
\langle \Delta I(\vec{p}_i, z_i) \Delta I(\vec{p}_o, z_o) \rangle \propto \left| \int d\vec{p}_o' \, g_{i o}(\vec{p}_o', z_o) g_{i o}(\vec{p}_i, z_i) \right|^2
\]

where \( g_{i o}(\vec{p}_o', z_o) \), \( g_{i o}(\vec{p}_i, z_i) \), \( g_{i o}(\vec{p}_l, z_l) \), \( g_{i o}(\vec{p}_L, z_L) \), are the Gaussian propagators, or Green’s functions propagating the field from the source plane to the one-to-one primary ghost image plane, from the primary ghost image plane to the output plane of the lens, and from the lens plane to the secondary ghost image plane, respectively,

\[
\langle \Delta I(\vec{p}_i, z_i) \Delta I(\vec{p}_o, z_o) \rangle \propto \left( \int d\vec{p}_o' \delta(\vec{p}_o' - \vec{p}_o) \right) \text{somb} \frac{\pi D}{s_o} |\vec{p}_o' - \vec{p}_i/\mu|^2
\]

indicating a point-to-spot correlation between the selected “slice,” or internal cross section, of the object and the secondary X-ray ghost image plane, namely the secondary X-ray ghost image-forming function. In Eq. (64), \( D \) is the diameter of the lens, \( s_o \) is the distance from the primary ghost image to the lens, \( s_i \) is the distance from the lens to the secondary ghost image, satisfying the Gaussian thin-lens equation \( 1/s_o + 1/s_i = 1/f \), where \( f \) is the focal length of the lens, and \( \mu = s_i/s_o \) is the magnification factor of the secondary ghost image. The 2-D detector array, \( D_1 \), is now placed on the image plane of the X-ray lens (secondary ghost image plane), \( z_i = z_j \). The position of \( D_1 \) defines the position of \( z_{gi} \) and thus defines the “slice” at \( z_o = z_{gi} \) of the internal cross section of the object by means of the Gaussian thin-lens equation. We then take into account the complex aperture function \( \Lambda(\vec{p}_o, z_o = z_{gi}) \) and the bucket detector \( D_2 \) into the calculation. A magnified secondary X-ray ghost image of the aperture function is then observed from the joint detection of the CCD (CMOS), \( D_1 \), and the bucket detector \( D_2 \),

Fig. 23. Schematic of an X-ray ghost microscope using an X-ray lens system. A beam splitter (most likely a crystal aligned to utilize Laue diffraction) creates two paths for the beam, one directed at the object substance and the second directed at the primary ghost image plane. An X-ray lens system is placed behind the primary X-ray ghost image to reproduce a significantly magnified secondary ghost image that is resolvable by a standard CCD or CMOS. Note: (1) due to the “ghost” nature of the primary ghost image, \( s_o \) of the microscope can be placed as close as possible to the primary X-ray ghost image to “force” the angular separation of nanometer scale inside the object greater than the angular resolution of the secondary imaging system; (2) \( \Delta \theta_{min} \approx 1.22 \lambda/D \) is the angular resolution of the secondary imaging device, where \( \lambda \) is the wavelength of the X-ray and \( D \) is the effective diameter of the X-ray lens.

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angular resolution of a millimeter diameter lens can achieve $10^{-7}$ rad, which is able to resolve a ghost image with $10^{-3}$ m spatial resolution because $s_0 < 10^{-2}$ m to the primary ghost image plane is easily achievable to force the angular separation of nanometer scale, $\Delta \theta = 10^{-9} / s_0 > 10^{-7}$ rad.

(III) Secondary ghost image reproduced by a scintillator-visible-light-lens assembly.

If the use of X-ray optics is not available or not preferred, one can place a scintillator on the primary ghost image plane $z_{gi}$ to convert it into the visible spectrum, allowing it to then be magnified by a visible-light lens system onto a secondary image plane. This design is suitable for higher energy X-ray imaging, such as $\gtrsim 20$ keV. The schematic design of this type of X-ray microscope is shown in Fig. 24. The setup is similar to that of Fig. 23, except that a scintillator is placed on the primary X-ray ghost image plane to convert the X-ray ghost image into the visible spectrum.

Unlike the X-ray ghost microscope shown in Fig. 23, which has a clear path for the two-photon amplitudes from the light source to the detectors, here it is not clear that the scintillator-lens system preserves the result of the two-photon interference, and thus the secondary ghost image. To understand how the secondary ghost image is preserved, we can say that the scintillator essentially “detects” the X-rays, thus establishing the presence of the X-ray ghost image on the scintillator plane. Although, prior to correlation, this plane is simply a distribution of quantum speckles from interfering photon pairs, the scintillator converts these X-ray fluctuations to the visible spectrum, $\Delta I_s(\vec{p}_{gi}, z_{gi}) \approx \Delta I_r(\vec{p}_{gi}, z_{gi})$. The lens system then images this distribution of fluctuations and produces a diffraction-limited magnified image of them on the secondary ghost image plane. The intensity fluctuation detected by the scintillator is the result of two-photon interference. The converted visible light from the

Fig. 24. Schematic of an X-ray ghost microscope using a scintillator-visible-light-lens assembly to magnify the primary X-ray ghost image. This is nearly identical to the setup of Fig. 23, but now a scintillator is placed on the ghost image plane to convert the X-ray ghost image into the visible spectrum. Now an optical lens (or lens system) of visible-light produces a magnified secondary ghost image.
scintillator then passes through a lens and is measured by $D_1$ on the secondary ghost image plane, $z_1 = z_i$,

$$\Delta I(\rho_1, z_i) = \sum_{m \neq n} E_m(z_1)E_n(\rho_1, z_i)$$

$$= \sum_{m \neq n} \int \psi_{\rho_1}(\rho_1, z_i) |g_{\rho_1}(\rho_1, z_i)||g_{\rho_1}(\rho_1, z_i)|$$

$$\times \int \psi_{\rho_1}(\rho_1, z_i) |g_{\rho_1}(\rho_1, z_i)||g_{\rho_1}(\rho_1, z_i)|$$

$$= \sum_{m \neq n} E_m(\rho_1, z_i)E_n(\rho_1, z_i) \text{somb}^2 \frac{\pi D}{\lambda o} |\rho_1 + \rho_i|/\mu |. \quad (66)$$

Here, $\lambda_0$ is the center wavelength emitted from the scintillator. It is interesting that the X-ray intensity fluctuations at $(\rho_1, z_i)$ of the primary ghost image plane are "propagated" to a unique point in the secondary image plane $(\rho_1, z_i)$, where $(\rho_1, z_i)$ is defined by the somb-function and the Gaussian thin-lens equation of the visible-light microscope. A magnified secondary ghost image is observable from the correlation measurement between the intensity fluctuations of the X-ray and the intensity fluctuations of the visible light,

$$\langle \Delta I(\rho_1, z_i) \Delta I_2 \rangle$$

$$\propto \left( \int d\rho_oA(\rho_o, z_o = z_g) \text{somb}^2 \frac{\pi D}{\lambda o} |\rho_o - \rho_1|/\mu | \right)^2$$

$$\propto \int d\rho_oA(\rho_o, z_o = z_g) \text{somb}^2 \frac{\pi D}{\lambda o} |\rho_o - \rho_1|/\mu |. \quad (67)$$

Even with a new dependence on visible light, pairing the scintillator with super-resolving visible-light imaging techniques will allow for nanometer resolution. More often, a standard compound microscope will be the best accessible option. Even with limited angular resolution, $\Delta \theta_{\text{min}} \approx 1.22 \lambda_0/D$, this setup would still have benefits of standard optical microscopy as it allows for imaging the deep internal structure of the object with limited angular resolution of the secondary imaging device.

One factor that may aid in the high resolving capabilities of the X-ray ghost microscope is the turbulence-resistant property of two-photon interference. It has been proved in early sections that ghost imaging and other two-photon interference phenomena can be set up to achieve turbulence-resistant measurements. This is achieved when the superposed two-photon amplitudes experience the same turbulence and medium vibrations along their optical paths, meaning any composition, density, length, refractive index, or medium vibration induced random phase variations along the optical paths do not have any effect on each individual two-photon interference. This also includes medium vibrations that would typically cause a blurred classical image of high resolution.

It should be emphasized that the above calculation is focused on second-order spatial correlation by assuming perfect second-order temporal coherence. When dealing with a broad spectrum thermal field, such as X-rays, (1) the second-order temporal coherence has to be taken into account: a higher degree of second-order temporal coherence must be achieved experimentally; (2) the time average of slow photodetectors and correlation circuit must also be taken into account. A short-pulsed tabletop X-ray source is a good solution for an X-ray microscope.

References