Sensitivity of ghost imaging compared to conventional imaging [Invited]

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Ghost imaging has been attracting more and more attention, which provides a way to obtain images of objects with only a single-pixel detector. Considering possible applications, it becomes urgent to clarify the sensitivity of ghost imaging. Due to the unique characteristics of single-pixel detectors, which collect photons without distributing them to multiple pixels, often outperforming array sensors, ghost imaging is believed to be more sensitive than conventional imaging. However, a systematic analysis on the sensitivity of ghost imaging is yet to be completed. In this paper, we present a method for quantitatively assessing the sensitivity of ghost imaging. A detailed comparison is provided between ghost imaging and conventional imaging, taking into account the particle nature of photons and the noise of detection. With the settings of the two imaging methods being the same to the most extent, the minimal required number of detected photons for images of a certain quality is considered. For the thermal source version, ghost imaging demonstrates enhanced sensitivity under practical situations, with noise considered. Employing an entangled source, ghost imaging surpasses conventional imaging techniques in terms of sensitivity obviously. In one word, ghost imaging promises higher sensitivity at low photon flux and noisy situations.

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1. Introduction

Imaging is an important method of information acquisition, with lots of ways developed for various kinds of scenarios, among which ghost imaging (GI) has been growing rapidly and fascinating researchers from different fields in recent years. In typical GI configuration, light from the object is collected into a single-pixel detector (called bucket detector), with the intensity/photon patterns of the illumination recorded or actively controlled. The image of the object then can be reconstructed via second-order correlation between the illumination patterns and the output of the bucket detector. The first experiment was demonstrated using entangled photons[1]. Several years later, theory and experiments of GI with a (pseudo-)thermal source were reported[2,3]. Along with the debates on whether GI is naturally quantum or classical[4-8], the theory of GI has become more clearly understood, and the related techniques have progressed significantly[9-16], with sources, sensors, imaging strategies, and algorithms developed[17-21]. Considering possible applications, the performance of GI was enhanced under scattering, turbulence, strong background noise, and other practical situations[22-26]. As an effective way of information acquisition, the idea of GI has been extended from spatial imaging to signal detection for different degrees of freedom, such as the reconstruction of temporal signal and Fourier imaging[29,30]. It was also extended for different waves, including infrared, THz, X-ray, microwave, and even matter waves[31-36].

Such prosperous progress has arisen based on the promising advantages of GI. Roughly speaking, GI surpasses conventional imaging (CI) techniques in sensitivity and robustness, as well as the ability of single-pixel imaging and lensless imaging[37-40]. First, for those wavebands that have large array sensors and/or imaging lenses that are yet to be developed, GI provides an
effective way for reconstructing high-resolution images with only an achievable single-pixel sensor, along with modulation in intensity distribution \cite{39,41}. Second, due to the fact that information is acquired by correlation, influence caused by those uncorrelated noises or disturbances can be greatly suppressed or reduced, such as scattering, strong background light, and weak turbulence\cite{23,37,42,44}. Third, the utility of bucket detection is deemed to provide higher sensitivity than CI. Since the photon flux is collected onto a single-pixel detector, instead of distributed onto thousands of pixels as done in CI systems, it appears easier to use GI to obtain the expected information with smaller photon flux. Besides, the performance of single-pixel detectors is usually better than array sensors, which also gain a technical advantage. Imaging of an object can even be achieved with an average number of detected photons per pixel lower than 1 with GI\cite{19,45,46}, but is inconceivable for CI.

From the perspective of resource consumption, it remains uncertain how many photons are necessary to create an image\cite{47} and which imaging method, GI or CI, consumes fewer resources. To discuss this issue, sensitivity of imaging is one of the possible measurements. However, the sensitivity of GI, as well as a comparison to CI, has yet to be systematically analyzed. It was supposed that, when the intensity is relatively strong, the imaging quality of GI has little relation with the light intensity\cite{48}, while it is closely related to the signal intensity in weak cases\cite{49}. It was also proposed that GI demands fewer photons, making it suitable for remote sensing and biomedical imaging\cite{39,50}, and GI with an entangled source performs better against shot noise\cite{51,52}. It remains unclear under what kind of conditions GI exhibits any advantage in terms of sensitivity and how much of an advantage it contains. Further, how to measure the sensitivity of GI is still an open problem.

In this paper, we try to illustrate this issue. We first introduce basic models for GI, taking the particle nature of photons and practical noise into account. Then, we suggest a definition of imaging sensitivity relying on estimating the output. With theoretical analysis and numerical simulation based on our model, the performances of GI and CI are carefully compared, emphasizing mainly the sensitivity of both techniques. From the results, it can be seen that GI shows higher sensitivity than CI in practical situations. GI with thermal sources performs better only with better sensitivity of a single-pixel detector in practical situations, while for GI with an entangled source, it behaves more sensitively under background noise in practical applications, even when the single-pixel detector has the same parameters as the array detector in CI.

The paper is arranged as follows. In Section 2, we establish the basic models. In Section 3, estimation of the output images and sensitivity of the measurement are elaborated. In Section 4, the sensitivity of GI with the thermal source is analyzed in detail, with comparison to CI under different conditions. In Section 5, GI with entanglements is discussed. Finally, some related discussions are made in Section 6, followed by a conclusion in Section 7.

2. Basic Models

Typical configurations of thermal GI\cite{2,3,53} and entangled GI\cite{1,51} are shown in Figs. 1(a) and 1(b). Light from the source is divided into two beams: one recorded by $D_b$ is called the reference arm; the other illuminates the object, and the echo reflected from the object is detected by a bucket detector ($D_b$), referred to as the object arm. It is worth noting that the optical field of the reference arm can also be calculated, as in computational GI. The image can be reconstructed through correlation or coincidence measurement between two arms.

For the thermal case shown in Fig. 1(a), the output from the beam splitter (BS) can be represented as

$$\hat{E}_1(\rho_1) = r\hat{E}(\rho) + \hat{v}(\rho), \quad \hat{E}_2(\rho_2) = t\hat{E}(\rho) + \hat{v}(\rho),$$

where $\hat{E}$ is the input thermal field, $\hat{v}$ is a vacuum field uncorrelated with $\hat{E}$, $\rho$ is the position in the transverse plane, and $t$ and $r$ are generated by SPDC. One of the photons interacts with the object and is recorded by $D_r$, referred to as the object arm. For entangled GI, entangled photon pairs are generated by SPDC. One of the photons interacts with the object and is received by $D_b$, while the other is detected by a photon counting array $D_r$. Then the image is obtained through correlation.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{The schematic diagram of typical (a) thermal and (b) entangled GI. In thermal GI, a laser is commonly used to illuminate a rotating ground glass plate, generating a pseudothermal light field. The intensity distribution of the light field irradiating on the object is recorded by a CCD camera ($D_s$). For computational GI, the actual CCD camera is unnecessary, with the intensity distribution being calculated. The outgoing light illuminates the object, and the echo reflected from the object is detected by a bucket detector ($D_b$). Then the reflectance of an object can be obtained by correlating the reference arm with the object arm. For entangled GI, entangled photon pairs are generated by SPDC. One of the photons interacts with the object and is received by $D_b$, while the other is detected by a photon counting array $D_r$. Then the image is obtained through correlation.}
\end{figure}
are the complex transmission and reflection coefficients of the BS.

Each output is directed through imaging systems characterized by their impulse response functions of \( h_1(\rho, \rho_1) \) and \( h_2(\rho_1, \rho_2) \), respectively. The fields at the reference detection plane and the object plane are given by

\[
\hat{E}_m(\rho_m) = \int d\rho h_m(\rho_m, \rho_1) \hat{E}_1(\rho_1),
\]

where \( \{m, l\} = \{r, 1\} \) or \( \{s, 2\} \). Then the second-order correlations between the object arm and the reference arm can be written as

\[
\langle \hat{E}_1^\dagger(\rho_1) \hat{E}_2(\rho_2) \rangle = \int d\rho_1 d\rho_2 \hat{E}_1^\dagger(\rho_1) \hat{E}_2(\rho_2) h_1(\rho_1, \rho_2) h_2(\rho_2, \rho_1) h_1(\rho_1, \rho_1).
\]

The fluctuation correlation is defined as

\[
G^{(2,2)}(\rho_1, \rho_2) = \langle I_B(\rho_1) I_B(\rho_2) \rangle - \langle I_B(\rho_1) \rangle \langle I_B(\rho_2) \rangle,
\]

where \( I_B(\rho) = \hat{E}_r(\rho) \hat{E}_r^\dagger(\rho) \) and \( I_s(\rho) = \hat{E}_s(\rho) \hat{E}_s^\dagger(\rho) \). The thermal source can be modeled by the Gaussian–Schell model, which is completely characterized by

\[
\langle \hat{E}_1(\rho_1) \hat{E}_2(\rho_2) \rangle = 0,
\]

\[
\langle \hat{E}_1^\dagger(\rho_1) \hat{E}_2(\rho_2) \rangle = \frac{2}{\pi a_0^2} e^{-\frac{a_0^2 \rho_1^2}{c^2}} e^{-\frac{a_0^2 \rho_2^2}{c^2}},
\]

where \( a_0 \) is the \( e^{-2} \) attenuation radius of the transverse intensity profile, and \( \rho_0 < a_0 \) is the transverse coherence radius. For thermal light, the Gaussian moment theorem establishes

\[
G^{(2,2)}(\rho_1, \rho_2) = \left| \int d\rho_1 d\rho_2 h_1(\rho_1, \rho_1) h_2(\rho_2, \rho_2) \langle \hat{E}_1^\dagger(\rho_1) \hat{E}_2(\rho_2) \rangle \right|^2.
\]

Taking the object arm as an example, the propagation function can be expressed as

\[
h_2(\rho_1, \rho_2) = \frac{k_0}{i 2\pi L} \exp \left[ ik_0 L + \frac{i k_0}{2L} (\rho_1 - \rho_2)^2 \right],
\]

where \( L \) represents the distance the beam travels. Through engineering design, we can achieve the transfer functions \( h_1(\rho_1, \rho_1) = h_2(\rho_1, \rho_1) \). For far-field propagation \((k_0^2/2L \ll 1)\), the second-order correlation result of Eq. (3) is proportional to

\[
G^{(2,2)}(\rho_1, \rho_2) \propto e^{-|\rho_1|^2 + |\rho_2|^2}/a_0^2 e^{-|\rho_1 - \rho_2|^2}/\rho_0^2,
\]

where the transverse intensity radius satisfying \( a_0 = 2L/k_0 \rho_0 \) and the transverse coherence radius is given by \( \rho_0 = 2L/k_0 a_0 \).

For the spatial integration, we assume that \( a_0 \) exceeds the transverse extent of the object by an amount sufficient to permit the approximation,

\[
G^{(2,2)}(\rho_1, \rho_2) \propto e^{-|\rho_1 - \rho_2|^2}/\rho_0^2.
\]

The basic reconstructed image of the object can be expressed as

\[
G(\rho) = \langle I_s(\rho) I_B \rangle - \langle I_s(\rho) \rangle \langle I_B \rangle,
\]

where \( I_B = \int d\rho T(\rho) I_s(\rho) \) represents the bucket detection signal.

By substituting \( I_B \) into Eq. (10) and combining it with Eq. (9), we can obtain the thermal case,

\[
G(\rho) = \int d\rho T(\rho) G^{(2,2)}(\rho_1, \rho_2) = \alpha_1 T(\rho_2) \ast e^{-|\rho_1 - \rho_2|^2}/\rho_0^2,
\]

where \( \ast \) denotes the convolution operation, \( T(\rho_2) \) represents the reflectivity of the object, and \( \alpha_1 \) is a coefficient related to the total reflectivity of the object and the illumination energy.

In the entangled case, the signal and the idler fields are generated by a spontaneous parametric down-conversion (SPDC) process. The input–output relations of the crystal are

\[
\hat{E}_i(\rho) = U_i(\rho) \hat{E}_0(\rho) + V_i(\rho) \hat{E}_0(-\rho), \quad i \neq j = 1, 2,
\]

where \( \hat{E}_i(\rho) \) are the signal \((i = 1)\) and idler \((i = 2)\) field envelope operators at the output face of the crystal. \( \hat{E}_0(\rho) \) are the corresponding fields at the input surface of the crystal, and \( U_i \) and \( V_i \) are gain functions.

Typically, the signal and idler fields can be distinguished by polarization under type-II phase matching. In the plane-wave pump approximation, the coincidence shows

\[
G_{\text{spdc}}(\rho) = \alpha_2 T(\rho_2) e^{-|\rho_1 + \rho_2|^2}/\rho_0^2,
\]

with the convolution of \( T(-\rho_2) \) and the second-order correlation function, where \( \alpha_2 \) represents a proportional coefficient and \( \rho_2 = 2L/k_0 a_0 \) is the transverse coherence length. Although thermal light can simulate a part of the correlation nature of entangled light, there exists a difference between the two sources. And such distinction causes the difference in the geometric and visibility of images.

The results above were obtained with a sufficient number of samples. However, in practice, the number of samples is always limited, and the echo signal can be very weak (the object arm in GI). Taking detection under a specific illumination pattern as a sampling, information extracted by \( N \)-sampling GI becomes
To focus on the sensitivity of imaging, a detection model is established that takes quantum efficiency, background noise, and detection noise into account. Dealing with a noisy detection model and considering the limited resources, we use statistical optics and semi-classical theory to calculate the probability density distribution of the imaging results for both GI and CI. In the case of GI, we can model and analyze the probability distribution of the imaging reference arm, denoted as \( P(I_R(\rho)) \), as well as the bucket detection represented by \( P(I_B) \). Moreover, this approach enables the derivation of their joint probability density distributions \( P(I_R(\rho), I_B) = P(I_R(\rho)) \otimes P(I_B) \). (15)

After obtaining the probability distribution, the mean \( \langle G(\rho) \rangle \) and variance \( \Delta^2 G(\rho) \) can be calculated for each point of GI. This evaluation enables us to assess the effectiveness of the imaging process, which can then be used to determine the sensitivity of imaging based on the required photon number. Specific details will be discussed in subsequent sections.

For clarity, we introduce some symbols throughout the paper.

- \( I_R(\rho) \) is the light intensity distribution of the reference arm in GI.
- \( I_B \) is the bucket detection in GI.
- \( I_0 \) is the light intensity of the light source.
- \( I_B \) is the light intensity of background noise.
- \( n_p \) is the number of signal photons arriving per pixel during each measurement.
- \( n_s \) is the number of photoelectrons produced by signal light per pixel during each measurement.
- \( n_b \) is the number of photoelectrons produced by background noise per pixel during each measurement.
- \( n_d \) is the number of photoelectrons generated by dark noise per pixel during each measurement.
- \( N \) is the number of measurements for imaging.
- \( k \) is the number of pixels occupied by the object.
- \( t \) is the single exposure time of the detector.
- \( T_0 = N \) is the total exposure time for imaging.
- \( p_t \) is the probability of light being reflected back to the detector by the object.
- \( \eta \) is the quantum efficiency of the detector.
- \( \mu_b = \tilde{n}_b \) and \( \mu_d = \tilde{n}_d \), which are the expected values of background noise and dark noise, respectively. And \( \mu = \mu_b + \mu_d \).
- \( q \) is the quantization interval of the detector.

### 2.1. Intensity distribution of the illumination

In a scenario with weak light where the echo is at the level of few photons, it is necessary to take the particle nature of photons into account. For a thermal source, the density operator can be described by

\[
G(\rho) = \frac{1}{N} \sum_{i=1}^{N} ((I_R(\rho) - \langle I_R(\rho) \rangle)(I_B - \langle I_B \rangle)).
\] (14)

For a certain point \( \rho \), the photon statistics follows Bose–Einstein distribution \(^{61}\),

\[
P_n = |c_n(\rho)|^2 = \frac{\langle n(\rho) \rangle^n}{(1 + \langle n(\rho) \rangle)^{n+1}},
\] (17)

where \( \langle n(\rho) \rangle \) is the average number of photons at position \( \rho \). Focusing on the primary goals, we assume the coherence length of the light field on the camera equals one pixel, which is equivalent to the spatial resolution of the imaging system. The photon number \( n(\rho) \) is statistically independent of each pixel.

When the light intensity is sufficiently strong, \( \langle n \rangle \gg 1 \), \( P_n \) is approximately a negative exponential distribution, as is usually referred to in GI,

\[
P_{n \rightarrow \infty} = \frac{1}{\langle n \rangle} e^{-n/\langle n \rangle},
\] (18)

with the approximation of \( \langle n \rangle^n/(1 + \langle n \rangle)^{n+1} \approx (1 - 1/\langle n \rangle)^n/\langle n \rangle \approx e^{-1/\langle n \rangle}/\langle n \rangle \).

On the other hand, the generated entangled state can be expressed as \(^{62}\)

\[
|\psi\rangle = \prod_{\rho} \sum_{n=0}^{\infty} c_n(\rho)|n, \rho\rangle_1|n, -\rho\rangle_2,
\] (19)

which is a superposition of number states at the position \( \rho \) in the idler field and \( -\rho \) in the signal field. It can be obtained that, in the plane wave pump approximation, the photon number distribution is also a Bose–Einstein distribution at each position \(^{56,63}\), as shown in Eq. (17). Since the efficiency of SPDC is low, namely \( \langle n(\rho) \rangle \ll 1 \), the state \( |\psi\rangle \) is usually reduced as

\[
|\psi\rangle = c_0|0\rangle_1|0\rangle_2 + c_1|1, \rho\rangle_1|1, -\rho\rangle_2.
\] (20)

At this time, GI with entangled photons is derived from the coincidences of photon counting. Observed on only one arm, the photon number distribution obeys the Poisson distribution \( P(n) = I_0^k e^{-I_0}/n! \), where \( I_0 \) represents the average light intensity.

### 2.2. Propagation and interaction with the object

In thermal GI, the signal of the reference arm is strong enough to result in a high signal-to-noise ratio (SNR). Besides, for computational GI, the reference arm is obtained by calculation \(^{64}\). Therefore, the detection noise in the reference arm can be disregarded. For the object arm, the primary considerations are the attenuation of the signal light in the propagation and the effect of...
reflection by the object. To explore the basic principle, the target is modeled as a planar 2D object, whose rough surface is quasi-Lambertian that sends light back into the hemisphere with intensity reflection ratio \( T(p) \), where \( (T^{-1}(p)T(p')) = T(p)\delta(p - p') \) and \( T(p) \) is the random field-reflection coefficient\[^{38,34,65}\]. The deterministic pattern \( T(p) \) is what we are trying to image.

For light source with intensity \( I_0 \), the probability that the light from the source is reflected back to the detector by the object is \( p_1 = p_1 T(p)p_2 \), where \( p_1 \) represents the probability of light reaching the object, and \( p_2 \) represents the probability of photons returning from the object to the detector. For distant objects, reaching the object, and tons. Since photons within a speckle are all the same and appear returning from the object to the detector. For distant objects, amplifying and digitizing for output.

\[ n_p \sim B(I_0 t_0/\hbar\nu, p_1) \]  \hspace{1cm} (21)

within the period \( t_0 \). The average number of photons reaching the detector is \( \bar{n}_p = (I_0 t_0/\hbar\nu)p_1 \). And the corresponding variance is \( \Delta^2 n_p = (I_0 t_0/\hbar\nu)p_1(1 - p_1) \). In this paper, to simplify the analysis, the light attenuation is the same in space, and the spatial distortion of the light field caused by atmospheric turbulence is not considered.

### 2.3. Detection

We will discuss two detection models: linear detection and Geiger detection.

#### 2.3.1. Linear mode

For a camera or photoelectric sensor, the process of signal detection can be sketched in Fig. 2. Different noises are introduced into the detection process. The first is the background noise from the environment rather than the interested scene, generating photoelectrons with a number of \( n_b \). The second is the quantum noise due to the quantum efficiency \( \eta < 1 \). The third is the dark noise \( n_d \), which is generated by the detector itself even when there is no light. And the last is the noise caused by quantization\[^{66}\].

Working in the linear zone of the sensor, the number of electrons increases linearly with the number of photons received. In practical cases, for quantum efficiency \( \eta < 1 \), the photoelectron statistics may be very different from that of the arrival photons. If the quantum state of the light field reaching the detection surface is \( |n_p> \), the probability of \( n_p \) photoelectrons generated obeys binomial distribution with the expectation being \( \eta n_p \), as

\[ P_{n_p} = \binom{n_p}{n_p} \eta^n (1 - \eta)^{n_p - n_p}. \]  \hspace{1cm} (22)

Considering the distribution of signal photon \( |c_{n_p}>^2 \), the number distribution of photoelectrons is

\[ P_{n_p} = \sum |c_{n_p}|^2 \binom{n_p}{n_p} \eta^n (1 - \eta)^{n_p - n_p}, \]  \hspace{1cm} (23)

with the expectation \( \bar{n}_p = \eta n_p \). Likewise, because the background noise obeys the Poisson distribution, the noise photoelectrons produced also obey the Poisson distribution with the expected value \( \bar{n}_b = (I_0 t_0/\hbar\nu)\eta \). To focus more on the physical aspects, we mainly consider the randomly fluctuating noise generated by thermal excitation in dark noise. Electrons thermally induced by dark noise are also Poisson distributed\[^{66}\]. The dark noise grows linearly with increasing exposure time \( n_d = n_d t \), where \( n_d \) represents the number of thermal electrons generated per unit of time. Before considering quantization, the mean of the output value \( y \) is

\[ y = K(\bar{n}_b + \bar{n}_d) = K[(I_0 t_0/\hbar\nu) p_1 \eta + (I_0 t_0/\hbar\nu) \eta + n_d t], \]  \hspace{1cm} (24)

where \( K \) is the magnification factor. From a physical point of view, the different noises are linearly independent and can be added linearly\[^{66}\], so the variance of the output value \( y \) is

\[ \Delta^2 y = K^2(\Delta^2 n_b + \Delta^2 n_d + \Delta^2 n_d) \]

\[ = K^2\left(\frac{I_0 t_0}{\hbar\nu} p_1 \eta (1 - p_1 \eta) + \frac{I_0 t_0}{\hbar\nu} \eta + n_d t\right). \]  \hspace{1cm} (25)

Finally, the analog-to-digital conversion introduces another kind of noise (quantization noise)\[^{66}\]. Assuming \( q \) represents the quantization interval, the results can be written as

\[ y = \left[ K\left(\frac{I_0 t_0}{\hbar\nu} p_1 \eta + \frac{I_0 t_0}{\hbar\nu} \eta + n_d t\right)/q\right], \]

\[ \Delta^2 y = K^2\left(\frac{I_0 t_0}{\hbar\nu} p_1 \eta (1 - p_1 \eta) + \frac{I_0 t_0}{\hbar\nu} \eta + n_d t\right)/q^2 + \Delta^2 q. \]  \hspace{1cm} (26)

where \( [.] \) means rounding down and \( \Delta^2 q \) represents the quantization noise. When \( q = 1 \), Eq. (26) then degenerates to
Eqs. (24) and (25). The effect of quantization noise is nonlinear, and we will discuss it in detail separately in Section 4.3.

### 2.3.2. Geiger mode

When the light is faint, it is necessary to use the Geiger mode, which gives only 0 and 1, corresponding to no photon and photons detected. Still, the photon number cannot be determined. For Geiger mode, we assume the probability of producing a dark count is \( \mu \), where \( \mu = \mu_0 + \mu_1 \) and \( 0 < \mu < 1 \). For a single photon, the probability that the detector outputs “0” or “1” is

\[
P(y = 0) = (1 - \eta)(1 - \mu), \quad P(y = 1) = \eta + \mu - \eta \mu, \tag{27}
\]

where \( 0 < \eta < 1 \). The credibility of the signal can be defined as \( \eta/(\eta + \mu) \), so people seek higher quantum efficiency and lower dark noise. In the same way, with \( n_p \) signal photons, the output distribution of the detector is

\[
P(y = 0) = (1 - \eta)^{n_p}(1 - \mu), \quad P(y = 1) = 1 - (1 - \eta)^{n_p}(1 - \mu). \tag{28}
\]

Having the distribution of the signal photons \( n_p \), we get the output distribution of Geiger detection.

### 2.4. Image reconstruction of GI and CI

GI collects echoes with a bucket detector and utilizes the second-order correlation property of the light field to acquire the image of the object. Some new algorithms were developed for different scenes\(^{[67,68]}\). In this paper, we aim to explore the fundamental difference between these two imaging methods. Therefore, we still use the basic and fundamental correlation imaging algorithm as Eq. (14), which can also be written as

\[
G(\rho) = \frac{1}{N} \sum_{i=1}^{N} \Delta I_{b_i}(\rho) \Delta I_{b_i}, \tag{29}
\]

representing the average over a number \( N \) of measurements, where \( \Delta I_{b_i}(\rho) = I_{b_i}(\rho) - \langle I_{b_i}(\rho) \rangle \), \( \Delta I_{b_i} = I_{b_i} - \langle I_{b_i} \rangle \).

For CI, the image of the object is acquired by a detector with spatial resolution. The result of CI becomes accumulated over exposure time \( T_0 \),

\[
C(\rho) = \frac{1}{T_0} \sum_{i=1}^{N} C(\rho), \tag{30}
\]

where \( T = NT \), which is the same as GI. In the case of pulsed illumination, the results of CI become a superposition of the individual pulse measurements,

\[
C(\rho) = \sum_{i=1}^{N} C(\rho). \tag{31}
\]

The accumulation time for CI equals the total sampling time for GI. When there is no quantization noise \( (q = 1) \), Eqs. (30) and (31) are equivalent. However, the two will be significantly different when \( q \gg 1 \). The specific cases related to quantization are also detailed in Section 4.3.

### 3. Evaluation of Imaging Performance

For imaging, the amount of information is contained in the number of photons\(^{[69]}\). Information for optical imaging acquired is related to the SNR and the sampling time, and it is physically affected by photon detection.

#### 3.1. SNR of detection

Due to the limited intensity of source, the echoes may be weak for remote sensing or objects with low reflectivity, even to the level of few photons. Equation (25) tells us that, with average illumination intensity \( I_0 \), the variance of the number of detected photoelectrons \( n_t \) is \( \Delta^2 n_t = (I_0/\eta)(1 - p_t) \eta / (\eta + p_t) \). Thus, when \( p_t \) is very small, the variance of the signal can be approximated as \( \Delta^2 n_t \approx (I_0/\eta)(1 - p_t) \eta / (\eta + p_t) \). In mathematics, when \( p_t \) is very small, the binomial distribution converges to a Poisson distribution. This also explains why the photon number distribution is Poisson when the number of returned photons is small. The SNR is \( n_t / \sqrt{\Delta^2 n_t} = (I_0/\eta)(1 - p_t) \eta / (\eta + p_t) \), which becomes lower as \( I_0 \) and \( p_t \) decrease. This means the correlation between two arms of GI decreases under weak echoes. \( n_t \) obeys Bose–Einstein distribution in time and space, as a result of convolution between Bose–Einstein distribution and the binomial distribution. But the specific distribution of \( n_t \) will be different from \( I_0 \). For random background noise \( I_o \), the noise photoelectrons produced obey Poisson distribution with the expected value \( \mu_b = (I_o/\eta)(1 - p_t) \eta / (\eta + p_t) \). So the variance of the detection signal can be written as

\[
\Delta^2 y = K^2 \left( \frac{I_o I_b}{\eta h_0} p_t \eta + \frac{I_o I_b}{\eta h_0} n_t + n_t \right) = K^2 (n_t + \Delta^2 n_b + \Delta^2 n_d). \tag{32}
\]

For CI, the SNR of the detection signal of each pixel in the array detector is

\[
D_{\text{SNR}}^C = \frac{K n_t}{\sqrt{K^2 (n_t + \Delta^2 n_b + \Delta^2 n_d)}} = \frac{n_t}{\sqrt{n_t + \Delta^2 n_b + \Delta^2 n_d}}. \tag{33}
\]

The denominator is the standard deviation of the entire detection, in which the first term is the effect of shot noise.

For GI, the bucket detector collects the echoes, which are distributed on the array detector in CI, and similarly, the SNR of the detection signal can be obtained as
\[ D_{\text{SNR}}^C = \frac{k \bar{n}_r}{\sqrt{k \bar{n}_r + \Delta^2 n_b + \Delta^2 n_d}}. \]  

(34)

The SNR of the bucket detector is higher than that of the array detector. Here, the ideal linear amplification \( K \) does not affect the SNR in Eqs. (33) and (34), so we assume \( K = 1 \) in the following.

### 3.2. CNR of GI and CI

To measure the image quality, the contrast-to-noise ratio (CNR) of GI is usually calculated as\[70\]

\[ G_{\text{CNR}} = \frac{\langle G(\rho_{\text{in}}) \rangle - \langle G(\rho_{\text{out}}) \rangle}{\sqrt{\Delta^2 G(\rho_{\text{in}}) + \Delta^2 G(\rho_{\text{out}})}}, \]  

(35)

where \( \rho_{\text{in}} \) represents the area occupied by the object in the field of view, called the object region; \( \rho_{\text{out}} \) represents the rest of the area, called the background region; and \( \Delta^2 G \) is the corresponding variance. Similarly, the CNR of CI is

\[ C_{\text{CNR}} = \frac{\langle C(\rho_{\text{in}}) \rangle - \langle C(\rho_{\text{out}}) \rangle}{\sqrt{\Delta^2 C(\rho_{\text{in}}) + \Delta^2 C(\rho_{\text{out}})}}, \]  

(36)

CNR is recognized as an effective indicator to evaluate the quality of binary images.

#### 3.2.1. Image quality of GI

For speckle illumination, suppose the object occupies \( k \) pixels, the bucket detection \( I_B \) is the sum of \( k \) independent speckle signals in the object region as \( I_B = \sum_{i=1}^{k} n_i + n_b + n_d \). The first term represents all signals, whose probability distribution is the convolution of the \( k \) variables\[71\],

\[ P\left( \sum_{i=1}^{k} n_i \right) = P(n_1^{(1)}) \otimes P(n_2^{(2)}) \otimes \cdots \otimes P(n_k^{(k)}). \]  

(37)

The probability distribution can be obtained as

\[ P\left( \sum_{i=1}^{k} n_i = n \right) = \frac{(n + k - 1)!}{(k-1)!n!} \frac{\bar{n}_n^n}{(1 + \bar{n}_n)^{n+k}}. \]  

(38)

For a pixel in the background region \( I_B(\rho_{\text{out}}) \), no correlation with the bucket detection \( I_B \) appears at all, so the joint probability distribution can be written as

\[ P_{\text{out}}(I_B, I_B(\rho_{\text{out}})) = P(I_B) \otimes P(I_B(\rho_{\text{out}})). \]  

(39)

For a pixel in the object region, the bucket detection contains the signal, so the signal \( I_B(\rho_{\text{in}}) \) is correlated with part of the bucket detection \( I_B \). But it has nothing to do with the rest of the bucket detection signal \( I_B' = I_B - I_B(\rho_{\text{in}}) \). So the joint probability distribution of \( I_B(\rho_{\text{in}}) \) and \( I_B \) can be replaced by\[60\]

\[ P_m(I_B, I_B(\rho_{\text{in}})) = P(I_B') \otimes P(I_B(\rho_{\text{in}})). \]  

(40)

In Eq. (40), \( I_B \) and \( I_B(\rho_{\text{in}}) \) are no longer statistically independent.

Then we can get the first-order and the second-order moments in the object region and background region of GI,

\[ \langle I_R(\rho)I_B \rangle = \sum \sum I_R(\rho)I_B \times P_j, \]  

\[ \langle I_R^2(\rho)I_B^2 \rangle = \sum \sum I_R^2(\rho)I_B^2 \times P_j, \]  

(41)

where \( I_R(\rho) \) represents \( I_R(\rho_{\text{in}}) \) and \( I_R(\rho_{\text{out}}) \), and \( P_j \) represents \( P_{\text{in}} \) and \( P_{\text{out}} \). The summation is used here because the function distribution \( P_j \) is discrete. By subtracting the mean from the reference arm and bucket detection results, we can obtain \( \langle \Delta I_R(\rho) \rangle \), \( \langle \Delta I_B \rangle \), and \( \langle \Delta I_R(\rho), \Delta I_B \rangle \) accordingly. According to

\[ G(\rho) = \langle \Delta I_R(\rho) \Delta I_B \rangle, \]  

\[ \Delta^2 G(\rho) = \langle \Delta I_R^2(\rho) \Delta I_B^2 \rangle - \langle \Delta I_R(\rho) \rangle^2 \langle \Delta I_B \rangle^2, \]  

(42)

the CNR of GI can be obtained by substituting Eqs. (41) and (42) into Eq. (35). The signal-related imaging quality can be judged by CNR.

#### 3.2.2. Image quality of CI

Equivalent to \( N \) measurements in GI, the corresponding photon number distribution for CI is \( P(n) = (NL_0)^ne^{-NL_0}/n! \) for Eqs. (30) and (31) because the sum of variables that obey the Poisson distribution also follows the Poisson distribution. Then the distribution of the photoelectron number of array detectors is

\[ P(C(\rho_{\text{in}}) = n) = \left( N \left( \frac{l_{\text{in}}}{\hbar \nu} \eta + \frac{l_{\text{in}}}{\hbar \nu} \eta + n_\text{I} t \right) \right)^n \]  

\[ \times e^{-N \left( \frac{l_{\text{in}}}{\hbar \nu} \eta + \frac{l_{\text{in}}}{\hbar \nu} \eta + n_\text{I} t \right)}/n!, \]  

\[ P(C(\rho_{\text{out}}) = n) = \left( N \left( \frac{l_{\text{in}}}{\hbar \nu} \eta + n_\text{I} t \right) \right)^n e^{-N \left( \frac{l_{\text{in}}}{\hbar \nu} \eta + n_\text{I} t \right)}/n!. \]  

(43)

The average photoelectrons numbers of the object region and the background region are

\[ \langle C(\rho_{\text{in}}) \rangle = N \left( \frac{l_{\text{in}}}{\hbar \nu} \eta + \frac{l_{\text{in}}}{\hbar \nu} \eta + n_\text{I} t \right), \]  

\[ \langle C(\rho_{\text{out}}) \rangle = N \left( \frac{l_{\text{in}}}{\hbar \nu} \eta + n_\text{I} t \right). \]  

(44)
The corresponding variances are
\[ \Delta^2 \mathcal{C}(\rho_{in}) = N \left( \frac{I_0 I_0}{\hbar \omega} p_0 \eta + \frac{I_0 I_0}{\hbar \omega} n_1 t \right), \]
\[ \Delta^2 \mathcal{C}(\rho_{out}) = N \left( \frac{I_0 I_0}{\hbar \omega} \eta + n_1 t \right). \]  

(45)

Similarly, the CNR of CI is
\[ C_{\text{CNR}} = \frac{N \frac{I_0 I_0}{\hbar \omega} p_0 \eta}{\sqrt{N \frac{I_0 I_0}{\hbar \omega} p_0 \eta + 2 N \left( \frac{I_0 I_0}{\hbar \omega} \eta + n_1 t \right)}}. \]  

(46)

For simplicity, Eq. (46) can be rewritten as
\[ C_{\text{CNR}} = \frac{\sqrt{N} \hat{n}_s}{\sqrt{\hat{n}_s + 2(\Delta^2 n_b + \Delta^2 n_d)}}. \]  

(47)

Without background noise and dark noise, Eq. (47) becomes a simple form as \( C_{\text{CNR}} = \sqrt{N} \hat{n}_s \).

3.3. Imaging sensitivity

For detectors, when the SNR reaches 1, the number of signal photons detected is defined as the sensitivity. As an analogy, we propose to define the required number of detected signal photons to achieve CNR = 1 as imaging sensitivity. The fewer photons that are needed to reach this standard, the higher the sensitivity of the imaging method and the stronger the imaging ability for weak echo. According to the definition of imaging sensitivity, it is possible to measure the capabilities of different imaging methods under weak signals.

4. Comparison between Thermal GI and CI

In this section, we analyze the sensitivity of GI under background noise and detection noise. Then GI is compared with CI under linear detection in Section 4.1. The quality of GI under Geiger detection is also analyzed in Section 4.2. The impact of the quantization is discussed in Section 4.3 separately.

In order to better compare the sensitivity of GI and CI, we make the following assumption.

(i) Both GI and CI obtain images with the same imaging time, which is defined as the sum of pulse durations for samplings.

(ii) Without special instructions, the ambient noise, quantum efficiency, dark noise, and quantization noise of the bucket detector in GI are set equal to that of one pixel of the array detector in CI.

(iii) In GI, the average size of speckle is the same as the pixel size, and resource consumption is measured in pixels. The pixels are independent of each other.

(iv) The light source of GI is thermal, which obeys Bose–Einstein distribution; the light source of CI is a common laser source, which obeys Poisson distribution.

(v) In thermal GI, with large intensity in the reference arm, the detection noise can be disregarded.

(vi) The spatial distribution of the light field has no distortion during the propagation process.

(vii) Assume a binary object.

Then the sensitivities of the two imaging methods is compared.

4.1. Noiseless case under linear detection

From the previous model, we can get the photoelectron number of the bucket detector, \( I_B = \sum_{i=1}^{N} n_i^2 + n_b + n_d \). Putting the probability distribution of \( I_B \) into Eq. (42), the CNR of GI can be obtained as
\[ G_{\text{CNR}} = \frac{\sqrt{N} \hat{n}_s}{\sqrt{(2k + 7) \hat{n}_s^2 + 2k \hat{n}_s + 2(\Delta^2 n_b + \Delta^2 n_d)}}. \]  

(48)

As analyzed before, both background noise and dark noise obey Poisson distributions, so \( \mu_b = \Delta^2 n_b, \mu_d = \Delta^2 n_d \). Since Poisson distributions are additive, \( \mu = \mu_b + \mu_d \), the result of Eq. (48) can be written as
\[ G_{\text{CNR}} = \frac{\sqrt{N} \hat{n}_s}{\sqrt{(2k + 7) \hat{n}_s^2 + 2k \hat{n}_s + 2\mu}}. \]  

(49)

In Eq. (49), \( (2k + 7) \hat{n}_s^2 + 2k \hat{n}_s + 2\mu \) in the denominator are the effects of irrelevant speckle signals in bucket detection, shot noise, and background and dark noise, respectively. Ideally, if \( \hat{n}_s \) is large, Eq. (48) approximately is
\[ G_{\text{CNR}} \approx \frac{\sqrt{N}}{2k + 7}. \]  

(50)

which is consistent with the result in Ref. [48]. That means, in the case of a high SNR, the imaging quality of GI is not related to the absolute intensity of the signal.

Figure 3(a) intuitively shows the variations in CNR with \( \hat{n}_s \) and \( k \), with \( N = 1000 \). The CNR will increase as \( \hat{n}_s \) increases and decrease as \( k \) increases. To analyze the trend more clearly, the CNR at different noise levels \( \mu \) and object sizes \( k \) is illustrated in Figs. 3(b) and 3(c). It can be seen in Fig. 3(b) that noise shows a significant impact on the image quality under weak light, and the sensitivity of GI also decreases. When CNR = 1, the average number of photoelectrons at noise levels \( \mu = 0, \mu = 5 \), and \( \mu = 10 \) is \( \hat{n}_s = 0.02, \hat{n}_s = 0.11 \), and \( \hat{n}_s = 0.15 \), respectively. With the increase in signal intensity, the influence of noise becomes smaller, and the imaging results in three cases tend to be the same. In Fig. 3(c), the effect of \( k \) is more obvious when the light is strong. Larger \( k \) corresponds to poorer image quality.

Then GI is compared with CI. When the parameters of the bucket detector for GI are the same as the parameters of a pixel in CI, the ratio of the CNR between them is
sents

\[
\mu \text{ of GI has a relatively small decline. However, CI requires fewer}
\]

\[
\text{increasing and is much faster than that of CI. Figure 4(b indicates the results of the two imaging methods in the presence of noise. The blue line represents}
\]

\[
\text{the results of GI in the absence of noise. In this situation, } \bar{n}_s \text{ shows the results of both imaging methods in the absence of noise. In this situation, } \bar{n}_s \text{ of GI increases rapidly with CNR increasing and is much faster than that of CI. Figure 4(b indicates the results of the two imaging methods in the presence of noise. The blue line represents } \mu = 10, \text{ and the red line represents } \mu = 20. \text{ The CNR of CI is drastically reduced, while that of GI has a relatively small decline. However, CI requires fewer}
\]

\[
\text{photons and is still more sensitive than GI. It should be noted that the imaging quality of CI is not affected by the pixel number } k \text{ occupied by the object, but the imaging quality of GI decreases as the object size } k \text{ increases. To illustrate this, the red diamond “○” in Fig. 4(b) represents the results of GI in the case of } \mu = 20 \text{ and } k = 20. \text{ So, in Eq. (51), the ratio } \frac{G_{\text{CNR}}}{C_{\text{CNR}}} \text{ is always less than } 1 \text{ for this case, and the sensitivity of GI is not as good as CI.}
\]

\[
\text{However, in fact, the sensitivity of a single-pixel detector can be better than that of an array detector. For example, it has higher detection efficiency and lower noise levels. Subsequently, we will analyze the sensitivity of GI and CI based on these two points.}
\]

\[
\text{Generally speaking, the detection efficiency is related to the photoelectric conversion material, the effective detection area, and the transfer efficiency of the circuit. For some industrial cameras, each pixel is integrated with a transfer circuit to improve the output frame rate. The effective detection area of pixels is significantly reduced, and the detection efficiency of array detectors is much lower than that of single-pixel detectors. Based on the above considerations, assuming the detection efficiency of each pixel in the array detector is 60% the single-pixel detector and the noise levels are the same, the results of both imaging methods are shown in Fig. 5. Figure 5(a) is the ideal case without noise, and CI still performs better than GI. Figure 5(b) shows the image results in the case of } \mu = 10 \text{ and } \mu = 20, \text{ with } N = 3000. \text{ Figure 5(b) indicates that the imaging sensitivity of GI is better than CI when noise exists. When } \text{CNR = 1} \text{ is achieved, the } \bar{n}_s \text{ required for GI and CI is 0.12 and 0.19, respectively. It can be concluded that GI is more sensitive than CI, with better detection efficiency in low-light conditions.}
\]

\[
\text{Corresponding imaging simulations were done to check these results. Figures 6(a) and 6(b) describe the simulation process of GI and CI, respectively. A speckle field } I_0(\rho) \text{ with a negative exponential distribution is generated, which is quantized to 0–255 as a reference arm } [I_0(\rho)]. \text{ The light intensity is strong so that the noise, which is tiny compared to the signal, can be ignored. Then the light intensity of speckle patterns is converted}
\]

\[
G_{\text{CNR}} = \frac{\bar{n}_s + 2\mu}{\sqrt{(2k + 1)\bar{n}_s^2 + 2k\bar{n}_s + 2}} \tag{51}
\]
into the number of photons, $I_0(\rho) t/(h\nu)$. For photons of such amount, we calculate the number of returned photons. Each photon reaches the object surface with probability $p_1$. The object is a binary object with a reflectivity of $T(\rho)$. And the reflected photon reaches the detector with probability $p_2$. According to the total number of photons, transmission probability, and reflectivity, the photon number distribution arriving at the detector can be obtained. Based on this probability distribution, a random number is taken as the number of returned photons. This is equivalent to investigating photon by photon, which starts from the light source and returns to the detector according to the calculated probability, being determined whether detected or not randomly. Similarly, considering detection efficiency, the corresponding signal is probabilistically obtained from arriving photons. The output of the detector can be calculated based on the detection model in Section 2.3. In GI, the bucket detection is obtained by collecting all the received photons. For CI in Fig. 6(b), direct imaging with an array detector is performed. Echo is received by a pixel at the corresponding position, and noise exists in all pixels.

Figure 7 shows the corresponding simulation results for the red line in Fig. 5(b). The upper row is the result of GI, and the lower row is the result of CI. The horizontal axis represents the change in $\bar{n}_s$. The field of view is $16 \times 16$ pixels, and the object is a letter “N” with $k = 22$. The simulation results are consistent with our theoretical analysis. When the signal is relatively strong, the imaging quality of CI is better than that of GI, but when the signal is weak ($\bar{n}_s < 0.9$), the imaging quality of GI is better than that of CI.

Meanwhile, the noise levels of single-pixel detectors are usually much lower than that of array detectors. The sensitivity of a single-pixel detector can be an order of magnitude higher than that of an array detector. In Fig. 8, we demonstrate the relationship between the number of detected photons and the CNR of reconstructed images under different noise conditions. The dotted green line, dotted blue line, dotted yellow line, dotted black line, and dotted purple line represent the imaging results of GI under the noise levels of $\mu = 2$, $\mu = 5$, $\mu = 10$, $\mu = 15$, $\mu = 20$, respectively. The intersection points of these curves and the blue chain line represent the imaging sensitivity with different noise levels. The imaging sensitivity of GI is improved with the decrease of the dark noise of the detector. With the lower dark noise, the number of signal photons needed for GI is less to achieve $\text{CNR} = 1$. For example, for GI with $\mu = 2$, the expected number of imaging photoelectrons to reach $\text{CNR} = 1$ is $\bar{n}_s = 0.04$, but for CI under $\mu = 20$, the expected number of imaging photoelectrons is $\bar{n}_s = 0.12$. So when the level of dark noise in the single-pixel detector is lower than that of a pixel in the array detector, the sensitivity of GI is higher than that of CI in weak light conditions.

The corresponding simulation imaging results are shown in Fig. 9. The upper row represents the results of GI with $\mu = 2$, and the lower row represents the results of CI with noise level $\mu = 20$. It can be seen that, when $\bar{n}_s \leq 0.5$, the imaging results of GI are better than that of CI. The imaging results also confirmed our conclusion that, when the sensitivity of a single-pixel detector is better than a pixel of the array detector, GI is more sensitive than CI in weak light.

Furthermore, for the total signal photoelectrons $n_{\text{sum}} = N\bar{n}_s$, the number of signal photoelectrons measured each time and the number of measurements are inversely related. The CNR of GI can be written as

$$\frac{n_{\text{sum}}}{\bar{n}_s} = \frac{N}{\bar{n}_s} = \frac{N\bar{n}_s}{\bar{n}_s} = N,$$

where $N$ is the number of measurements. The CNR of GI can be written as

$$\text{CNR}_\text{GI} = \frac{N\bar{n}_s}{\bar{n}_s} = \frac{N\bar{n}_s}{\bar{n}_s} = N.$$  

Fig. 6. Flow chart of the simulation process of (a) GI and (b) CI.

Fig. 7. The simulation imaging results of GI and CI in Fig. 6(b), with $\mu = 20$.

Fig. 8. The imaging sensitivity of CI and GI under different noise levels. GI: $\mu = 2, 5, 10, 15, 20$, and CI: $\mu = 20$.  

Fig. 9. The imaging results of GI and CI under different noise levels. GI: $\mu = 2, 5, 10, 15, 20$, and CI: $\mu = 20$.  

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Fig. 9. The simulation imaging results of GI and CI in Fig. 8, with noise levels $\mu = 2$ and $\mu = 20$, respectively.

$G_{\text{CNR}} = \frac{n_{\text{sum}}}{\sqrt{2(k + 7)n_s + 2kn_{\text{sum}} + 2\mu n_{\text{sum}}/n_s}}.$ \hfill (52)

It means there is an optimal signal intensity for the best image quality. The best image quality can be obtained when $n_s = \sqrt{2\mu n_{\text{sum}}/(2k + 7)}$. At this time, the optimal number of measurements is $N = \sqrt{(2k + 7)n_{\text{sum}}/(2\mu)}$. This has an impact on the performance of GI when total energy is limited, which may further improve the sensitivity of GI.

4.2. GI under Geiger mode detection

In low-light conditions, in pursuit of higher sensitivity and the limitations of current technology, it is necessary to use the Geiger mode in bucket detection. In this section, the imaging quality in Geiger mode is analyzed and compared to the linear mode. For the sum of $k$ speckles, combining Eqs. (28) and (38), the response probability of the Geiger mode bucket detector is

$$P(I_B = 0) = \frac{1}{(1 + \bar{n}_s)^k} (1 - \mu) = \frac{1 - \mu}{(1 + \bar{n}_s)^k},$$

$$P(I_B = 1) = 1 - \frac{1 - \mu}{(1 + \bar{n}_s)^k}. \hfill (53)$$

The bucket detection provides only two values, with the dynamic range significantly reduced. So the joint probability distribution of $I_B(\rho)$ and $I_B$ can be divided into two cases,

$$P(I_B(\rho), I_B) = \begin{cases} P(I_B(\rho), I_B = 1), \\ P(I_B(\rho), I_B = 0). \end{cases} \hfill (54)$$

Then the mean and variance of the final imaging results can be calculated. Because $\bar{n}_s \ll 1$, we use Taylor’s formula to expand the formula $1/(1 + \bar{n}_s) \approx 1 - \bar{n}_s + \bar{n}_s^2$ in the calculation process. After calculation and bringing the results into Eq. (14), the CNR is

$$G_{\text{CNR}}^\text{geiger} = \frac{\sqrt{N}A_1}{\sqrt{A_2 + A_3}}, \hfill (55)$$

$$A_1 = (\bar{n}_s - 4\bar{n}_s^2) \frac{1 - \mu}{(1 + \bar{n}_s)^{k-1}},$$

$$A_2 = \frac{1 - \mu}{(1 + \bar{n}_s)^k} - \left(\frac{1 - \mu}{(1 + \bar{n}_s)^k}\right)^2,$$

$$A_3 = -\left(1 - 2\frac{1 - \mu}{(1 + \bar{n}_s)^k}\right)\left(\frac{1 - \mu}{(1 + \bar{n}_s)^{k-1}} (1 - 3\bar{n}_s + 14\bar{n}_s^2)\right).$$

For such “click or not” response, one can easily take the coincidence counting as

$$G(x) = \langle I_B(\rho)I_B \rangle. \hfill (56)$$

Only the speckle field with the bucket detection $I_B = 1$ remains, so Eq. (56) actually becomes $G(x) = \frac{1}{N} \sum_{i=1}^{N} I_B(\rho)$. The CNR result of Eq. (56) is

$$G_{\text{CNR}}^\text{geiger} = \frac{\sqrt{N}B_1}{B_2 + B_3}, \hfill (57)$$

where

$$B_1 = \frac{1 - \mu}{(1 + \bar{n}_s)^k} - \left(\frac{1 - \mu}{(1 + \bar{n}_s)^k}\right)^2,$$

$$B_2 = 1 - \left(\frac{1 - \mu}{(1 + \bar{n}_s)^k}\right)^2,$$

$$B_3 = 2 - (2 - 2\bar{n}_s + 24\bar{n}_s^2) \frac{1 - \mu}{(1 + \bar{n}_s)^{k-1}}$$

$$- \left(1 - \frac{1 - \mu}{(1 + \bar{n}_s)^{k-1}} (1 - 2\bar{n}_s + 6\bar{n}_s^2)\right)^2.$$
time, fluctuations in the photon number will never be reflected in the detection results, so the imaging quality will gradually tend to zero. Moreover, in Geiger mode, the speckle field with $I_B = 0$ also contains the information of the object, which is lost in Eq. (56). When the bucket detection provides 0, it can be regarded as negative fluctuation caused by the object. For fluctuation correlation, all the information of bucket detection is used, so the amount of information brought by the fluctuation correlation in Eq. (14) is more significant than the general coincidence counting in Eq. (56).

In Fig. 10(b), GI in Geiger mode is compared to linear mode with the same detection parameters, and both use fluctuation correlation in Eq. (14). The solid blue line represents linear mode detection, and the solid red line represents Geiger mode detection. And the asterisk represents the simulation imaging results. When the light signal is very weak, the imaging results of linear detection and Geiger detection are the same. As the signal strength increases, the amount of information obtained by Geiger detection is reduced, and the imaging quality of linear detection gradually increases. The simulation imaging results are in good agreement with the theoretical results in linear mode. For Geiger mode, there is a slight difference between the simulation results and theoretical curve. The reason is that we did an approximation in theoretical calculations.

So, with the same detection efficiency, the utilization of Geiger mode actually limits the acquisition of information in GI. On the other hand, the detection efficiency is always higher in Geiger mode than linear mode, reducing the required number of signal photons $\bar{n}_p$ in low-light conditions. Therefore, when evaluating Geiger detection, it is crucial to consider its sensitivity and dynamic range in relation to the specific situations.

### 4.3. Influence of quantization

The quantization process of the signal introduces quantization noise, and the inevitable digital threshold makes weak signals undetectable. The impact of quantization on GI and CI is discussed in this section.

#### 4.3.1. GI under quantization

Considering the detector with a response threshold $q$, the output of $y$ photoelectrons becomes $\lfloor y/q \rfloor$, where $\lfloor \cdot \rfloor$ means rounding down. For the bucket detection, the result after quantization is $I_B^q = \lfloor I_B/q \rfloor = \lfloor (\sum n_i \mu_n + n_b + n_\theta)/q \rfloor$. The process of quantization is a nonlinear and irreversible operation. With strong light intensity $I_B \gg q$, the expected and variance of bucket detection approximately are

$$E(I_B^q) = E(I_B)/q, \quad D(I_B^q) = D(I_B)/q^2.$$  

(58)

In this case, the final CNR is not affected by $q$, and the influence of threshold can be ignored.

As the signal decreases even compared to the threshold $I_B \sim q$, the impact of $q$ becomes significant. The relationship between the CNR of GI and $q$ cannot be expressed by a simple formula.

Given the parameters, we can calculate the probability distribution function with $q$,

$$P(I_B^q = n) = \sum_{x=nq+1}^{x=(n+1)q} P(I_B = x).$$  

(59)

The first-order and second-order moments of the bucket detection distribution can be obtained as in Eq. (60),

$$\{I_B^q\} = \sum_{i=1}^{\infty} I_B^q(i) P(I_B^q(i)),$$

$$\{(I_B^q)^2\} = \sum_{i=1}^{\infty} (I_B^q(i))^2 P(I_B^q(i)).$$  

(60)

Then the mean and variance of the bucket signal can be obtained. During the calculation of the CNR, we used an approximation,

$$\langle I_R[I_B] \rangle = \langle I_R I_B \rangle \frac{\{I_B\}}{\{I_B\}}.$$  

(61)

Figure 11 shows the results of theoretical [Fig. 11(a)] and imaging simulation [Fig. 11(b)]. The horizontal axis is $\bar{n}_p$. The blue, red, and yellow lines indicate that the number of pixels occupied by the object is $k = 20$, $k = 40$, and $k = 80$, respectively, with $\mu = 0$ and $N = 1000$.

Generally speaking, for a pixel $I(q)\rho$ in the reference arm, when the corresponding signal $n_i$ in bucket is much smaller than the threshold $q$, the fluctuation of this signal will not exceed the quantitative interval, which is challenging to detect. However, the results show that even if the signal of one pixel is much smaller than the threshold, the whole echo is still greater than $q$ after being accumulated by bucket detector. At this time, the image can still be obtained through correlation. In Fig. 11(a), when the light intensity is strong, the smaller $k$ leads to the higher CNR as usual. However, as the light intensity decreases, the image quality of objects with smaller $k$ decreases faster, and the CNR of the small object becomes smaller than the big one. The reason is that, when the signal is comparable to the threshold, the higher overall bucket signal intensity provides greater resistance to the threshold’s influence, resulting in better quality for ghost images with larger $k$. The simulation imaging results

![Fig. 11. The variation of CNR with different object sizes $k$ with $q = 10$. (a) Theoretical results. (b) Simulation imaging results.](image-url)
under the same condition are plotted in Fig. 11(b). The results of simulation are basically the same as theoretical results. The slight difference may be due to the influence of our approximation in Eq. (61) of the theoretical calculation.

4.3.2. CI under quantization

For CI accumulating over the time of exposure in Eq. (30), the result with threshold $q$ is

$$C(\rho) = \left[ \frac{1}{q} \int_{0}^{1} C(\rho) \right].$$

(62)

The probability distributions of output results are

$$P(I_q(\rho_{in}) = n) = \sum_{x=nq+1}^{x=(n+1)q} \frac{(N(\rho n + \mu))^x}{x!} e^{-N(\rho n + \mu)}.$$  

$$P(I_q(\rho_{out}) = n) = \sum_{x=nq+1}^{x=(n+1)q} \frac{(N(\rho n + \mu))^x}{x!} e^{-N(\rho n)}.$$  

(63)

For the case of pulsed illumination in Eq. (31), the result becomes

$$C(\rho) = \sum_{i=1}^{N} [C_i(\rho)].$$

(64)

The corresponding probability distributions of output results become

$$P(I_q(\rho_{in}) = n) = N \sum_{x=nq+1}^{x=(n+1)q} \frac{(\tilde{n}_i + \mu)^x}{x!} e^{-(\tilde{n}_i + \mu)},$$

$$P(I_q(\rho_{out}) = n) = N \sum_{x=nq+1}^{x=(n+1)q} \frac{\mu^x}{x!} e^{-\mu}.$$  

(65)

According to

$$\langle C_q(\rho) \rangle = \sum_{i=1}^{I_q} I_q P(I_q(\rho)),$$

$$\langle C_q^2(\rho) \rangle = \sum_{i=1}^{I_q} I_q^2 P(I_q(\rho)),$$

(66)

where $I_q$ represents $I_q(\rho_{in})$ or $I_q(\rho_{out})$, the CNR of CI in both cases can be obtained by Eq. (36).

4.3.3. Comparison of GI and CI under quantization

The performance of GI and CI under quantization is compared in Fig. 12. The horizontal axis is the CNR, and the vertical axis is $\bar{n}_s$, with $k = 20$ and $N = 1000$. The solid red line represents GI, and the dotted blue line represents CI. CI in Figs. 12(a) and 12(b) is the result of exposure time accumulation in Eq. (62), and CI in Figs. 12(c) and 12(d) is the result of pulsed illumination in Eq. (64). In Figs. 12(a) and 12(b), the imaging sensitivity of CI is always better than that of GI. Without noise, the increase in the CNR requires few imaging photons. With the noise $\mu = 10$, the CNR of both GI and CI decreases, while the decline of CI is more pronounced.

In Fig. 12(c), when the expected number of signal photons is extremely small, the signal photoelectrons $n_s$ of CI cannot exceed the threshold during the exposure time. In contrast, bucket detection collects the signal in GI, which makes it easier to exceed the threshold level in a single measurement and achieve a higher detection SNR. So in the case of Fig. 12(c), when the signal is weak, GI exhibits higher imaging sensitivity compared to CI, resulting in a sensitivity advantage. In Fig. 12(d), the noise level is equal to the quantization interval. For both GI and CI, the change of signal can be obtained by the detector. At this time, the imaging sensitivity of CI is improved in weak signals, which is better than that of GI.

Figure 13 shows the simulation imaging results of Fig. 12(c). It can be seen that, when $\tilde{n}_s \leq 4$, the imaging quality of CI is indeed worse than that of GI, but as the signal increases, the imaging quality of CI increases rapidly and surpasses that of GI. These results are consistent with our theoretical analysis.

5. Comparison between Entangled GI and CI

In thermal light GI, the impact of shot noise is significant in weak light, which seriously affects the correlation between the reference arm and the bucket detection. However, the entangled
For entangled light sources, combining Eqs. (17) and (28), the response probability of the detector in the reference arm is 

\[ P(I_B = 0) = \frac{1 - \mu}{1 + \eta \bar{n}_p}, \]

\[ P(I_B = 1) = 1 - \frac{1 - \mu}{1 + \eta \bar{n}_p}. \]  

The coincidence count is calculated when two detectors click at the same time. For the pixels in background region, the joint probability distribution between \( I_i \) and \( I_B \) is 

\[ P(I_B I_i = 1) = p(I_B = 1)P(I_i = 1) = \left(1 - \frac{1 - \mu}{1 + \eta \bar{n}_p} \right) \times \frac{\eta \bar{n}_p + \mu}{1 + \eta \bar{n}_p}, \]

\[ P(I_B I_i = 0) = 1 - p(I_B I_i = 1). \]  

Then we can get the first moment and second moment,

\[ \langle I_B I_i \rangle = \left(1 - \frac{1 - \mu}{1 + \eta \bar{n}_p} \right) \times \frac{\eta \bar{n}_p + \mu}{1 + \eta \bar{n}_p}, \]

\[ \langle I_B^2 I_i^2 \rangle = \left(1 - \frac{1 - \mu}{1 + \eta \bar{n}_p} \right) \times \frac{\eta \bar{n}_p + \mu}{1 + \eta \bar{n}_p}. \]  

In the same way, for the pixels in the object region, the joint probability distribution between \( I_i \) and \( I_B \) is 

\[ P(I_B I_i = 1) \]

\[ = \frac{\mu}{1 + \eta \bar{n}_p} \times \left(1 + \frac{1 - \mu}{(1 + \eta \bar{n}_p)^{k-1}} \right) \times \frac{\eta \bar{n}_p + \mu}{1 + \eta \bar{n}_p} \times \left(\eta + (1 - \eta) \left(1 - \frac{1 - \mu}{(1 + \eta \bar{n}_p)^{k-1}} \right)\right), \]

\[ P(I_B I_i = 0) = 1 - p(I_B I_i = 1). \]  

If \( \mu = 0 \) and \( \eta = 0 \), then \( P(I_B I_i = 1) = \bar{n}_p/(1 + \bar{n}_p) \), which is consistent with our previous results. Then the CNR can be obtained as 

\[ G_{\text{CNR}} = \frac{\sqrt{N} (\mu - 1) \eta \bar{n}_p (\mu + \eta (1 - \eta) \bar{n}_p - \eta)}{(1 - \mu) (C_1 + C_2 \times C_3 + C_4)}, \]  

where 

\[ C_1 = (\mu + \eta \bar{n}_p)(\mu - 1 + (1 + \eta \bar{n}_p)^k) \]

\[ C_2 = \mu + \eta \bar{n}_p + \mu \eta \bar{n}_p + \eta \bar{n}_p(\eta \bar{n}_p - 1) - \eta^2 \bar{n}_p^2 + (1 + \eta \bar{n}_p)^k, \]

\[ C_3 = \mu^2 (1 + \bar{n}_p) + \bar{n}_p(-1 + \eta - \bar{n}_p + \eta \bar{n}_p + \eta^2 \bar{n}_p^2 + (1 + \eta \bar{n}_p)^k), \]

\[ C_4 = \mu(-1 + \eta^2(\bar{n}_p - 1) \bar{n}_p - \eta \bar{n}_p^2 + (1 + \eta \bar{n}_p)^k). \]  

Figure 13 shows the change of the CNR with \( \bar{n}_p \) at different \( \eta \) and \( \mu \), with \( k = 20 \) and \( N = 1000 \). The higher the quantum efficiency \( \eta \) and the lower the noise level \( \mu \), the better the imaging quality. As the number of signal photons \( \bar{n}_p \) increases, the CNR increases first, then decreases, and finally tends to zero. The reason is that, with low light power, the number of coincidence
counts is tiny, and the image quality increases as the number of counts increases. However, as the number of photon pairs increases to a certain extent, there will be more than one entangled photon pair in each pulse. The correlation between each photon pair decreases, resulting in the decline of imaging quality.

Nowadays, detectors develop rapidly, and some sensors can already resolve photons. Then, the imaging quality of entangled light with the linear detection mode is analyzed, which can resolve the number of photons for each pulse. In general, the increase of photon pairs brings the rise of information. In entangled GI, both arms use the same detection methods, which are affected by quantum efficiency and dark noise. The reference arm can be written as \( I_R (\rho) = I_R' (\rho) + n_d \). The background noise only exists in the bucket detection, and the bucket detection can be written as \( I_B = I_B' + n_b + n_d \). The probability distributions of \( I_B' \) and \( I_R' \) are, respectively,

\[
P(I_B' (\rho) = n) = \frac{(\eta \bar{n}_p)^n}{(1 + \eta \bar{n}_p)^{n+1}},
\]

\[
P(I_R' = n) = \frac{(n + k - 1)!}{(k - 1)!n!} \frac{(\eta \bar{n}_p)^n}{(1 + \eta \bar{n}_p)^{n+k}}.
\]

Same as the model above, for the background region, the joint probability distribution between \( I_B \) and \( I_R \) is

\[
P(I_B (\rho_{out}), I_R) = \frac{(I_B - \mu_b - \mu_d + k - 1)!}{(k - 1)! (I_B - \mu_d - \mu_d)!} \times \frac{(\eta \bar{n}_p)^{\mu_b + \mu_d - 2\mu_d}}{(1 + \eta \bar{n}_p)^{\mu_b + \mu_d - 2\mu_d + k + 1}}.
\]

For a specific point in the object region \( I_B (\rho_{in}) \), the joint probability distribution of \( I_B \) and \( I(\rho_{in}) \) can be obtained as

\[
P(I_B (\rho_{in}), I_B) = \frac{(I_B - I_B - \mu_b + k - 2)!}{(k - 2)!} \frac{(\eta \bar{n}_p)^{I_B - \mu_b}}{(1 + \eta \bar{n}_p)^{I_B - \mu_b + k}}.
\]

After calculation, the image quality from Eq. (73) becomes

\[
G_{\text{CNR}} = \frac{\sqrt{N \eta \bar{n}_p (1 + \eta \bar{n}_p)}}{\sqrt{2 \mu_d^2 + D_1 + D_2}},
\]

where

\[
D_1 = \eta \bar{n}_p (1 + \eta \bar{n}_p) (1 + 2 \mu_b + (7 + 2k) \eta \bar{n}_p + (7 + 2k) \eta^2 \bar{n}_p^2),
\]

\[
D_2 = 2 \mu_d (\mu_b + (1 + k) \eta \bar{n}_p (1 + \eta \bar{n}_p)).
\]

The image quality under noise is shown in Fig. 15, with \( k = 20 \) and \( N = 1000 \). The solid red line represents the change of the CNR with signal intensity under ideal circumstances. The solid green line represents the results under quantum efficiency \( \eta = 60% \). The influence of quantum efficiency on imaging quality is relatively slight. The black and yellow dashed lines represent the results under background noise \( \mu_d = 1 \) and \( \mu_b = 2 \), respectively, whose impact is also insignificant. The blue and purple dotted lines represent the imaging quality with dark noise \( \mu_d = 1 \). The dark noise, which exists in both the reference arm and the bucket detector, has a greater impact on the imaging quality. The noise seriously reduces the imaging quality under low-light conditions.

Then the imaging results of GI and CI are compared under the same noise condition in Fig. 16. The horizontal axis represents CNR, and the vertical axis represents \( \bar{n}_p \). The solid line represents GI, and the dotted line represents CI, with \( k = 20 \) and \( N = 1000 \). The chain line in dark blue in Fig. 16 represents \( CNR = 1 \). Figure 16(a) shows the influence of dark noise \( \mu_d \). The red, green, and blue lines represent the imaging quality of GI and CI in the case of \( \mu_d = 0 \), \( \mu_d = 1 \), and \( \mu_d = 2 \), respectively. Entangled GI does not have advantage in sensitivity at this time. The influence of \( \mu_b \) is shown in Fig. 16(b). The red, green,
and blue lines represent the imaging quality of GI and CI in the case of \( \mu_b = 0, \mu_b = 1, \) and \( \mu_b = 2, \) respectively. It can be seen that the imaging results of CI are greatly affected by background noise, while the imaging results of GI are less affected by \( \mu_b. \) As the signal strength decreases, the imaging sensitivity of CI decreases rapidly, while the imaging sensitivity of GI decreases slowly. For example, with \( \mu_b = 2, \) the imaging sensitivity of GI is better than that of CI. So in the case of background noise, GI shows obvious advantages in sensitivity.

Figure 17 shows the simulation imaging results with \( \mu_b = 2 \) in Fig. 16(b). When \( \bar{n}_s > 0.3, \) the imaging quality of GI is not as good as CI, while for \( \bar{n}_s < 0.3, \) the imaging quality of GI is better than that of CI. The result is the same as the theoretical analysis. For entangled light, even if the parameters of the bucket detector (GI) and array detector (CI) are the same, GI is more sensitive than CI in weak light under background noise conditions. It also shows that the reason for image quality degradation for thermal light GI is mainly the decrease of correlation caused by shot noise in weak signal. The recent article verified our results to a certain degree\(^{[72]}\).

### 6. Discussion

The entangled photon pairs have strong correlation, making them superior for imaging in weak light conditions compared to thermal light. However, (pseudo-)thermal light is easy to produce and provides higher intensity, allowing for measurements at greater distances. In practical applications, it could achieve higher imaging sensitivity using a (pseudo-)thermal light source with a higher-sensitivity detector. This allows for enhanced detection capabilities and improved imaging ability.

In this paper, we considered GI from the perspective of resource consumption. The minimal number of required photons to obtain the image of an object is defined as sensitivity. For imaging, as is well known, many parameters, such as resolution and illumination intensity, can affect certain values of imaging quality. This article primarily analyzes the fundamental differences in the sensitivity between the two methods, with the main kinds of noise included. Image artifacts and distortions resulting from environmental factors were not taken into account. To make the comparison fair and reduce possible obstruction, most parameters are set as the same for both methods, except that: (i) a bucket detector is used for GI, while an array sensor is used for CI; (ii) the object is illuminated by entangled photons or a pseudo-thermal source for GI and ordinary laser for CI; (iii) the image is reconstructed with correlation calculation for GI while it is directly accumulated for CI. For sure, our results cannot answer all related problems. Some factors are not included, such as atmospheric turbulence and scattering. Nevertheless, these environmental factors can affect both imaging methods, and they remain open questions, worthy of further studies.

To reveal the fundamental distinction, we employed the basic fluctuation correlation algorithm for image reconstruction in GI. Advanced algorithms can be compared with general correlation algorithms to explore their differences. For instance, the imaging results of differential ghost imaging (DGI) and normalized ghost imaging (NGI) are presented in Fig. 18, in the case of \( u = 10, k = 10 \) depicted in Fig. 4. The error bars are obtained from 100 simulations. From the intersection point with the blue chain line, it can be inferred that the difference in algorithms does not cause much difference in imaging sensitivity. Although the imaging sensitivities of DGI and NGI are slightly improved compared to GI, the results do not influence our main conclusion. For nonlinear algorithms such as compressive sensing and machine learning, which might also enhance the quality of GI, they depend on specific scenarios and are usually sensitive to noise. Additionally, corresponding algorithms can also be applied in CI. Therefore, to explore the fundamental principles, we utilized the most basic algorithms of two imaging methods to acquire information of the target.

To verify our results, subsequent experiments can be carried out in the future. For comparison on sensitivity, typical GI and typical CI setups are required, with comparable configurations.
Average intensity of illumination, working distance, and objects of interest should be the same for both imaging methods. The quantum efficiency, sensitivity, and noise level of the bucket detector used for GI, as well as those of the array sensor used for CI, are expected to be the same, respectively. To obtain the influence of those parameters, the performances of detectors are also expected to be (effectively) adjustable. Then, an experimental comparison can be conducted to evaluate the sensitivity of the two imaging methods.

7. Conclusion

With the increasing sensitivity of detector, assessing the sensitivity of GI and comparing it to CI becomes increasingly significant. This paper defines the number of photons required for CNR = 1 as the imaging sensitivity and clarifies the relationship between imaging sensitivity and echo signal intensity, background noise, and detection noise. Then the sensitivities of GI and CI are carefully analyzed and compared.

For the thermal source, we find that GI does not show significance for noiseless situations, but under practical noise, GI shows better sensitivity with the higher detection efficiency or the lower noise levels for bucket detection in low-light conditions. In fact, a single-pixel detector can perform better than a pixel in array detector, in terms of such as detection efficiency and noise suppression. In addition, with the detection threshold, the array detector may not respond when the signal is very weak.Bucket detection in GI can improve the SNR by collecting signals, which can also achieve higher imaging sensitivity. This paper defines the number of photons required for bucket detection in low-light conditions, and noise suppression. In addition, with the detection threshold, the array detector may not respond when the signal is very weak. At this time, bucket detection in GI can improve the SNR by collecting signals, which can also achieve higher imaging sensitivity. For the entangled source, due to the strong correlation of entangled photon pairs, GI can maintain high imaging quality in extremely low-light conditions. Although the bucket detector in GI and the array detector in CI have the same parameters, entangled GI has sensitivity advantages in weak light conditions over CI in a specific range with background noise. Above all, the simulation imaging results agree with the theoretical results.

For some scenes, GI can perform even better. For example, if a low reflection object moves at high speed, CI can no longer obtain an image by accumulating photons. GI can achieve a clear image by introducing a moving compensation strategy under specific coding in the illumination source, which can help accumulate photons among multiple measurements[69]. In addition, information extraction can be further enhanced by some strategies for weak echoes, such as first photon GI[19]. Moreover, specific algorithms also can help further improve the imaging quality and reduce the number of imaging photons required[23].

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